New Method for Determining $|V_{ub}|/|V_{ts}|$ by the Processes $\overline{B} \to \rho l \overline{\nu}$ and $\overline{B} \to K^* l \overline{l}$

A. I. Sanda
Department of Physics, Nagoya University, Chikusa-ku, Nagoya, 464-01 Japan

Atsushi Yamada
Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo, 113 Japan
(Received 10 July 1995)

The differential decay width of the process $\overline{B} \to \rho l \overline{\nu}$ is related to that of the process $\overline{B} \to K^* l \overline{l}$ by using the SU(3)-flavor symmetry and heavy quark approximation. The ratio of the Kobayashi-Maskawa matrix elements is obtained in the zero recoil limit of ρ and K^*, allowing a determination of $|V_{ub}|/|V_{ts}|$, which in turn determines $|V_{ub}|$.

PACS numbers: 12.15.Hh, 13.20.He

A precise test of unitarity of the Kobayashi-Maskawa matrix [1] is essential for further investigations of the quark mass matrix and understanding the origin of *CP* violation. This is most conveniently performed in the *B* meson systems because of the large *CP* violation predicted in this system [2]. Strategies for an accurate determination of Kobayashi-Maskawa matrix elements in *B* decay is required.

An important number is one of the sides of the unitarity triangle $|V_{ub}|$ [3]. A well known method is to study the leptonic spectrum at the kinematical point where the charm quark cannot be produced. Since there are always questions as to what extent the obtained result is independent of theoretical interpretations, it is important to get at the number in as many independent ways as possible. In this Letter, we propose a strategy to get at $|V_{ub}|$. Complementing the analysis of the leptonic spectrum, we propose to obtain $|V_{ub}|/|V_{ts}|$ from $\overline{B} \to \rho l \overline{\nu}$ and $\overline{B} \to K^* l \overline{l}$. Then $|V_{ub}|$ can be accurately evaluated, since $|V_{ts}|$ is well determined by the unitarity condition. Our analysis uses the SU(3)-flavor symmetry and heavy quark approximation.

Consider the zero recoil limit of the K^* and ρ mesons. Using the heavy quark approximation, it will be shown that the matrix element of the hadronic currents describing the decay $\overline{B} \to K^* l \overline{l}$ can be expressed by the same form factor appearing in the decay $\overline{B} \to \rho l \overline{\nu}$. The form factors in each process are equated by the use of the SU(3)-flavor symmetry, and the ratio of the Kobayashi-Maskawa matrix elements is obtained by the ratio of these differential decay widths.

First consider the semileptonic decay $\overline{B} \to \rho l \overline{\nu}$. This process is described by the invariant amplitude

$$M = \frac{4G_F}{\sqrt{2}} V_{ub} \overline{u}_L \gamma_\mu b_L \overline{l}_L \gamma^\mu \nu \,. \tag{1}$$

The hadronic matrix elements required for this process are

$$\langle \rho(p', \epsilon) | \overline{u} \gamma_\mu b | \overline{B}(p) \rangle = i g^\rho \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p + p')^\rho (p - p')^\sigma, \tag{2}$$

$$\langle \rho(p', \epsilon) | \overline{u} \gamma_\mu \gamma_5 b | \overline{B}(p) \rangle = f^\rho \epsilon^*_\mu + a^\rho_+ (\epsilon^* \cdot p)(p + p')_\mu + a^\rho_- (\epsilon^* \cdot p)(p - p')_\mu . \tag{3}$$

The form factors f^ρ, $a^\rho_\pm$, and g^ρ are Lorentz invariant functions of the invariant mass squared $q^2 = (p - p')^2$ of the two leptons. The ρ meson polarization vector ϵ^* is given by

$$\epsilon_L = \left(\frac{p_\rho}{m_\rho}, 0, 0, \frac{E_\rho}{m_\rho} \right), \quad \epsilon_\perp = (0, \epsilon_x, \epsilon_y, 0). \tag{4}$$

Hereafter we study the decay in the rest frame of the $\overline{B}$ meson, so that $p^\mu = (m_{\overline{B}}, \vec{0})$, $p'^\mu = (E_\rho, \vec{p}_\rho)$, and the momentum of the ρ meson $p_\rho = |\vec{p}_\rho|$ is given by

$$p_\rho = \frac{1}{2m_{\overline{B}}} [(m^2_{\overline{B}} - m^2_\rho - q^2)^2 - 4m^2_\rho q^2]^{1/2}. \tag{5}$$

