Soft Photons from Off-Shell Particles in a Hot Plasma

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Considering the propagation of off-shell particles in the framework of thermal field theory, we present the general formalism for the calculation of the production rate of soft photons and dileptons from a hot plasma. This approach is first illustrated with an electrodynamic plasma. The photon production rate from strongly interacting quarks in the quark-gluon plasma, which might be formed in ultrarelativistic heavy ion collisions, is also calculated. We utilize an effective field theory incorporating dynamical chiral symmetry breaking and obtain results in the previously inaccessible regime of photon energies of the order of the plasma temperature.

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The main goal of the present experimental effort invested in ultrarelativistic heavy ion collisions (URHIC) is to observe an excursion of the system into the phase of a quark-gluon plasma (QGP) [1]. A direct way to "see" this short-lived state would be by observing photons or dileptons emitted from the hot plasma; see the recent discussion in [2]. Since these probes interact only electromagnetically, this signal is not distorted by later interactions as are others. However, the potential signal competes with a background of photons from other processes such as π^0 decays and hadronic reactions [3]. The knowledge of the thermal spectrum from theoretical calculations would help disentangle the various sources and identify the phases reached during the collision.

This problem represents a challenge to theory, due to the nonperturbative nature of the photon emission process: Multiple rescattering of the emitting particles and the Landau-Pomeranchuk-Migdal (LPM) effect play an important role in the low energy sector for photon energies $E_\gamma \leq T$ [4].

In the present paper, we improve on existing calculations of the photon production rate by accounting for thermal scattering and subsequent off-shell particle propagation. The problem is addressed in the framework of thermal field theory. Results are given for a QED plasma as well as for a QGP within a model incorporating dynamical chiral symmetry breaking.

In lowest order, the production of photons proceeds via $q\bar{q}$ annihilation and Compton processes. Summing up these two processes with thermal quark (q), $\bar{q}$, and gluon distributions of temperature T gives the production rate R (per unit volume element) to lowest order as

$$R^0 = E\frac{dN_\gamma^0}{d^3\boldsymbol{p}} = \frac{5}{9}\,\frac{\alpha\alpha_s}{2\pi^2}\,T^2 e^{-E/T}\left[\ln\frac{ET}{m^2} + c^0\right], \quad (1)$$

with some constant c^0. This rate diverges when $m \to 0$, which is the crucial limit of chiral symmetry restoration for strongly interacting quarks approaching the phase transition temperature. A shielding of this unphysical divergence requires the calculation of *medium effects* on the emission process.

The application of the Braaten-Pisarski method of hard thermal loops [5] to this problem has been studied recently [3,6]. In the rates obtained with this approach, the term in the square brackets of Eq. (1) is replaced by $\ln(E/T)$, which clearly shows the limitations of the approach to the region of $E_\gamma \gg T$.

The dominant physical process for quarks emitting soft photons with energies $E_\gamma \leq T$ is the scattering in the medium, which results in an energy uncertainty as the quark propagates. Formally, we describe this off-shellness by a finite width of the quark in analogy to the decay width of an excited state. Now, after an interaction with the medium which sets the quark off-shell, instead of interacting again with the medium, the quark may emit a real photon. Thus the quark width is directly related to the emission rate of soft photons. Taking into account such a spectral width naturally removes the infrared divergences mentioned before, and enables us to give production rates for soft photons.

We now present a brief summary of the formalism. As argued above, effective field theory in a hot system requires the use of physical states with a nonzero width [7,8]. It is calculated from the imaginary part of the retarded quark self-energy $\Sigma_q^R(p)$. The real part of this self-energy function is absorbed into the mass of the quark, by the approximation

$$\begin{aligned}[S^R(p)]^{-1} &= p_\mu\gamma^\mu - m_q^0 - \Sigma_q^R(p)\\ &\approx (p_0 \pm i\gamma_q)\gamma_0 - \boldsymbol{p}\cdot\boldsymbol{\gamma} - m_q \end{aligned} \quad (2)$$

