Doppler Peaks from Cosmic Texture

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We compute the angular power spectrum of temperature anisotropies on the microwave sky in the cosmic texture theory, with standard recombination assumed. The spectrum shows "Doppler" peaks analogous to those in scenarios based on primordial adiabatic fluctuations such as "standard cold dark matter," but at quite different angular scales. There appear to be excellent prospects for using this as a discriminant between inflationary and cosmic defect theories.

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The cosmic microwave background (CMB) anisotropy is the cleanest probe we have of structure formation in the Universe. The COBE observation [1] provided the first solid evidence of anisotropy, but due to its limited angular resolution it provided little more than an overall normalization and rough evidence of scale invariance. Higher resolution measurements of the angular power spectrum and statistical properties of the anisotropies will provide far more powerful constraints on theories.

Existing theories of the origin of structure are based either upon the amplification of quantum fluctuations during inflation or upon symmetry breaking and field ordering. The latter category includes cosmic strings, global monopoles, and textures. While techniques for computing perturbations in inflationary theories are well established [2], greater uncertainty is attached to field ordering theories because they are nonlinear and non-Gaussian.

Nevertheless, there has been progress, particularly in those theories described by the nonlinear sigma model [3]. Recently the degree scale CMB anisotropies were calculated for the cosmic defect theories under the simplifying assumption that the Universe was fully ionized [4]. Reionization is more likely in defect theories than in Gaussian theories because large density perturbations around the defects could cause early star formation, releasing ionizing radiation. But the extent of this effect is uncertain because it depends on the efficiency of star formation, which is poorly understood.

In this Letter we compute the power spectrum of CMB anisotropies in the texture theory, assuming instead standard recombination. In particular, we wish to see whether acoustic oscillations of the photon-baryon-electron (PBE) fluid produce "Doppler peaks" analogous to those in inflationary theories. The calculation is harder than in the reionized case, but simplifies if we concentrate on the power spectrum alone rather than attempting to make sky maps. While the non-Gaussianity of such maps is undoubtedly a key signature for cosmic defects, the CMB anisotropy power spectrum is likely to be measured first.

The power spectra of perturbations depend only on the two-point correlations of the field stress-energy tensor, $\Theta_{\mu\nu}$, and one can hope to model these in a simple way. We use three-dimensional field simulations to construct a model for the $\Theta_{\mu\nu}$ correlations, which are then fed into linear perturbation codes to compute density perturbations and anisotropy spectra. As a check of our model, we perform full 3D fluid simulations of the source fields, gravity, and matter in the tight coupling epoch, and compare the perturbation spectra and cross correlations in the simulations with those predicted by the model.

There are a number of advantages to this hybrid approach. A single 3D simulation has very limited dynamic range, but many such simulations can be combined to help construct a model valid over all relevant length scales. Such a model allows one to calculate ensemble averaged power spectra without the noise inherent in 3D simulations. It can also be used in a Boltzmann calculation to compute the anisotropies for arbitrary ionization histories.

In field ordering theories, the simplest assumption is that the Universe began in a homogeneous and isotropic initial state. When a symmetry breaking phase transition occurs, some field with "angular" degrees of freedom is given a nonzero vacuum expectation value, $\vec{\phi}^2 = \phi_0^2$. The source for perturbations is $\Theta_{\mu\nu}$, the stress-energy tensor of the ordering fields, which interacts with the matter and radiation only gravitationally. Since the field ordering process is causal, it follows that the correlations of the fluctuating part of $\Theta_{\mu\nu}$ are strictly zero for space-time points whose past light cones extrapolated back to the time of the phase transition do not overlap. We work in synchronous gauge, setting all perturbation variables zero before the symmetry breaking phase transition. The metric and matter perturbation variables h_{ij} and δ_N obey causal evolution equations. Since the power spectrum is the Fourier transform of the correlation function, it follows that all these variables have "white noise" power spectra on superhorizon scales at all times.

