Kondo Effect in a Luttinger Liquid: Exact Results from Conformal Field Theory

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We report on exact results for the low-temperature thermodynamics of a spin-$\frac{1}{2}$ magnetic impurity coupled to a one-dimensional interacting electron system. By using boundary conformal field theory, we show that there are only two types of critical behaviors consistent with the symmetries of the problem: *either* a local Fermi liquid *or* a theory with an anomalous response identical to that recently proposed by Furusaki and Nagaosa. Suppression of backscattering off the impurity leads to the same critical properties as for the two-channel Kondo effect.

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Electron correlations often play an important role in condensed matter of reduced dimensionality. A key issue, raised by experiments on mesoscopic quantum dots and wires [1], is how to describe the interplay between impurity and correlation effects. For electrons in one dimension (1D), it has long been known that any finite concentration of impurities leads to Anderson localization [2], but, as shown recently, even a *single* potential scatterer may dramatically influence the physics in the presence of repulsive e-e interactions: at $T = 0$ the scatterer acts as a perfectly reflecting barrier [3].

The case of a *dynamical* scatterer, like a magnetic impurity, is less well understood. The 3D analog, with noninteracting quasiparticles representing the electrons (*Fermi liquid*), is that of the Kondo problem [4]. By symmetry, it can be modeled by a 1D gas of free chiral particles coupled to a magnetic impurity, allowing for an exact solution [5]. In contrast, the case of fully interacting 1D electrons in the presence of a magnetic impurity largely remains to be explored. Laboratory studies of artificial potential defects ("antidots") in quantum wires have recently been reported [6], and the experimental study of an interacting electron system coupled to a *spinful* defect may soon be within reach. (A possible realization is a quantum dot containing two spin levels and coupled to two narrow leads [7].) This poses a challenge to the theorist. In 1D the e-e interaction removes the single-particle spectrum, and the electrons effectively get replaced by new collective excitations, separately carrying spin and charge (*Luttinger liquid*) [8]. A magnetic impurity, on the other hand, couples to individual electrons, and it is *a priori* not clear how to incorporate its description in that of the spin-charge separated modes.

The problem was recently considered by Furusaki and Nagaosa [9], expanding on earlier work by Lee and Toner [10]. These authors studied a Tomonaga-Luttinger model [11], with the electrons coupled to a local magnetic moment via a Kondo exchange. Using "poor man's scaling," an infinite-coupling fixed point was identified, suggesting a completely screened impurity at low temperatures. The impurity specific heat, as well as the conductance, was argued to exhibit an anomalous temperature dependence, with a leading term $T^{(1/K_\rho)-1}$, K_ρ being the Luttinger liquid "charge parameter" [8]. However, the validity of the result remains unclear, as it relies upon a perturbative expansion in a strong coupling region where perturbation theory in fact loses its meaning.

In this Letter we study the problem using boundary conformal field theory (BCFT) [12]. The heart of the method, pioneered by Affleck and Ludwig [13,14], is to replace the impurity by a scale invariant boundary condition. Combined with the machinery of BCFT this approach has proven very powerful, and has opened up an entirely new vista on quantum impurity problems. As with any application of conformal theory, the method gives a classification of *possible* critical behaviors. Being exact, this information is extremely valuable as it places strong constraints on any constructive theory of an impurity problem. In the present case several new features appear, making the identification of boundary condition less obvious. Still, by exploiting symmetry arguments we arrive at an exact result showing that the problem must renormalize to one of only two possible fixed-point theories.

We describe the electrons by a Tomonaga-Luttinger Hamiltonian with repulsive interaction ($g > 0$) [11]:

$$\mathcal{H}_{\rm TL} = \frac{1}{2\pi} \int dx \Big\{ v_F \Big[:\psi^\dagger_{L,\sigma}(x) i \frac{d}{dx} \psi_{L,\sigma}(x): - :\psi^\dagger_{R,\sigma}(x) i \frac{d}{dx} \psi_{R,\sigma}(x): \Big] + \frac{g}{2} :\psi^\dagger_{k,\sigma}(x)\psi_{k,\sigma}(x)::\psi^\dagger_{l,-\sigma}(x)\psi_{l,-\sigma}(x): + g :\psi^\dagger_{R,\sigma}(x)\psi_{L,\sigma}(x)\psi^\dagger_{L,-\sigma}(x)\psi_{R,-\sigma}(x): \Big\}, \quad (1)$$