in the vicinity of $p_0 = \pm\sqrt{\boldsymbol{p}^2 + m_q^2}$. Here, $\gamma_\mu = (\gamma_0, \boldsymbol{\gamma})$ is the four-vector of Dirac matrices. This equation also determines the quark width γ_q, which then enters the spectral function $\mathcal{A}(E,\boldsymbol{p})$. Up to a factor, this function is the imaginary part of the retarded full propagator of the quark field and deviates from its vacuum (perturbative) form $(E\gamma^0 + \boldsymbol{p}\boldsymbol{\gamma} + m_q)\,\mathrm{sgn}\,(E)\,\delta(E^2 - \boldsymbol{p}^2 - m_q^2)$. With the above approximation we obtain

$$\mathcal{A}_q(E,\boldsymbol{p}) = \frac{\gamma_q}{\pi}\,\frac{\gamma_0[E^2 + \Omega_q(\boldsymbol{p})^2] - 2E\boldsymbol{\gamma}\boldsymbol{p} + 2Em_q}{[E^2 - \Omega_q(\boldsymbol{p})^2]^2 + 4E^2\gamma_q^2}\,, \quad (3)$$

where $\Omega_q(\boldsymbol{p})^2 = \boldsymbol{p}^2 + m_q^2 + \gamma_q^2$. One may regard this double Lorentzian spectral function as the generalization of the standard energy-momentum relation $E^2 = \boldsymbol{p}^2 + m_q^2$ to a broader distribution for thermally scattered particles. Retarded as well as causal propagators are defined as dispersion integrals of $\mathcal{A}_q$, and we refer to [8] for a formulation of thermal field theory in terms of spectral functions.

In a loop expansion, the one-loop (Fock) diagram is the lowest order term with a nonvanishing imaginary part. In the following, we restrict ourselves to this lowest order. We consider a model where quarks are coupled to different types of bosons, to be specified later. The calculation of the Fock self-energy with full propagators is straightforward [8] and gives

$$\begin{aligned}\mathrm{Im}\Sigma^R(p_0,\boldsymbol{p}) = -\pi \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \int_{-\infty}^{\infty} dE\, \Gamma_\mu \mathcal{A}_q(E,\boldsymbol{k}) \\ \times \Gamma_\nu\, \mathcal{A}_B^{\mu\nu}(E - p_0, \boldsymbol{k} - \boldsymbol{p}) \\ \times [n_q(E) + n_B(E - p_0)]. \qquad (4)\end{aligned}$$

Here, $\mathcal{A}_B$ is the boson spectral function, Γ_μ and Γ_ν are the interaction matrices at the vertices, and n_B (n_q) is the standard thermal equilibrium Bose (Fermi) distribution functions at temperature T.

The width calculated from the quark self-energy diagram now enters the photon polarization Π at finite temperature. The imaginary part of the retarded one-loop polarization function Π^R is [8]

$$\begin{aligned}\mathrm{Im}\Pi^R_{\mu\nu}(k_0,\boldsymbol{k}) = -\pi\, e_q^2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \int_{-\infty}^{\infty} dE \\ \times \mathrm{Tr}[\gamma_\mu \mathcal{A}_q(E + k_0, \boldsymbol{p} + \boldsymbol{k}) \gamma_\nu \mathcal{A}_q(E,\boldsymbol{p})] \\ \times [n_q(E) - n_q(E + k_0)], \qquad (5)\end{aligned}$$

where e_q is the electric charge of the quark. However, this one-loop polarization tensor violates current conservation and gauge invariance: $k_\mu \Pi^{\mu\nu} \neq 0$. Therefore, the standard sum over the photon polarizations $\epsilon_\mu \epsilon_\nu \Pi^{\mu\nu} = \Pi^\mu_\mu$ is not gauge invariant and does not give a meaningful photon production rate.

This can be traced back to the fact that for an effective field theory the conserved electromagnetic current is different from the naive $\bar{\psi}\gamma_\mu\psi$. Because of the nonlocal nature of the effective Lagrangian of such a model, the conserved current associated with local gauge invariance acquires a correction term, which makes the current-current correlation function $\tilde{\Pi}^{\mu\nu}$ different from the above polarization tensor, such that $k_\mu \tilde{\Pi}^{\mu\nu} = 0$. The correction is of higher loop order than $\Pi^{\mu\nu}$ and amounts to the introduction of a vertex correction into Eq. (5).