The linearized Einstein equations may be decomposed into scalar, vector, and tensor parts: In Fourier space, a symmetric tensor, $T_{ij}(x) = \Sigma_{\mathbf{k}} T_{ij}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$, can be written as

$$T_{ij}(\mathbf{k}) = \tfrac{1}{3}\delta_{ij}T + (\hat{k}_i\hat{k}_j - \tfrac{1}{3}\delta_{ij})T^S + (\hat{k}_i T_j^V + \hat{k}_j T_i^V) + T_{ij}^T, \qquad (1)$$

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where $T_i^V k_i = k_i T_{ij}^T = T_{ij}^T k_j = T_{ii}^T = 0$. A complete set of evolution equations for the metric perturbation variables is

$$\frac{\dot{a}}{a}\dot{h} + \frac{k^2}{3}h^- = 8\pi G a^2 \sum_N \rho_N \delta_N + 8\pi G \Theta_{00}, \quad (2)$$

$$k^2 \dot{h}^- = -24\pi G i k \sum_N (p_N + \rho_N) v_N + 24\pi G \Pi, \quad (3)$$

$$\ddot{h}_{ij}^T + 2\frac{\dot{a}}{a}\dot{h}_{ij}^T + k^2 h_{ij}^T = 16\pi G \Theta_{ij}^T, \quad (4)$$

$$\ddot{h}_i^V + 2\frac{\dot{a}}{a}\dot{h}_i^V = 16\pi G \Theta_i^V, \quad (5)$$

where $\Pi \equiv \partial_i \Theta_{0i}$. The metric perturbations are defined by $ds^2 = a^2(\tau)\{-d\tau^2 + [\delta_{ij} + h_{ij}(x,\tau)]dx^i dx^j\}$, with τ conformal time and $a(\tau)$ the scale factor, and $h^- \equiv h - h^S$. The variables ρ_N, p_N, δ_N, and v_N refer to the density, pressure, density contrast, and velocity of each fluid—photons, baryons, cold dark matter, and neutrinos.

We wish to model $\Theta_{\mu\nu}$ while respecting stress-energy conservation [5],

$$\dot{\Theta}_{00} + \frac{\dot{a}}{a}(\Theta_{00} + \Theta) = \Pi,$$

$$\dot{\Pi} + 2\frac{\dot{a}}{a}\Pi = -\frac{k^2}{3}(\Theta + 2\Theta^S). \quad (6)$$

The underlying dynamics is that of nonlinearly coupled scalar fields. A crude description is that spatial gradients in the scalar fields are "frozen" outside the horizon, and redshift away after horizon crossing. This may be viewed as a change in the effective equation of state: If there were only spatial gradients outside the horizon, we would have $\Theta = -\Theta_{00}$. If these gradients redshifted away like radiation, we would have $\Theta_{00} = +\Theta$ inside the horizon.

In our model we treat the source as a fluid with a scale-dependent equation of state: $\Theta = \gamma(k,\tau)\Theta_{00}$ and $\Theta^S = \gamma^S(k,\tau)\Theta_{00}$, where γ and γ^S are determined from field simulations. In the pure matter or radiation epochs they have scaling form, being functions only of $k\tau$. With γ and γ^S fixed, Eqs. (6) determine the evolution of all scalar parts of $\Theta_{\mu\nu}$.

The main simplification this model makes is that the evolution equations for $\Theta_{\mu\nu}$ are linear. While a given mode is outside the horizon it quickly settles into a "scaling" solution specified by a single amplitude. It follows that in the long time limit unequal time cross correlations factorize, i.e.,

$$\langle A_{\mathbf{k}}(\tau) B_{-\mathbf{k}}(\tau')\rangle = \langle A_{-\mathbf{k}}(\tau) B_{\mathbf{k}}(\tau')\rangle = a(k,\tau) b(k,\tau'), \quad (7)$$

where $a(k,\tau)$ and $b(k,\tau)$ are "master" functions satisfying the same linear equations that the random field modes $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ do. Setting $B_{\mathbf{k}} = A_{\mathbf{k}}$ and $\tau = \tau'$, one sees that $a^2(k,\tau)$ is just the power spectrum of $A_{\mathbf{k}}(\tau)$. The functions γ and γ^S are given by $\langle\Theta_{00}\Theta\rangle/\langle\Theta_{00}^2\rangle$ and $\langle\Theta_{00}\Theta^S\rangle/\langle\Theta_{00}^2\rangle$, which we measure in 3D simulations and fit to specify the model [6].