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and coupled to a spin-$\frac{1}{2}$ impurity by

$$\mathcal{H}_{\text{el-imp}} = \lambda_{kl} : \psi^{\dagger}_{k,\sigma}(0)\tfrac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}\psi_{l,\mu}(0): \cdot \boldsymbol{S}\,. \tag{2}$$

Here $\psi_{L/R,\sigma}(x)$ are the left or right moving components of the electron field $\Psi_\sigma(x)$, expanded about the Fermi points $\pm k_F$, and we implicitly sum over repeated indices for spin $\sigma, \mu = \uparrow, \downarrow$ and handedness $k, l = L, R$. Normal ordering :: is carried out with respect to (w.r.t.) the filled Dirac sea. The couplings $\lambda_F \equiv \lambda_{LL} \equiv \lambda_{RR}$ and $\lambda_B \equiv \lambda_{LR} \equiv \lambda_{RL}$ are the amplitudes for forward and backward electron scattering off the impurity $\boldsymbol{S}$, respectively. For the physically relevant case $\lambda_F = \lambda_B$ (*Kondo interaction*), $\mathcal{H}_{\text{TL}} + \mathcal{H}_{\text{el-imp}}$ contains the long-wavelength physics of a small-U Hubbard chain off half filling, and coupled to a single spin-$\frac{1}{2}$ impurity. Then $g = Ua/2\pi$ and $v_F = 2at \sin ak_F$, with U and t the usual Hubbard parameters and a the lattice spacing.

The bulk Hamiltonian $\mathcal{H}_{\text{TL}}$ can be written on diagonal Sugawara form [15], using the charge and spin currents

$$j_{L/R}(x) = \cosh\theta : \psi^{\dagger}_{L/R,\sigma}(x)\psi_{L/R,\sigma}(x): + \sinh\theta : \psi^{\dagger}_{R/L,\sigma}(x)\psi_{R/L,\sigma}(x):, \tag{3a}$$

$$\boldsymbol{J}_{L/R}(x) = : \psi^{\dagger}_{L/R,\sigma}(x)\tfrac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}\psi_{L/R,\mu}(x):, \tag{3b}$$

with $\tanh 2\theta = g/(v_F + g)$. Dropping a marginally irrelevant term $-(g/\pi)\boldsymbol{J}_L \cdot \boldsymbol{J}_R$, one obtains the critical bulk Hamiltonian

$$\mathcal{H}^*_{\text{TL}} = \int dx \left\{ \frac{v_c}{8\pi} : j_l(x) j_l(x): + \frac{v_s}{6\pi} : \boldsymbol{J}_l(x) \cdot \boldsymbol{J}_l(x): \right\}. \tag{4}$$

The spin and charge separation in (4) yields two dynamically independent theories, each Lorentz invariant with a characteristic velocity, $v_c = v_F(1 + 2g/v_F)^{1/2}$ and $v_s = v_F - g$. The currents $j_l(x)$ and $\boldsymbol{J}_l(x)$ satisfy the (level-2) U(1) and (level-1) $\text{SU}(2)_1$ Kac-Moody algebras, respectively, i.e., $\mathcal{H}^*_{\text{TL}}$ is invariant under the chiral symmetry $\text{U}(1)_L \times \text{U}(1)_R \times \text{SU}(2)_{1,L} \times \text{SU}(2)_{1,R}$.

To cast the problem on a form where BCFT applies, we use a representation where the impurity location $x = 0$ defines a *boundary*. For this purpose we confine the system to the finite interval $x \in [-\ell, \ell]$, fold it in half to $[0, \ell]$, identify the two points $x = \pm \ell$, and introduce new currents for $x \geq 0$: $j^1_{L/R}(x) \equiv j_{L/R}(x)$, $j^2_{L/R}(x) \equiv j_{R/L}(-x)$, and analogously for $\boldsymbol{J}_l(x)$. We thus arrive at a representation with doubled degrees of freedom on half the interval. In 2D Euclidean space-time $\{z = v\tau + ix\}$, with $v = v_c$ (v_s) for charge (spin), we interpret the time axis as a boundary where

$$j^1_{L/R}(\tau, 0) = j^2_{R/L}(\tau, 0), \quad \boldsymbol{J}^1_{L/R}(\tau, 0) = \boldsymbol{J}^2_{R/L}(\tau, 0)\,. \tag{5}$$

By analytic continuation, this is equivalent to a chiral (left-handed) theory on $[-\ell, \ell]$. The Hamiltonian then takes the form (4), but with the sum over handedness replaced by a sum over channels 1 and 2 of left-handed currents only.