However, in our approach we consider the particular case of a quark width which is only dependent on the temperature T but not on the quark momentum, as appropriate for a slow quark embedded in the medium. In this case, only the j^0 component of the current is modified and, correspondingly, only the components $\tilde{\Pi}^{0\nu} = \tilde{\Pi}^{\nu 0}$ are different from the one-loop result. It is crucial to realize that the spacelike components are not modified, $\tilde{\Pi}^{ij} = \Pi^{ij}$. With constant retarded quark self-energy and photon momentum $k_\mu = (k,0,0,k)$, the gauge invariant sum over the polarizations reduces to $\tilde{\Pi}^\mu_\mu = \tilde{\Pi}^{00} - \tilde{\Pi}^{ii} = \tilde{\Pi}^{00} - \Pi^{ii} = -(\Pi^{11} + \Pi^{22})$. In particular, this may be calculated with the unmodified polarization tensor from Eq. (5). The photon emission rate out of the hot plasma then is

$$\begin{aligned}R(E_\gamma, T) = E_\gamma \frac{dN_\gamma}{d^3\boldsymbol{p}} = 2\, \frac{n_B(E_\gamma, T)}{8\pi^3}\, \mathrm{Im}(\Pi^R_{11} + \Pi^R_{22}) \\ = \frac{i}{8\pi^3} (\Pi^<_{11} + \Pi^<_{22}) . \qquad (6)\end{aligned}$$

In the present work we focus on soft photons emitted from a QGP. Dynamical chiral symmetry breaking and its restoration at the phase transition temperature T_c plays an important role and has to be incorporated in a realistic description of the quark dynamics. We do so by considering the Nambu–Jona-Lasinio (NJL) model [9] in the SU(2) version on the quark level. See [10] for a review and the notations used in the following.

This effective field theory models the chiral symmetry properties of QCD in the nonperturbative regime by a quartic self-interaction of quarks. At small temperature, the dominant contribution to the quark self-energy is the tadpole (Hartree) term, which is expressed in terms of the spectral function as

$$\begin{aligned}\Sigma^H = -\, 2GN_C N_f \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \int_{-\infty}^{\infty} dE\, \mathrm{Tr}[\mathcal{A}_q(E,\boldsymbol{p})] \\ \times n_q(E) . \qquad (7)\end{aligned}$$

Like any nonrenormalizable model, the NJL model requires a momentum cutoff Λ, which can be motivated as a crude incorporation of asymptotic freedom at large Q^2. For the present generalization, this cutoff is shifted to the energy integration, $\Lambda_E = \sqrt{\Lambda^2 + m_q^2(T)}$ for the above diagram. See [11] for details.

Usually, the temperature dependent quark mass $m_q(T)$ is the solution of the gap equation $m_q = m_0 + \Sigma^H(m_q)$. With appropriate parameters, this describes the scenario of spontaneous chiral symmetry breaking, i.e., the transition from a current quark mass $m_0 \approx 5$ MeV to the constituent quark mass $m_q \approx 1/3 \times$ (the nucleon mass), and its restoration at a transition temperature T_c. The only parameters were chosen as $m_0 = 5$ MeV, $\Lambda = 0.65$ GeV, and $G = 4.73\ \mathrm{GeV}^{-2}$, which result in $T_c = 160$ MeV and $m_\pi = 137$ MeV.

The Fock self-energy is the next-to-leading order contribution in a $1/N_c$ expansion [12]. Since we consider quarks with three-momentum $\boldsymbol{p} = 0$, we can decompose the Fock contribution to the self-energy in a (complex) scalar and a vector part as $\Sigma^{\mathrm{Fock}} = \Sigma^S + \gamma_0 \Sigma^V$. These are added to the Hartree self-energy and, instead of the

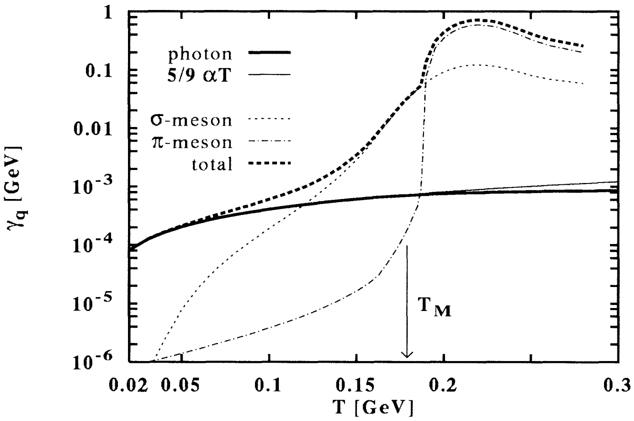

FIG. 1. Contributions to the width γ_q of a quark embedded in a QED and QCD plasma as a function of the temperature T.