The initial conditions for the model are determined by scaling and causality. Scaling implies that all correlators should be specified in terms of a single scale τ alone. Dimensional analysis then gives $\langle|\Theta_{00}(\mathbf{k},\tau)|^2\rangle = f(k\tau)\phi_0^4/V\tau$, where V is a fiducial comoving volume. Causality implies a white noise spectrum for Θ_{00}, so that $f(k\tau) = \text{const}$ for $k\tau \ll 1$. Thus for each k the master function for Θ_{00} should be a fixed constant times $\tau^{-1/2}$ at early times. Scaling of Θ and Eqs. (6) fix $\gamma(0) = -\frac{1}{2}$ in the radiation era.

Let us make two comments on the most obvious limitations of the model. Examination of the large N expressions [7] shows that the Fourier modes of the energy momentum tensor far outside the horizon ($k\tau \ll 1$) or far inside the horizon ($k\tau \gg 1$) are dominated by interference terms involving horizon wavelength modes, where the most important dynamics is taking place. These interference effects are unlikely to be well represented by the dynamics of a fluid in which all Fourier modes decouple. However, the dominant perturbations in a given Fourier mode of the metric and matter variables are produced by the source around horizon crossing, so all we really require is that the model adequately represents the source at this time. At horizon crossing, only dynamics on a single length scale are involved, and these may be reasonably well described by fluid equations.

A second concern regards the source unequal time correlations, which completely determine all perturbation power spectra. Equation (7) shows that the model actually builds in *maximal* unequal time correlations, leading to an overestimate of the coherence in time of the source terms [8]. We argue again, however, that around horizon crossing there is only one length and time scale involved, and the time scale for decoherence of the source is of the same order as that for the source to redshift away. Nevertheless it is of some importance to check the model predictions against a full 3D simulation to see whether the predicted coherent oscillations are really there.

Figure 1 shows the power in the variables h^-, δ_γ, δ_C, and v_γ from the 3D code against those predicted by model, at the instant of matter-radiation decoupling. We have similarly checked the power spectra of the source terms Θ_{00} and Π. As mentioned, we are particularly concerned to check whether the radiation oscillations evident in the model are really present, since these give rise to the Doppler peaks. The power spectra for δ_γ and v_γ show evidence of coherent oscillations, but these are less pronounced than in the model. An important question is whether this smoothing is due to real incoherence in the perturbations, or instead is a numerical artifact.

A positive definite quantity like the power is especially sensitive to numerical noise and sampling errors. A cleaner check of whether phase-coherent oscillations are present in the 3D simulations is to measure the cross

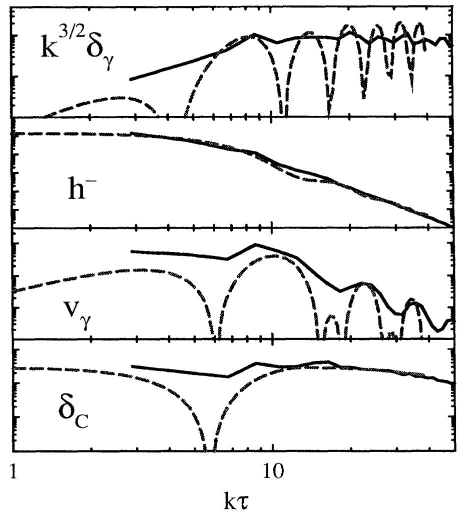

FIG. 1. Power spectra from three-dimensional simulations (solid lines) compared to those predicted by the equation of state model (dashed lines). The vertical scales are arbitrary: large ticks are separated by an order of magnitude.

correlation of δ_γ with some variable that does *not* have oscillatory behavior, such as h^-. Figure 2 shows $\langle \delta_\gamma h^- \rangle$ measured in the simulations compared with the model predictions. If the δ_γ oscillations were incoherent, this cross correlation would not follow the model as it does. Similarly, we have checked $\langle \delta_C h^- \rangle$ and confirmed that a single sign change takes place just as the model predicts.