It is instructive to first study the case of only *forward scattering* off the impurity, i.e., $\lambda_{LR} = \lambda_{RL} = 0$ in (2):

$$\mathcal{H}_F = \lambda_F[\boldsymbol{J}^1_L(0) + \boldsymbol{J}^2_L(0)] \cdot \boldsymbol{S}\,. \tag{6}$$

As the two currents are coupled via $\boldsymbol{S}$, $\mathcal{H}_F$ breaks the $\text{SU}(2)_1 \times \text{SU}(2)_1$ symmetry of $\mathcal{H}^*_{\text{TL}}$ down to the diagonal level-2 subalgebra $\text{SU}(2)_2$ spanned by $\boldsymbol{J}(x) = \boldsymbol{J}^1_L(x) + \boldsymbol{J}^2_L(x)$. To adopt to this fact we use the Goddard-Kent-Olive construction [16] to write the spin part of the Hamiltonian as a sum of an $\text{SU}(2)_2$ Sugawara Hamiltonian and an Ising model. We can then absorb $\mathcal{H}_F$ into $\mathcal{H}^*_{\text{TL}}$ by the canonical transformation $\boldsymbol{J}(x) \to \boldsymbol{J}'(x) \equiv \boldsymbol{J}(x) + \boldsymbol{S}\delta(x)$, $\boldsymbol{J}'(x)$ being the spin current of electrons *and* impurity. The impurity thus disappears from the Hamiltonian, and as a consequence (5) gets "renormalized." This new, renormalized boundary condition is most easily defined by the *selection rule* that prescribes how the $\text{U}(1) \times \text{U}(1)$, $\text{SU}(2)_2$, and Ising degrees of freedom recombine at the boundary *after* the shift $\boldsymbol{J}(x) \to \boldsymbol{J}'(x)$.

Consider first the unperturbed problem with $\lambda_F = 0$. The charge eigenstates organize into a product of two U(1) conformal towers, one for each channel, and labeled by two integer quantum numbers $(Q, \Delta Q)$, the sum and difference of net charge in the two channels (w.r.t. the ground state). These eigenstates are in 1-1 correspondence to the scaling operators in the charge sector, of dimensions

$$\Delta_c = \tfrac{1}{4}(q_1^2 + q_2^2) + N_c\,, \tag{7}$$

with

$$q_i = Q\,\frac{e^{\theta}}{2} - (-1)^i \Delta Q\,\frac{e^{-\theta}}{2}\,, \tag{8}$$

and $N_c \in \mathbb{N}$. Similarly, the eigenstates in the $\text{SU}(2)_2$ and *Ising* sectors appear in conformal towers labeled by the spin quantum numbers $j = 0, \frac{1}{2}, 1$, and the Ising primary fields $\phi = \mathbb{1}, \sigma, \epsilon$, respectively:

$$\Delta_S = \frac{1}{4} j(j+1) + N_S\,, \tag{9}$$

$$\Delta_{\text{Ising}} = 0(\mathbb{1}), \frac{1}{16}(\sigma), \frac{1}{2}(\epsilon) + N_{\text{Ising}}\,, \tag{10}$$

where $N_S, N_{\text{Ising}} \in \mathbb{N}$. The complete set of conformal towers is accordingly labeled by $(Q, \Delta Q, j, \phi)$ and the spectrum of scaling dimensions is $\Delta = \Delta_c + \Delta_S + \Delta_{\text{Ising}}$. The selection rule for combining quantum numbers can be extracted from comparison with *Bethe ansatz* results for the Hubbard model [17] (of which $\mathcal{H}_{\text{TL}}$ is the long-wavelength effective theory), and one finds $(j, \phi) = (0, \mathbb{1})$ or $(1, \epsilon)$ for $Q, \frac{1}{2}(Q + \Delta Q)$ even; $(0, \epsilon)$ or $(1, \mathbb{1})$ for Q even and $\frac{1}{2}(Q + \Delta Q)$ odd; and $(\frac{1}{2}, \sigma)$ for Q odd.

