Apparent Inconsistency of Observed Composite Fermion Geometric Resonances and Measured Effective Mass

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New ultrahigh frequency surface acoustic waves (SAW) on a high density 2D electron system demonstrate large principle and secondary geometric resonances of the cyclotron orbits of composite fermions (CF) with the acoustic waves. The effective mass as derived from dc transport produces a CF cyclotron frequency near the resonance structures comparable to the SAW frequency. A simplistic model of noninteracting CF's in semiclassical motion with such a cyclotron frequency is inconsistent with observation of the resonances.

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Our understanding of the correlated two-dimensional electron system (2DES) was recently elevated by the description of the gauge transformed, or composite, fermion and the experimental demonstration of these quasiparticles forming a Fermi surface at filling factor $\nu = \frac{1}{2}$. The magnetoresistance spectrum of the low-disorder 2D electron system is dominated by the fractional quantum Hall effect [1], with a principal series of states at $\nu = p/(2p + 1)$ and their particle-hole counter parts at $\nu = (p + 1)/(2p + 1)$. While the Laughlin [2] wave function elucidated the $\frac{1}{3}$ state as an incompressible electron liquid, the higher order fractions were not readily assigned wave functions. The principal series was later described by Jain [3] using a set of trial wave functions based on a picture of noninteracting composite fermions (CF), which are electrons bound to an even number of magnetic flux quanta: The fractional quantized states correspond to integer quantized Hall states for the composite fermions. It was formally shown by Lopez and Fradkin [4] that the electron system at $\nu = p/(2p + 1)$ can be transformed into a system of fermions interacting with a Chern-Simons gauge field, such that the ground state is indeed a system of p filled Landau levels for the transformed fermions in the mean-field approximation.

The fermion Chern-Simons picture of Lopez and Fradkin was further developed by Halperin, Lee, and Read (HLR) [5] to study even denominator filling factors, and it is within this work that a crucial prediction was made. HLR proposed that the composite fermions will fill a Fermi sea and at $\nu = \frac{1}{2}$ there should be a well-defined Fermi surface for these quasiparticles. As the B field is tuned away from $\nu = \frac{1}{2}$, the fermions will move in an effective magnetic field $\Delta B = B - B(\nu = \frac{1}{2})$ and will execute cyclotron motion with radius $R_c^* = \hbar k_F/e\Delta B$, and $k_F = (4\pi n_e)^{1/2}$; n_e is the electron density. The theory computed an anomaly in surface acoustic wave propagation that agreed qualitatively with an extensive set of previously unexplained experimental results [6]. Surface acoustic waves (SAW) can be propagated on the surface of a heterostructure with the 2D electron conductivity, $\sigma_{xx}(q)$, determining the transmitted SAW amplitude and velocity. It had been found that SAW showed an anomalous drop in amplitude and velocity at $\nu = \frac{1}{2}$ (and $\frac{1}{4}$), reflecting an enhanced conductivity of the 2D electron system. Following HLR, this anomaly is understood to occur when the SAW wavelength is roughly smaller than the composite fermion mean free path, so that the conductivity is enhanced since the CF can move in the direction of the piezoelectric field of the SAW without scattering. The width of the anomaly in the B field is determined by the CF cyclotron motion; the enhanced conductivity is cut off as the quasiparticle moves more laterally with respect to the SAW propagation direction as ΔB increases.

While these findings provided strong support for the fermion Chern-Simons picture, more definitive evidence for the presence of a Fermi surface is the observation of geometric resonance of the quasiparticle cyclotron motion with the acoustic waves. This commensurability of the orbits and the wavelengths allows measurement of the quasiparticle k_F. Such geometric resonances were indeed observed [7] in SAW measurements. Following these findings, a static array of antidots in a 2DES was also used to demonstrate a Fermi surface effect [8].