In this frame, either $(p + p')^\rho$ or $(p - p')^\sigma$ in Eq. (2) should be the spatial components $\pm \vec{p}^{\,i}_\rho$, so the right-hand side of Eq. (2) is proportional to p_ρ. Furthermore, since $\epsilon^* \cdot p = \epsilon^{*0} m_{\overline{B}}$ and only the longitudinal polarization vector ϵ^L has a nonzero time component p_ρ/m_ρ, the second and third terms in Eq. (3) are also proportional to p_ρ. Thus, the hadronic matrix elements are expanded as

$$\langle \rho(p', \epsilon) | \overline{u} \gamma_\mu b | \overline{B}(p) \rangle = \mathcal{O}(p_\rho) \,, \tag{6}$$

$$\langle \rho(p', \epsilon) | \overline{u} \gamma_\mu \gamma_5 b | \overline{B}(p) \rangle = f^\rho \epsilon^*_i + \mathcal{O}(p_\rho) \,, \tag{7}$$

in the vicinity of the zero recoil ρ meson, $p_\rho \simeq 0$, or, equivalently, the maximum q^2, $q^2 \simeq q^2_{\rm max} = (m_{\overline{B}} - m_\rho)^2$. The q^2 distribution of the decay width is computed as

$$\frac{d\Gamma(\overline{B} \to \rho l \overline{\nu})}{dq^2} = |V_{ub}|^2 \frac{G_F}{32\pi^3 m^2_{\overline{B}}} |f^\rho|^2 q^2 p_\rho + \mathcal{O}(p^3_\rho) \,. \tag{8}$$

Next we consider the flavor changing neutral decay $\overline{B} \to K^* l \bar{l}$. In the standard model, this decay takes place at the loop level via penguin and box diagrams [4]. The QCD corrected effective Hamiltonian describing this decay is

$$H = \frac{4G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^* \{C_7(m_b) O_7 + C_8^{\text{eff}}(m_b) O_8 + C_9(m_b) O_9\}, \tag{9}$$

where the operators O_7, O_8, and O_9 are defined as

$$\begin{aligned} O_7 &= \frac{e^2}{16\pi^2} m_b \bar{s}_L i \sigma_{\mu\nu} (q^\nu / q^2) b_R \bar{l} \gamma^\mu l, \\ O_8 &= \frac{e^2}{16\pi^2} \bar{s}_L \gamma_\mu b_L \bar{l} \gamma^\mu l, \\ O_9 &= \frac{e^2}{16\pi^2} \bar{s}_L \gamma_\mu b_L \bar{l} \gamma^\mu \gamma_5 l. \end{aligned} \tag{10}$$

Here the term $m_s \bar{s}_R i \sigma_{\mu\nu} (q^\nu / q^2) b_L$ has been neglected compared to $m_b \bar{s}_L i \sigma_{\mu\nu} (q^\nu / q^2) b_R$. The QCD corrected Wilson coefficients $C_i(m_b)$ are dependent on the top quark mass. In the standard model, the vector and axial vector current operators O_8 and O_9 yield the dominant contributions to the Hamiltonian (9) and the contributions of the magnetic moment type operator O_7 is less than 10% of O_8 and O_9. The coefficient $C_8^{\text{eff}}(m_b)$ contains the contributions from the $c\bar{c}$ continuum obtained from the electromagnetic penguin diagram and the long distance contributions due to the J/ψ and ψ' poles [5], and therefore it is dependent on q^2. In general, the long distance contributions are significant. However, they are small in the regions of q^2 relevant for our analysis; $q^2 \gtrsim 0.6 m_{\overline{B}}^2$. [See, e.g., Figs. 3(a) and 3(b) of Lim *et al.* in Ref. [5].] The analytic expressions of the Wilson coefficients and their numerical values are given in Ref. [6].

The hadronic matrix elements of the magnetic moment type operator and the vector and axial vector currents are necessary to evaluate the Hamiltonian (9). The vector and axial vector currents are expressed in terms of the form factors as

$$\langle K^*(p', \epsilon) | \bar{s} \gamma_\mu b | \overline{B}(p) \rangle = i g^{K^*} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p + p')^\rho (p - p')^\sigma, \tag{11}$$

$$\langle K^*(p', \epsilon) | \bar{s} \gamma_\mu \gamma_5 b | \overline{B}(p) \rangle = f^{K^*} \epsilon_\mu^* + a_+^{K^*} (\epsilon^* \cdot p)(p + p')_\mu + a_-^{K^*} (\epsilon^* \cdot p)(p - p')_\mu, \tag{12}$$