When $\lambda_F \neq 0$ we absorb $\mathcal{H}_F$ into $\mathcal{H}^*_{\text{TL}}$ by redefining the spin current as that of electrons *and* impurity. Effectively, this adds an extra spin-$\frac{1}{2}$ degree of freedom to the $\text{SU}(2)_2$ towers, which, as a result, get shifted according to the conformal field theory *fusion rules*: $j = 0 \to \frac{1}{2}$, $\frac{1}{2} \to 0$ or 1, $1 \to \frac{1}{2}$. The selection rule describing the new content of possible boundary scaling operators is obtained by applying fusion *twice* to the previous selection rule [14]. This gives for forward scattering: $(j, \phi) = (0$ or 1, or $\epsilon)$ for Q even; $(\frac{1}{2}, \sigma)$ for Q odd.

The low-temperature thermodynamics is now governed by the *leading correction-to-scaling boundary operator (LCBO)*. As this must preserve all symmetries of $\mathcal{H}_{\rm TL}^* + \mathcal{H}_F$, the forward scattering selection rule together with invariance under chiral U(1), $\mathrm{SU}(2)_2$, and channel exchange ($1 \leftrightarrow 2$), imply a unique LCBO given by the first descendant in the $j = 1$ tower: $\boldsymbol{J}_{-1} \cdot \boldsymbol{\phi}$. This is the same LCBO that drives critical scaling in the two-channel Kondo effect for noninteracting electrons [14]. Specifically, the impurity contributions to the specific heat δC and spin susceptibility $\delta\chi$ are given to leading order by

$$\delta C = \frac{\mu_F^2 9\pi^2}{v_s^3} T \ln\left(\frac{1}{\tau_0 T}\right), \tag{11a}$$

$$\delta \chi = \frac{\mu_F^2 18}{v_s^3} \ln\left(\frac{1}{\tau_0 T}\right), \tag{11b}$$

as $T \to 0$. Here μ_F is the scaling field conjugate to $\boldsymbol{J}_{-1} \cdot \boldsymbol{\phi}$ and τ_0 a short-time cutoff. With the known bulk response for the Tomonaga-Luttinger model, $C = \pi(v_c^{-1} + v_s^{-1})T/3$ and $\chi = 1/2\pi v_s$ [18], we predict a Wilson ratio

$$R_W = \frac{\delta\chi/\chi}{\delta C/C} = \frac{4}{3}\left(1 + \frac{v_s}{v_c}\right). \tag{12}$$

For $g \to 0$ ($v_c, v_s \to v_F$), this reduces to the universal number 8/3 characterizing the usual two-channel Kondo effect [14].

Let us now include backward scattering off the impurity, $\lambda_B \equiv \lambda_{LR} \equiv \lambda_{RL} \neq 0$. The corresponding terms in (2) break the chiral SU(2) *and* chiral U(1) invariance of $\mathcal{H}_{\rm TL}^*$. As a consequence ΔQ is no longer restricted to zero, and the charge sector makes nontrivial contributions to the content of scaling operators. The lowest dimension operator with $\Delta Q \neq 0$ allowed by the forward scattering selection rule is obtained from $(Q, \Delta Q, j, \phi) = (0, \pm 2, 0, \mathbb{1})$, and has dimension $\Delta = \frac{1}{2}e^{-2\theta} \leq \frac{1}{2}$. Back scattering is thus a relevant perturbation and drives the system to a new fixed point. When the flows of λ_F and λ_B converge, this is the fixed point for *Kondo scattering* in a Luttinger liquid.