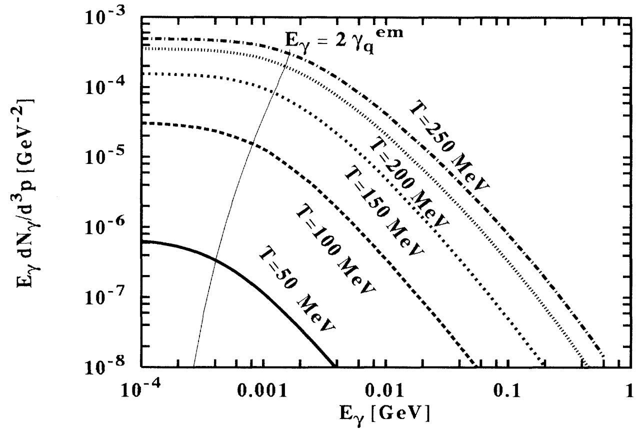

FIG. 2. Photon production rate R_γ from a QED plasma as a function of the photon energy E_γ. Photon energies below 1 MeV are plotted only to demonstrate the cutoff of the infrared divergence.

gap equation, we solve Eq. (2) for the mass and width of the effective quark field.

As an illustrative example, we first consider a coupling to photons only, with a T-dependent mass from the NJL model. We perform this calculation self-consistently using real and imaginary parts of the Fock self-energy (4) for massless free photons. This amounts to including the multiple scattering and LPM effects mentioned before within a QED plasma.

The quark spectral width as a function of temperature is plotted in Fig. 1. Because of the smallness of the coupling constant one may approximate it very well by the lowest order, which gives

$$\gamma_q^{em}(T) \approx \frac{5}{9}\,\alpha T\left[1 - \frac{\mathrm{Re}\Sigma^V}{m_q}\right] \approx \frac{5}{9}\,\alpha T\left[1 - \frac{10}{9}\,\frac{\alpha}{\pi}\,\frac{\Lambda T}{m_q^2}\right] \sim \frac{5}{9}\,\alpha\, T\,. \quad (8)$$

The photon production rate we obtain from Eq. (6) with this width is shown in Fig. 2 for various typical values of the temperature. For small photon energies, i.e., very soft photons, we find a saturation of the rate below values of $E_\gamma = 2\gamma_q^{em}$.

The physical interpretation of this effect is obvious: The emission of low-energy photons requires unperturbed propagation of the emitter over the wavelength of the photon. Along its path, however, the quark is subject to thermal perturbations, and this hinders the photon emission for $E_\gamma < 2\gamma_q$. Our result agrees with the conjecture of Weldon [4], and we obtain the dominant suppression scale at twice the spectral width of the emitting particle.

The photon production rate may be approximated as

$$R_{\mathrm{fit}}^{\gamma} = \frac{4\gamma_q}{E_\gamma^2 + 4\gamma_q^2}\, e^{-E_\gamma/T} z[T]\,,$$
$$z[T] \propto \begin{cases} T^2 & \text{for} \quad E_\gamma \ll 2\gamma_q\,, \\ T & \text{for} \quad E_\gamma \gg 2\gamma_q\,. \end{cases} \quad (9)$$

For all temperatures, the limit $E_\gamma \rightarrow \infty$ is determined by the Boltzmann factor $e^{-E_\gamma/T}$.

Finally, we calculate the soft photon production rate from a QGP. Now, the boson in the Fock self-energy diagram of the quark is a collective strongly interacting excitation of the system. Within the NJL model, these are regarded as effective, T-dependent π and σ mesons. In addition to the standard NJL results, we consider also the temperature-dependent finite width of these effective mesons and describe them by generalized Lorentzian spectral functions. Both mass and width of the respective meson are determined by the complex dispersion relation, as in the fermionic case, Eq. (2).