in the same way as in Eqs. (2) and (3). Again, only the term $f^{K^*} \epsilon_i^*$ remains nonzero in the limit of zero recoil K^*. As for the magnetic moment type operator, since $q^\nu = (m_{\overline{B}} - m_{K^*}, \vec{0}) + \mathcal{O}(p_{K^*})$, only the components $\bar{s} i \sigma_{0i} b$ and $\bar{s} i \sigma_{0i} \gamma_5 b$ are relevant in the same limit. The hadronic matrix elements of these operators can be related to those of the vector and axial vector currents (11) and (12) by the static heavy quark approximation. In this approximation, the b quark in the $\overline{B}$ meson stays on-shell throughout the reaction, and we can set the equation of motion for the b quark, $\gamma_0 b = b$, which leads to the relations [7]

$$\langle K^*(p', \epsilon) | \bar{s} i \sigma_{0i} b | \overline{B}(p) \rangle = \langle K^*(p', \epsilon) | \bar{s} \gamma_i b | \overline{B}(p) \rangle, \tag{13}$$

$$\langle K^*(p', \epsilon) | \bar{s} i \sigma_{0i} \gamma_5 b | \overline{B}(p) \rangle = -\langle K^*(p', \epsilon) | \bar{s} \gamma_i \gamma_5 b | \overline{B}(p) \rangle. \tag{14}$$

The right-hand sides of Eqs. (13) and (14) can be expressed in terms of the same form factors that appeared in Eqs. (11) and (12), and only the term $f^{K^*} \epsilon_i^*$ remains nonzero in the zero recoil limit. Thus, the hadronic matrix elements required for $\overline{B} \to K^* l \bar{l}$ are described only in terms of the form factor f^{K^*} in the vicinity $p_{K^*} \simeq 0$. The q^2 distribution of the decay width is given by

$$\frac{d\Gamma(\overline{B} \to K^* l \bar{l})}{dq^2} = |V_{tb} V_{ts}^*|^2 \frac{G_F}{32\pi^3 m_{\overline{B}}^2} |f^{K^*}|^2 \Big(\frac{\alpha_{\text{QED}}}{4\pi}\Big)^2 2(C_V^2 + C_A^2) q^2 p_{K^*} + \mathcal{O}(p_{K^*}^3), \tag{15}$$

$$C_V = -C_8^{\text{eff}}(m_b)_{q^2 = q_{\max}^2} + m_b \frac{m_{\overline{B}} - m_{K^*}}{q_{\max}^2} C_7(m_b), \qquad C_A = -C_9(m_b).$$

Now we extract $|V_{ub}|/|V_{ts}|$ from the q^2 distributions (8) and (15), applying the SU(3)-flavor symmetry to the ρ and K^* mesons. The question arises as to where we expect the form factors f^ρ and f^{K^*} to be nearly equal. This can be settled experimentally by studying the q^2 distribution near the $p_{\rho, K^*} \to 0$ limit. For now, the best guess is that the SU(3)-flavor symmetry holds when u and s quarks in respective b decays to be at rest in the B meson rest frame,

$$f^\rho(q_{\max}^{2\overline{B} \to \rho}) = f^{K^*}(q_{\max}^{2\overline{B} \to K^*}), \tag{16}$$

which is expected to be valid in the region $q_{\max}^{2\overline{B} \to \rho} = (m_{\overline{B}} - m_\rho)^2$ and $q_{\max}^{2\overline{B} \to K^*} = (m_{\overline{B}} - m_{K^*})^2$, respectively. Then the ratio $|V_{ub}|^2 / |V_{tb} V_{ts}^*|^2$ is extracted as

$$\frac{|V_{ub}|^2}{|V_{tb} V_{ts}^*|^2} = \frac{q_{\max}^{2\overline{B} \to K^*}}{q_{\max}^{2\overline{B} \to \rho}} \Big(\frac{p_{K^*}}{p_\rho}\Big)_{\lim} \Big(\frac{\alpha_{\text{QED}}}{4\pi}\Big)^2 2(C_V^2 + C_A^2) \Big[\frac{d\Gamma(\overline{B} \to \rho l \bar{\nu})}{dq^2}\Big]_{q^2 \to q_{\max}^{2\overline{B} \to \rho}} \Big/ \Big[\frac{d\Gamma(\overline{B} \to K^* l \bar{l})}{dq^2}\Big]_{q^2 \to q_{\max}^{2\overline{B} \to K^*}}. \tag{17}$$

Here $(p_\rho/p_{K^*})_{\rm lim} = \sqrt{m_\rho/m_{K^*}}$. In the limit $p_{\rho,K^*} \to 0$, i.e., $q^2 \to q^2_{\rm max}$, the q^2 distributions vanish due to the phase space suppression. However, the numerical values of the coefficients of p_{ρ,K^*} in Eqs. (8) and (15) can be precisely extracted in experiments. In fact, CLEO Collaboration has accurately determined the value of $|V_{cb}|f(q^2_{\rm max})$ for the process $\overline{B} \to D^* l \overline{\nu}$ [8]. In a similar manner, the right-hand side of Eq. (17) can be determined by experiments.