In the approximation of instantaneous recombination one can write the full expression for the microwave anisotropy in a direction $\mathbf{n}$ as the sum of "intrinsic," "Doppler," and "integrated Sachs-Wolfe" pieces,

$$\frac{\delta T}{T}(\mathbf{n}) \equiv \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi) = \frac{1}{4}\delta_\gamma - \mathbf{v}_\gamma \cdot \mathbf{n} - \frac{1}{2}\int_i^f d\tau\, \dot{h}_{ij} n^i n^j. \quad (8)$$

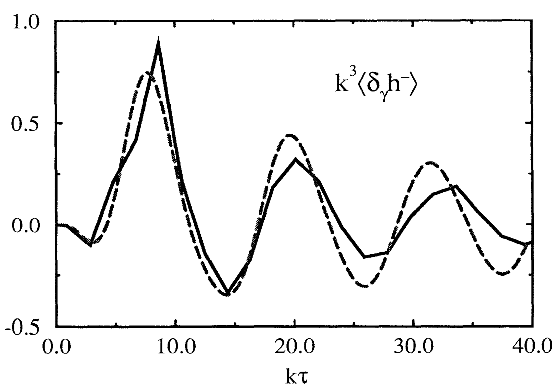

FIG. 2. The cross correlation $k^3\langle \delta_\gamma(k) h^-(k) \rangle$ measured in a single simulation (solid) is compared to the predictions of our model (dashed line). This provides strong evidence that coherent oscillations exist in the photon baryon fluid.

Converting this into a spectrum of C_l's involves ensemble averaging, which is simple in the factorization approximation. The scalar expression is well known (see, e.g., [9]); the vector and tensor contributions are even simpler. We define the vector master function by $\langle \dot{h}_i^V(\mathbf{k}) \dot{h}_j^V(-\mathbf{k}) \rangle = \dot{h}^V(k,\tau)\dot{h}^V(k,\tau')(\delta_{ij} - \hat{k}_i\hat{k}_j)$, and the tensor function by $\langle \dot{h}_{ij}^T(\mathbf{k}) \dot{h}_{kl}^T(-\mathbf{k}) \rangle = \dot{h}^T(k,\tau)\dot{h}^T(k,\tau')(\delta_{ik}\delta_{jl} + \cdots)$ where $\cdots$ denotes terms imposed by the transverse-traceless and symmetry conditions. Some algebra then shows that

$$\langle |a_{lm}^V|^2 \rangle = \frac{2}{\pi}\int k^2 dk\, l(l+1) \times \left[\int_i^f d\tau \frac{d}{d\tau}\left(\frac{j_l(k\Delta\tau)}{k^2\Delta\tau}\right)\dot{h}^V(k,\tau)\right]^2, \quad (9)$$

$$\langle |a_{lm}^T|^2 \rangle = \frac{1}{2\pi}\int k^2\, dk \frac{(l+2)!}{(l-2)!} \times \left(\int_i^f \frac{d\tau}{(k\Delta\tau)^2} j_l(k\Delta\tau)\dot{h}^T(k,\tau)\right)^2, \quad (10)$$

where $\Delta\tau \equiv \tau_f - \tau$. The latter equation agrees with the formula given by Abbott and Wise [10]. The functions h^V and h^T are obtained by solving the Einstein equations (4) and (5) numerically, using the master functions for Θ^V and Θ^T inferred from 3D simulations, computed in the same "factorization" approximation.

The final CMB anisotropy power spectrum is presented in Fig. 3, along with various components. The results shown were computed using a fluid code, in the "instantaneous recombination" approximation. We have checked the scalar and tensor calculations using a full Boltzmann code, with very good agreement. The cosmological parameters used were $h = 0.5$, $\Omega = 1$, and $\Omega_B = 0.05$.