We do not treat the mesons self-consistently. Only their contribution to γ_q is considered, while their influence on the quark mass is neglected [12]. Physically, our approach amounts to considering photon emission processes, which are initiated by the interaction of the quark with a *single* hot meson. The resulting quark width γ_q is plotted in Fig. 1. For low temperatures, we again find $\gamma \propto T$ for each of the mesonic channels. Because of the quasi Goldstone mode of the pion, its contribution remains negligible up to the Mott temperature $T_M = 179$, which is defined by $m_\pi(T_M) = 2m_q(T_M)$ as the point where the pion can dissociate in a $q\bar{q}$ pair. For $T > T_M$, the pionic contribution to the quark spectral width is actually dominant. The strong increase in γ_q around T_M is a result of critical opalescence of the system in this temperature range. Towards higher temperatures, the competing effect of an increase of the mass of the π (now a resonance) turns the width downwards.

The results for the photon emission rate, Eq. (6), are similar to those of Fig. 2, apart from the much higher saturation scale $\gamma_q \gg \gamma_q^{em}$ at temperatures $T > T_M$. In Fig. 3, we show the photon emission rate at three different

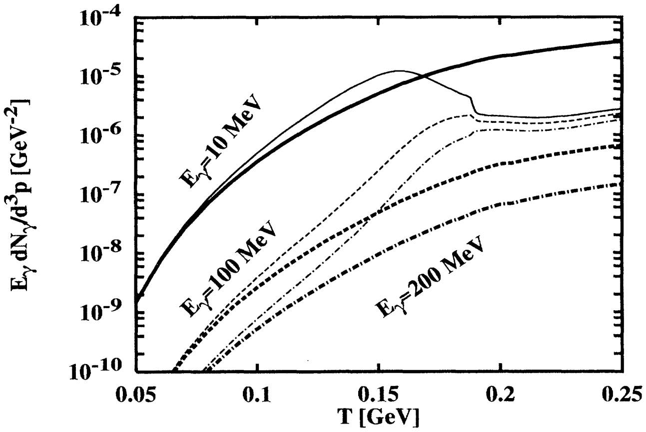

FIG. 3. Photon production rate R from a QGP as a function of temperature T. Thick lines: γ_q purely electromagnetic, thin lines: meson contributions from Fig. 1 added.

photon energies as a function of temperature. Comparing the electromagnetic case to the model including the quark-meson interaction, we find a surprising result: In the region of the chiral phase transition, the low-energy photon production rate *drops* with increasing temperature. The radiation rate is degenerate for all energies $E_\gamma < 2\gamma_q$ (see the flat behavior of the curves in Fig. 2). In view of Eq. (9), this is understood as a dominance of the saturation effect over the increase of temperature.

The validity of these results is restricted to the validity range of the effective field theory we are using. In temperature, this covers the region up to and around $T \sim T_c$. In energy, due to the cutoff $\Lambda_E \geq 0.65$ GeV we are using, our results are restricted to photon energies below this value. This is complementary to the validity range of results obtained using the technique of hard thermal loops as in [3,6], which is restricted to $E_\gamma \gg T \geq T_c$. An extension of the present work beyond its present range by using perturbative QCD with similar nontrivial spectral functions at $T > T_c$ has been performed in [11], including a detailed comparison to the previous calculations [3,6].

Let us emphasize that the qualitative properties of the soft photon rates, such as the saturation effect towards low temperatures, follow from general physical considerations as we discussed and are independent of the particular model we used. The decrease of the production rate of soft photons with temperature in the region of the phase transition might have important observable consequences.

Finally, we remark that the results presented for photons are immediately related to the rates for the production of soft dileptons from a hot plasma by use of the soft photon approximation [13], $E^+E^- dN^{\gamma\gamma\to l^+l}/d^3p^+d^3p^- = (\alpha/2\pi^2M^2)EdN^\gamma/d^3p$. Thus the present work can be applied to a calculation of soft dilepton yields from URHIC, which is of great experimental relevance [14].

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