Within the simplistic picture of integer quantum Hall states for noninteracting composite fermions at $\nu = p/(2p + 1)$, it might be expected that the gaps would scale as the cyclotron energy $\hbar\omega_c = \hbar e\Delta B/m^*c$, with CF effective mass m^*. Subsequent to prediction of the Fermi surface at $\nu = \frac{1}{2}$, linear increase of the energy gap with ΔB was observed by Du *et al.* [9], demonstrating an enlarged effective mass; $m^* \sim 10m_b$, where m_b the bare mass in GaAs. Another analysis of the magnetotransport oscillations around $\nu = \frac{1}{2}$ was performed by Leadley *et al.* [10], where the fractional quantum Hall effect (FQHE) is treated as Shubnikov–de Haas oscillations of the CF. The amplitudes of the oscillations are used to yield the CF effective mass, which was found as well to be enlarged. A similar subsequent analysis by Du *et al.* [10] derived an

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enlarged CF mass ($m^* \sim 15m_b$) and was used to suggest a divergence of the mass approaching $\nu = \frac{1}{2}$.

We present in this Letter new ultrahigh frequency SAW measurements using high density samples. We observe not only commensurability of the quasiparticle orbit with a single acoustic wavelength but also with two SAW wavelengths. In a model of noninteracting quasiparticles moving in semiclassical orbits the necessary condition for observation of the geometric resonance is that the SAW should present a temporally static, spatially periodic potential to the CF: $\omega_c^* \gg \omega_{\mathrm{SAW}}$ and $v_F^* \gg v_s$; ω_{SAW} is the SAW frequency, v_s is the sound velocity, and v_F^* is the quasiparticle Fermi velocity. Using the noninteracting CF model, these conditions are apparently violated using the cyclotron frequency derived in these samples from dc transport.

These experiments used a high mobility ($\mu \lesssim 5 \times 10^6$) GaAs/AlGaAs wafer with electron density $n_e \sim 1.6 \times 10^{11}$ cm^{-2}. In contrast to previous measurements, smaller SAW wavelengths facilitated better resolution of the geometric resonances, and high density samples were used to maximize the CF enhanced mass. SAW were generated and detected as described previously [6,7]. The transducer set of interest in these experiments produces SAW at 10.7 GHz, corresponding to a wavelength of about 2700 Å. This is the highest reported SAW frequency on GaAs.

The SAW traverses the 2DES where it is attenuated and slowed by the interaction of its piezoelectric field and the 2D electrons, with the sound velocity shift monotonically decreasing with increasing wave-vector-dependent sheet conductivity; $\Delta v/v = (\alpha^2/2)/\{1 + [\sigma_{xx}(q)/\sigma_m]^2\}$, where $\alpha^2/2 = 3.2 \times 10^{-4}$ is the piezoelectric coupling constant for GaAs, and $v_s = \omega_{\mathrm{SAW}}/q$. σ_m is used as a parameter [11] to optimize the fit of $\Delta v/v[\sigma_{xx}(\mathrm{dc})]$ to measured $\Delta v/v$ over the full B-field range.

Figure 1 shows the sound velocity shift versus B field around $\nu = \frac{1}{2}$, with the geometric resonance structure prominent. The principle resonances, which correspond to a commensurability of the cyclotron orbit and a single acoustic wave, are the large structures symmetrically spaced about $\frac{1}{2}$ at $\Delta B \sim \pm 6$ kG. Closer to $\frac{1}{2}$, broader structure is seen again symmetrically spaced about $\frac{1}{2}$: These features correspond to the secondary resonances and represent the cyclotron orbit resonating with two wavelengths. These secondary resonances were not resolved in previous measurements at smaller SAW q, and the principle structure was not nearly as prominent. SAW measurements comparing attenuation and sound velocity shift preliminarily suggest that no imaginary component to the conductivity is present at the resonances, as might be expected for a distinct mode crossing.