To study this case we consider the bare Kondo interaction

$$\mathcal{H}_K = \lambda \sum_{k,l=L,R} :\psi_{k,\sigma}^\dagger(0)\tfrac{1}{2}\boldsymbol{\sigma}_{\sigma\mu}\psi_{l,\mu}(0): \cdot \boldsymbol{S}, \tag{13}$$

obtained from (2) by choosing $\lambda_{kl} = \lambda$, i.e., $\lambda_F = \lambda_B = \lambda$. With no e-e interaction [$g = 0$ in (1)] we have a free bulk Hamiltonian $\mathcal{H}_0$ together with $\mathcal{H}_K$. Passing to a basis spanned by definite-parity fields $\psi_{\pm,\sigma}(x) = [\psi_{L,\sigma}(x) \pm \psi_{R,\sigma}(-x)]/\sqrt{2}$, $\mathcal{H}_0 + \mathcal{H}_K$ transforms into a two-channel theory, but with the impurity coupled to the electrons in only one of the channels. This renormalizes to a local Fermi liquid (like the ordinary 3D Kondo problem), with response functions scaling analytically with temperature [19]. However, a different approach must be used for the interacting problem since $\mathcal{H}_{\rm TL}^*$ is nonlocal in this basis. Here we exploit the expectation that *any* local impurity interaction, including the Kondo interaction $\mathcal{H}_K$, can be substituted by a renormalized boundary condition on the critical bulk theory [20]. The equivalent selection rule defines a fixed point, and by demanding that any associated LCBO must respect the symmetries of the problem *and* correctly reproduce the noninteracting limit as $g \to 0$, the possible critical theories can be deduced. (Note that a selection rule here defines a *boundary* fixed point, and is valid for all values of the marginal bulk coupling g. Hence, given a selection rule, Fermi-liquid scaling must emerge in the limit $g \to 0$.)

To have a generally applicable formalism we introduce a notation that does not make an implicit relation between the two diagonalized charge towers (as Q and ΔQ do), and denote a combination of conformal towers by $(C_1, D_1; C_2, D_2; j; \phi)$. Hence (C_i, D_i) replace Q and ΔQ, such that the scaling dimensions in the charge sector are now given by (7), with

$$q_i = C_i \frac{e^\theta}{2} - (-1)^i D_i \frac{e^{-\theta}}{2} \tag{14}$$

replacing (8). The corresponding states are seen to be global U(1) invariant if $q \equiv q_1 + q_2 = 0$, and chiral U(1) invariant if $q = \Delta q \equiv q_1 - q_2 = 0$. This is consistent with our previous notion of global and chiral U(1) invariance in terms of Q and ΔQ, as the former selection rules implied the relation $C_1 = C_2$ and $D_1 = D_2$. The crucial point to realize is that Q and ΔQ are not sufficient to label *all* combinations of U(1) conformal towers, whereas q and Δq are well defined for *any* selection rule. Hence, at the new fixed point, the signature of breaking chiral U(1) invariance is to allow operators with $\Delta q \neq 0$. Global U(1) invariance, on the other hand, respected by $\mathcal{H}_K$, requires $q = 0$. Together with invariance under channel exchange ($1 \leftrightarrow 2$), this leaves only two possibilities for the charge part of the LCBO [21]: (i) $C_1 = C_2 = 0$, $D_1 = D_2 =$ even integer $\Rightarrow \Delta_c = \frac{1}{2}p^2 e^{-2\theta} + N_c$, and (ii) $C_1 = -C_2 =$ even integer, $D_1 = D_2 = 0 \Rightarrow \Delta_c = \frac{1}{2}p^2 e^{2\theta} + N_c$, with $p, N_c \in \mathbb{N}$.

The complete scaling dimensions are obtained by coupling the $\mathrm{SU}(2)_2$ and Ising conformal towers to the pairs of U(1) towers in (i) and (ii). Starting with the $\mathrm{SU}(2)_2$ sector, the $j = \frac{1}{2}$ tower is expelled by global SU(2) invariance. Turning to the $j = 1$ tower, the primary operator $\boldsymbol{\phi}$ is excluded by the same reason. The lowest-dimension $\mathrm{SU}(2)_2$ singlet operator from this tower is $\boldsymbol{J}_{-1} \cdot \boldsymbol{\phi}$. However, this is the same operator that drives critical scaling in the forward scattering problem. It produces a diverging impurity susceptibility as $T \to 0$, in conflict with the known Fermi liquid scaling in the $g \to 0$ limit. The $j = 1$ tower is therefore expelled, and the only contribution from the $\mathrm{SU}(2)_2$ sector is the identity and its descendants. Next we note that no relevant scaling operators are allowed, since at $g = 0$ the fixed point is known to be stable, being that of the ordinary Kondo problem. As g is the *only* tunable parameter in $\mathcal{H}_{\mathrm{TL}}^{*}$ (with a renormalized boundary condition replacing H_K), this is true also for $g \neq 0$ since otherwise the theory would become noncritical. Hence, starting with (i) and $p = 0$, only $\mathbb{1}$ from the Ising sector is permissible, as any other choice would produce a relevant operator. For $p = 1$, all choices lead to relevant operators, whereas for $p \geq 2$ the converse is true. Summarizing, the possible couplings of $\mathrm{SU}(2)_2$ and Ising towers to the U(1) towers selected by (i) yield the following candidate LCBO dimensions:

$$\Delta_{\mathrm{LCBO}} = 1, \tfrac{1}{2}p^2 e^{-2\theta} + \{0, \tfrac{1}{16}, \tfrac{1}{2}\}, \tag{15}$$

with $p \in \mathbb{N} + 2$. Here $\Delta_{\mathrm{LCBO}} = 1$ is the dimension of the first U(1) Kac-Moody descendants $j_L^{1,2}$, allowed by the broken particle-hole symmetry of the underlying lattice model. Turning to (ii), and employing the same reasoning as above, one finds a second class of possible LCBO dimensions:

$$\Delta_{\mathrm{LCBO}} = 1, \tfrac{1}{2}e^{2\theta} + \tfrac{1}{2}, \tfrac{1}{2}p^2 e^{2\theta} + \{0, \tfrac{1}{16}, \tfrac{1}{2}\}, \tag{16}$$

and with p as above.

Each entry in (15) and (16) defines an effective scaling Hamiltonian $\mathcal{H}_{\mathrm{scaling}} = \mathcal{H}_{\mathrm{TL}}^{*} + \mu \mathcal{O}(0)$, with $\mathcal{O}(0)$ the corresponding LCBO conjugate to the scaling field μ. Using $\mathcal{H}_{\mathrm{scaling}}$, the finite-size corrections at the fixed point can be calculated perturbatively in μ, and by treating temperature as an inverse length the corrections to the bulk thermodynamics due to the impurity are accessible via finite-size scaling. Given (15) and (16), and requiring Fermi-liquid scaling for the impurity specific heat δC and susceptibility $\delta \chi$ as $g \to 0$, we find that there are only *two* possible types of critical behavior. When $\Delta_{\mathrm{LCBO}} = 1$ or $\Delta_{\mathrm{LCBO}} > \frac{3}{2}$ Fermi-liquid scaling persists for $g \neq 0$, whereas a non-Fermi-liquid behavior emerges when $\Delta_{\mathrm{LCBO}} = \frac{1}{2}(e^{2\theta} + 1)$:

$$\delta C = c_1(1/K_\rho - 1)^2 T^{1/K_\rho - 1} + c_2 T, \tag{17a}$$

$$\delta \chi = c_3 T^0, \tag{17b}$$

as $T \to 0$. Here $K_\rho = (1 + 2g/v_F)^{-1/2}$ and $c_{1,2,3}$ are amplitudes depending on the scaling fields and velocities. The LCBO driving the anomalous scaling in (17) is given by the composite operator $\mathcal{O}_{\mathrm{LCBO}} = [V_{2,0}^1 \times V_{-2,0}^2 + V_{-2,0}^1 \times V_{2,0}^2] \times \epsilon$ where $V_{C,D}^i$ is a U(1) primary (vertex) operator in channel i, and ϵ the Ising energy density. This scaling (17) agrees exactly with that proposed by Furusaki and Nagaosa [9], in support of a non-Fermi-liquid scenario. However, a simplified model (neglecting backward spin diagonal and forward spin off-diagonal Kondo scattering) suggests that in fact the other scenario (Fermi liquid) may be realized [22]. Note that in none of the two cases does the e-e interaction influence $\delta\chi$: the impurity remains completely screened for $g \neq 0$.

In summary, we have shown that the symmetries of the problem restrict the possible critical theories to *either* a local Fermi liquid (as for free 3D electrons) *or* a non-Fermi-liquid with thermodynamic response as in (17). The BCFT approach as presented here is quite general and can be used to derive the finite-size energy spectrum at the non-Fermi-liquid fixed point, as well as transport properties. Details will be published elsewhere [21].

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