Expression (17) is our main result. In this expression, $|V_{ts}|$ itself is not directly measured; however, it is well determined by the unitarity condition, so $|V_{ub}|$ can be precisely evaluated.

Let us discuss the number of events needed for an analysis of this type. From the figure shown in Lim *et al.* [5], we can read off

$$\int_{0.6}^{0.8} ds \frac{dB(B \to e^+e^- + \text{anything})}{ds} \sim (3 - 5) \times 10^{-7}, \tag{18}$$

where $s = q^2/m_b^2$. For luminosity such that 3×10^7 $B\overline{B}$ pairs are produced, there should be 72–120 $B \to e^+e^-$ or $\mu^+\mu^-$ + anything events in the kimematic region $0.6 < s < 0.8$ in one year of running the B factory. In this kinematic region, *anything* should be dominated by K^*. If the higher end of the estimate is valid, barring unexpected background or systematic problems, we hope to perform a 10% level measurement of $|V_{ub}|$.

The theoretical uncertainty in the derivation of Eq. (17) lies in Eq. (16), stemming from the breaking of the SU(3)-flavor symmetry in the ρ and K^* mesons. This is expected to be small. For example, the difference of the Fermi motion of the b quark in the decays $\overline{B} \to \rho l \overline{\nu}$ and $\overline{B} \to K^* l \overline{l}$ may give rise to the error of order $(m_s^2 - m_u^2)/(M_{\overline{B}} - m_b)m_b$ [9]. Also, we may guess that the ratio of the wave functions of ρ and K^* mesons is estimated by the ratio of their decay constants [10]:

$$\frac{g_{K^*K\pi}}{g_{\rho\pi\pi}} = 1.08 \pm 0.02. \tag{19}$$

A bit more SU(3) breaking (~20%) is expected between $g_\rho = 2f_\pi^2 g_{\rho\pi\pi}$ and $g_{K^*} = 2f_\pi f_K g_{K^*K\pi}$. Further theoretical study of the ratio $f^{K^*}(q^{2\overline{B}\to K^*}_{\rm max})/f^\rho(q^{2\overline{B}\to\rho}_{\rm max})$ will allow a more precise determination of $|V_{ub}|$.

Corrections to the relations (13) and (14) are of order $\Lambda_{\rm QCD}/m_b$ in the zero recoil K^* limit. The uncertainty due to these corrections is significantly reduced in the level of the q^2 distribution (15), because the contributions of the operator O_7 are numerically less than 10% of those of O_8 and O_9, and accordingly we expect this error to be of order $\Lambda_{\rm QCD}/m_b \times 10\% - 0.4\%$ and is negligible.

In general, $C_{V,A}$ in Eq. (17) may be sensitive to the parameters of new physics beyond the standard model. This fact provides us with an interesting possibility that a value of $|V_{ub}|$ extracted in our strategy will play a role in probing for new physics, by comparing values of $|V_{ub}|$ determined by other methods.

We have used the q^2 distributions of $\overline{B} \to \rho l \overline{\nu}$ and $\overline{B} \to K^* l \overline{l}$ at their respective $q^2 \to q^2_{\rm max}$ limit to determine $|V_{ub}|/|V_{ts}|$. Studies on the forward-backward asymmetry of the leptons [11] and the polarization of ρ and K^* mesons may be also useful. The forward-backward asymmetry is described by the form factor $f^\rho g^\rho$ in the decay $\overline{B} \to \rho l \overline{\nu}$, and is described by $(f^{K^*})^2$, $(g^{K^*})^2$, and $f^{K^*} g^{K^*}$ for $\overline{B} \to K^* l \overline{l}$. The terms $(f^{K^*})^2$ and $(g^{K^*})^2$ come from the magnetic moment type operator O_7, when its hadronic matrix elements are related to those of O_8 and O_9 in the heavy quark approximation.

Along a similar line, an analysis similar to ours might be made using the radiative decay $\overline{B} \to K^*\gamma$ [12]. The long distance contributions to this decay are small [13], and the effects of the breaking of the SU(3)-flavor symmetry are similar to our analysis. However, the theoretical prediction of this decay rate suffers from uncertainty due to the large recoil momentum of the K^* meson.

One of us (A. Y.) would like to thank K-I. Izawa, N. Kitazawa, T. Morozumi, M. Tanabashi, and S. Uno for useful discussions, and L. T. Handokoo for sending his computer program of the QCD corrected Wilson coefficients.

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