A cursory comparison of the measured $\Delta v/v$ to theory [5] is also shown in Fig. 1. If the CF mean free path is ℓ, $J_n(x)$ is the Bessel function and $X = qR_c^*$, then HLR predict $\tilde{\sigma}_{yy}(q) = (2/\rho_0)\sum_{n=-\infty}^{\infty}[dJ_n(X)/dX]^2/[1 + n^2(q\ell/X)^2]$ with $\sigma_{xx}(q) = \rho_{yy}(q)/\rho_{xy}^2$, and $\rho_{yy}(q) \sim 1/\tilde{\sigma}_{yy}$ near $\frac{1}{2}$, in the zero SAW frequency case. The resonance peak position gives a measure of the quasiparticle k_F, with minima in $\Delta v/v$ occurring close to the zeros of $J_1(qR_c^*)$, or at $\Delta B/B = q/K_{1,n}\,k_F$, where $K_{1,n}$ is the position of the nth zero of J_1. To assess the CF mean free path ℓ we recall [5,6] that the transition from q independent to q dependent conductivity occurs roughly at $q\ell \sim 2$; over the range of q's tested we estimate ℓ to be ~ 0.5 μm. The electron density inhomogeneity smears the resonance structure, and it is estimated that an inhomogeneity of about 1% is present over the length scale of the ultrasound path. Using [11] the SAW q and the estimated ℓ, then convolving this with approximately a 1% F.W.H.M. Gaussian density distribution gives the theory trace of Fig. 1.

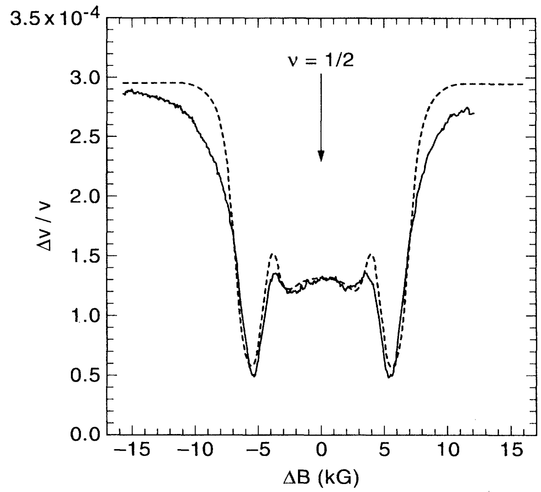

FIG. 1. Sound velocity shift versus magnetic field for 10.7 GHz surface acoustic waves near filling factor $\frac{1}{2}$. Both principle and secondary resonances are present. Temperature is $\sim$ 130 mK. The dashed line shows the theoretical fit to the data using parameters defined in the text.

We now turn to the conditions necessary for observation of the resonances, *which are independent of the theoretical fit.* We model the system as noninteracting composite fermions with semiclassical motion as determined by $\Delta B, R_c^*$: We refer to this as the simple model CF. As known from ultrasound attenuation in metals [12], geometric resonances may be observed if the SAW appears to the charge carrier to be a *static spatial wave;* the SAW provides a non-time-varying grating with which the quasiparticles can resonate. As such, the CF Fermi velocity should be much larger than the sound velocity, $v_F^* \gg v_s$ and the cyclotron frequency should be much larger than the SAW frequency, $\omega_c^* \gg \omega_{\mathrm{SAW}}$. If the CF cyclotron frequency is of the order or less than ω_{SAW}, or if v_F^* is not substantially larger than v_s then the spatial wave will vary during the period of the quasiparticle cyclotron orbit resulting in smearing or obliteration of the resonance features. *Within the limits of the simple model CF,* by virtue of our observation of the geometric resonances we know the conditions $\omega_c^* \gg \omega_{\mathrm{SAW}}$ and $v_F^* \gg v_s$ have been met. Our observations therefore set constraints on ω_c^*. We will now derive the CF effective mass from dc transport as done previously and compare the results to the constraints established by observation of the resonances, considering the limitations of the simple model CF.