We have found the *shape* of the spectrum for $l > 100$ to be remarkably insensitive to the details of our

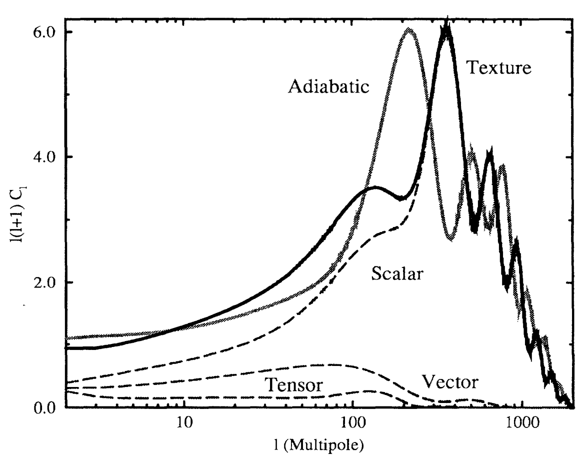

FIG. 3. The predicted anisotropy power spectrum $C_l = \langle |a_{lm}|^2 \rangle$, decomposed into scalar, vector, and tensor contributions. Shown for comparison (grey curve) is the prediction of the simplest CDM inflationary theory with the same cosmological parameters.

model: plausible variations change the results by less than 10%. Likewise the qualitative shape of the vector and tensor contributions is fairly model independent. We attach greater uncertainty (of order 50%) to the relative amplitude of the perturbations on larger (COBE) scales, partly because of statistical uncertainties in the vector and tensor power spectra extracted from the 3D code, but also because the scalar power on large scales *is* sensitive to variations in the "equations of state." As a check, however, we have measured the power in δ_γ at decoupling in the 3D code, and compared this with the large scale anisotropy as computed in Ref. [3]. The ratio is consistent with the model predictions to within 30%.

The most striking difference between the texture and inflationary spectra is that Doppler peaks and troughs are almost "out of phase"—where inflation predicts a maximum texture gives a minimum, and vice versa. This behavior is reminiscent of that found in "isocurvature" models [11], and it occurs for a similar reason [12]. The radiation oscillations are driven by the metric perturbations: $\ddot{\delta}_\gamma + c_S^2 \delta_\gamma \sim -\frac{2}{3}\ddot{h}$. The phase of the oscillations is determined by the behavior of $\ddot{h}$ as a mode crosses the horizon. In "adiabatic" theories like the simplest inflationary models, one has superhorizon curvature perturbations, in which the source term $\ddot{h}$ is to a first approximation constant. In the usual isocurvature theories, where there are instead superhorizon perturbations in the baryon-to-photon ratio, the source term is not constant, and this leads to a relative phase shift in the oscillations.

In the field ordering theories, there are no perturbations in either the space curvature or the baryon-to-photon ratio on superhorizon scales. This leads to yet another behavior of the $\ddot{h}$ forcing term around horizon crossing. Scale invariance and dimensional analysis imply that $\langle |h(\mathbf{k})|^2 \rangle = g(k\tau)\tau^3/V$, with g a dimensionless function. In standard "adiabatic" theories, $h \propto \tau^2$ so $g \propto k\tau$ for small $k\tau$. But for field ordering theories, the small k behavior of g is fixed instead by causality: h must have a white noise power spectrum, so $g = \text{const}$ and $h \propto \tau^{3/2}$ at small $k\tau$. Thus the forcing term decreases as $\tau^{-1/2}$. It follows that δ_γ reaches its first maximum sooner than in the adiabatic theories. It is intriguing that, at least in the context of theories without superhorizon baryon-to-photon fluctuations, the small scale angular power spectrum, and, in particular, the phase of the Doppler peaks, could be telling us something as fundamental as whether the perturbations were generated causally within the standard big bang, or were necessarily generated in a preceding inflationary epoch.

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