

Pechukas Replies: Alicki [1] ably summarizes the issues. It is unfortunate that the only “assignment maps” meeting all three conditions (a)–(c) are product maps, inappropriate outside the weak coupling regime. It is a shame that insisting on (a) and (b)—consistent assignments that preserve mixtures—means accepting that reduced dynamics is properly defined on a subset of possible states of S , those derived from physical—i.e., positive—states of $S + R$. Still, experimentalists will not be much troubled; they study fast processes in strongly coupled systems by relaxation methods, looking only at a subset of system states, those not far out of equilibrium.

I do not think it helps to give up (b), and in a sense it *cannot* be done. Mixture-preserving maps of the form recommended by Alicki take the entire space of system states ρ_S to a convex subset of states of $S + R$; that subset in turn maps back to a convex subset of system states under the partial trace operation tr_R :

$$\rho_S \xrightarrow{\Phi} \Phi \rho_S \xrightarrow{\text{tr}_R} \rho'_S.$$

The second map, if one to one, generates a mixture-preserving consistent assignment $\rho'_S \rightarrow \Phi \rho_S$; if many to one, it generates a family of such assignments. The domain of the consistent assignments is typically not all of the system states; in Alicki’s case, it is the range of T , where T is a completely positive map, which is typically a contraction.

What is important for reduced dynamics? Each initial state $\Phi \rho_S$ of $S + R$ generates a trajectory in the space of system states, $\rho_S(t) = \text{tr}_R(U_t \Phi \rho_S U_t^*)$, and at the beginning, $t = 0$, the trajectory issues from ρ'_S , not ρ_S . It is interesting that the entire trajectory can be regarded as the unfolding of a history of completely positive transformations acting on some fixed state ρ_S lying off the trajectory, but surely what is important for answering the question, what can happen with time in an open system?, is the nature of the transformation $\rho_S(0) \rightarrow \rho_S(t)$. It

is not obvious that *this* transformation must always be completely positive, even with the assignment maps that Alicki recommends.

Alicki properly emphasizes that state preparation is the critical issue here and properly criticizes the mathematically convenient assignment $\rho_S \rightarrow (\rho_S \rho_S^{\text{eq}-1} \rho_{SR}^{\text{eq}} + \rho_{SR}^{\text{eq}} \rho_S^{\text{eq}-1} \rho_S)/2$ as difficult to realize experimentally. His maps are not easy on the experimentalist either. In the simplest case in which T is the identity, the experimentalist must make the reservoir state $\text{tr}_S(\rho_{SR}^{\text{eq}})$. If $S + R$ is an aqueous solution of ions and S the ions, $\text{tr}_S(\rho_{SR}^{\text{eq}})$ is pure water still carrying, in its local structure, the ghostly imprint of the ions that have been traced out.

“Perturbed equilibrium” states are easier to make; for example, $\rho_{SR} = \exp[-\beta(H_{SR} + H_S)]/\text{tr}\{\exp[-\beta(H_{SR} + H_S)]\}$, where H_S is a “perturbation” depending only on system variables. In first order with respect to H_S this generates in the obvious way a consistent mixture-preserving assignment map. In general, for arbitrary strength of H_S , this is a realization of Alicki’s suggestion that we abandon (a) and accept nonlinearity as a feature of reduced dynamics outside the weak coupling regime.

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[1] R. Alicki, preceding Comment, Phys. Rev. Lett. **75**, 3020 (1995).