We confirm here that the heterostructures used in our experiments display an enlarged effective mass near $\frac{1}{2}$ consistent with previous results [9,10]. To assess m^* we first employ the analysis used by Du *et al.* [9] for the series of FQHE states at $\nu = (p + 1)/(2p + 1)$: The series at $\nu = p/(2p + 1)$ was not in our available B-field range. By measuring the temperature dependence of ρ_{xx} at filling factors $\frac{7}{13}$ through $\frac{3}{5}$ and using $\rho_{xx} = \rho_0 \exp[-E_g/2kT]$, the energy gap values E_g were derived and are displayed in Fig. 2. Unlike the data of Du *et al.* [9], the E_g do not fall on a line for $\nu \gtrsim \frac{4}{7}$. In the spirit of Ref. [9] we use the slope of the data at the smaller filling factors to evaluate the effective mass, and likewise find $m^* \approx 12m_b \approx 0.8m_e$, where m_e is the electron mass. To further assess the CF effective mass we followed the methods of Leadly *et al.* [10] and later Du *et al.* [10], where the principle series of fractions is analyzed as Shubnikov–de Haas oscillations. In this model the oscillations follow the conventional Ando formula [13]: $\Delta R_{xx} \sim (A_T/\sinh A_T)\exp(-\pi/\omega_c\tau)$, where $A_T = 2\pi^2 kT/\hbar\omega_c$. With the temperature dependence of R_{xx} for the fractional states and resistance maxima from $\frac{3}{5}$ to $\frac{7}{13}$, the CF's cyclotron energy and mass can be derived as a function of B. As seen previously [10], we derive a CF mass comparable to the electron mass (Fig. 2).

This measured mass m^* is now compared to the SAW results at 10.7 GHz. We use in the following discussion the smaller enlarged mass derived from the activation energy measurements ($m^* \sim 12m_b$) as opposed to the mass derived from the Shubnikov–de Haas analysis ($m^* > 15m_b \gtrsim m_e$). The cyclotron energy scale of the enhanced mass is $e\Delta B/m^*c \simeq (1.7\ \mathrm{K})\Delta B$, ΔB in Tesla. The SAW energy is $\hbar\omega_{\mathrm{SAW}} \simeq 0.5$ K for 10.7 GHz, therefore the CF cyclotron frequency and SAW frequency are equal at $\Delta B \simeq 0.3$ T. This is roughly the position of the secondary resonances in ΔB: see also plots of ω_c^* in Fig. 2. If one now applies the simple model CF, then observation of both the principle and secondary resonances violates the supposition requiring $\omega_c^* \gg \omega_{\mathrm{SAW}}$. Consider here a nontraveling but oscillating wave, with frequency ω_{SAW}. By the values of ω_{SAW} and ω_c^* just described, at the secondary resonance ΔB the SAW will complete an oscillation in the time the CF completes a cyclotron orbit, thus the field applied by the SAW averaged over the orbit period is zero. This does not produce a resonance in the semiclassical picture, since the SAW is the driving field for the CF motion. At the principal resonance, the CF orbit occurs over a SAW potential that experiences a π phase change during one cyclotron orbit, therefore reversing the sign of the driving field. We conclude that an inconsistency exists between the cyclotron frequency deduced from dc transport and the observation of the geometric resonances if we employ the simple model CF. We reiterate that this discussion used the smallest mass derived from the transport measurements and arrived at an inconsistency with the SAW data: if instead the larger effective mass derived from the Shubnikov–de Haas analysis is used then the inconsistency is substantially aggravated (see Fig. 2). If the proposed divergence [10] of the mass approaching $\nu = \frac{1}{2}$ is assumed, the situation worsens further.

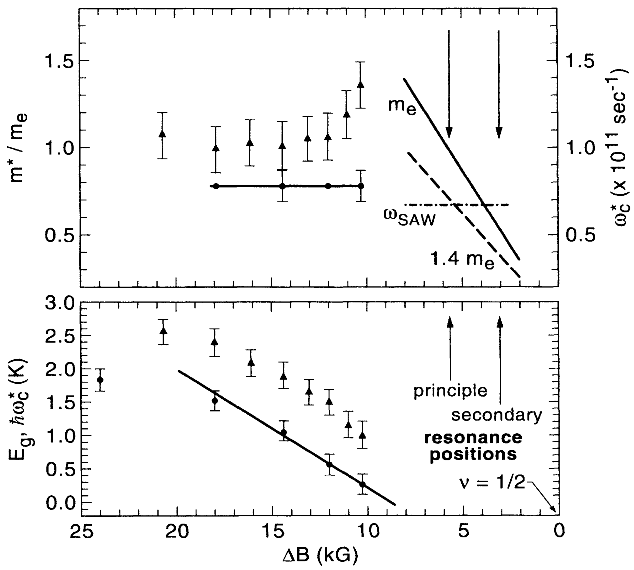

FIG. 2. Lower panel shows activation energy gap data (circles) for the series of fractions from $\nu = \frac{7}{13}$ to $\frac{3}{5}$. Triangles are the quasiparticle cyclotron gap data derived from the Shubnikov de Haas analysis (see text). Upper panel shows effective masses from activation energies (circles) and from Shubnikov–de Haas data (triangles). Cyclotron frequencies using the range of masses derived from the Shubnikov–de Haas data are plotted to the right in comparison to the SAW frequency.

The second condition necessary for observation of the geometric resonances, $v_F^* \gg v_s$, is also not satisfied when using the transport derived m^*. With $m^* = 12m_b$ the condition of a static spatial potential wave is violated; $v_F^*/v_s \sim 4$. The cyclotron period of the CF is $2\pi R_c^*/v_F^*$. In this time the wave will travel a distance $2\pi R_c^*/4 > R_c^*$. Therefore the SAW will not provide a static spatial wave to the cyclotron orbit, but rather will not move more than the radius of the orbit.

Our fundamental finding is that the ω_c^* derived from dc transport at the principal series of fractions does not agree with the observation of geometric resonances in the simple model of semiclassical motion of noninteracting composite fermions. This inconsistency was not apparent in previous SAW measurements [7] demonstrating resonances, both due to the lower $\omega_{\rm SAW}$ that could not resolve both principle and secondary resonances and due to the lower density samples employed then. The density used previously [6,7] would have resulted in a mass smaller by more than a factor of 2 [9].

If we continue to use the simple model CF, then we may heuristically derive an upper limit to the CF effective mass using the SAW results. Resonance observation should minimally require that $\omega_c^* \sim 4\omega_{\rm SAW}$, which states that the cyclotron orbit will roughly be complete within one quarter SAW period, thus predominantly maintaining the same piezoelectric field gradient. If this is the case, and the presence of the secondary resonance suggests this condition is at least marginally obeyed at that ΔB, then $\omega_c^* = 4\omega_{\rm SAW}$ at the secondary resonance ΔB, resulting in $m^* \sim 3m_b$.

The origin of the inconsistency is unclear. Three possibilities exist: (a) the mass from dc transport is incorrect, (b) the ω_c^* derived from dc transport is not the relevant frequency for the dynamic response to the SAW, and/or (c) the static wave criteria is not correct. It may be a fundamental error to assume a simple analogy to the near zero B-field electron problem by using dc transport to derive a CF mass. Assigning a single CF cyclotron frequency to a system of noninteracting fermions for both the FQHE oscillations and the dynamic response to the SAW field must also be questioned. Using free quasiparticles of enlarged m^* to produce the quantized Hall states at the principal fractions, with single fermion excitations the response to the SAW, neglects residual interactions between the fermions. The quasiparticle response to the SAW piezoelectric field may actually involve a particle-hole excitation, such that the composite particles are not free, but interacting, with a consequent shift up in the CF cyclotron frequency [14]. Finally, the static wave criteria for observation of the resonances may not be correct if the simple model CF is incorrect. Instead of this semiclassical result, it might be expected that the SAW ω and q are crossing the magnetoroton modes of the CF at high-order fractions [15]. Lack of a clear imaginary component to the conductivity and no obvious shift in the B-field resonance positions leave this an open question.

It is clear that the SAW frequencies employed here are sufficiently large such that a finite-ω calculation is needed. An analysis of properly σ_m scaled SAW data [11] with such a theory could give important constraints on m^* and on the Fermi liquid parameters. Effects of Fermi-liquid interactions on CF dynamic properties at $\frac{1}{2}$ have been considered [15], but only the consequences of the effective mass parameter and of the direct Chern-Simons and Coulomb interactions were examined.

In conclusion, we observe with high resolution both principle and secondary geometric resonances of the cyclotron motion of the composite fermion and surface acoustic waves. The CF mass derived from dc transport in these samples, and the cyclotron energy and Fermi velocity using this mass, violate the conditions necessary for geometric resonance within the limitations of a noninteracting, semiclassical quasiparticle model. This inconsistency is at present not explained.

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