Stationary Striations Developed in the Ionospheric Modification

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A nonlinear theory determining the conditions of existence and the structure of the stationary striations generated in the ionospheric modifications by powerful radio waves is proposed. The structure of the density depletions and its characteristic length, width, and depth are shown to be in agreement with observations. A strong enhancement of the electron temperature inside the striations is predicted.

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One of the most important new physical phenomena discovered during ionospheric modification by powerful radio waves was the generation of small-scale striations which represent plasma density depletions strongly elongated along the Earth's magnetic field. The striations determine both the effective artificial field-aligned scattering of uhf and vhf radio waves (AFAS) and the anomalous wideband attenuation (WBA) in the disturbed region [1–3]. A close connection of the striations with high frequency stimulated electromagnetic emission (SEE) and low frequency emissions (LFE) of the disturbed ionosphere [4–6] is also established. There exist some interesting peculiarities of the behavior of the striations under the modification of the ionosphere by powerful radio waves with frequencies near the multiple gyrofrequency [7,8].

According to the theory, striations are due to a local heating of the anisotropic ionospheric plasma by upper hybrid resonance waves, which are generated by a linear transformation of the pump radio wave on the striation density depletions. This process leads to a resonance (or thermal parametric) instability [9,10]. The resonance instability is nonlinear and has an explosive character [11–14]. These main features of instability are confirmed in ionospheric experiments [15].

A fundamental problem is the nonlinear stationary state which sets in after a full development of resonance instability. It is precisely this stationary state that determines the main characteristics of various nonlinear phenomena in the ionosphere, which were explored using various radio methods (AFAS, WBA, SEE, LFE). Moreover, striations were recently observed directly in experiments *in situ* on board rockets [16,17]. Striations appear as essentially local stationary depletions of plasma density, with scales of the order of 10 m across and several kilometers along the magnetic field lines.

In the present paper, the theory of the stationary state of striations in the ionospheric plasma is developed.

As mentioned above, striations are local depletions of the density of plasma particles ($N - N_0 = N_1 < 0$) created as a result of the local heating of electrons ($T - T_\infty > 0$) by the field of upper hybrid plasma waves E_1 generated by a powerful pump radio wave E_0. Consequently, the stationary state of striations is described by a system of nonlinear stationary equations for transport of particle density N_1, electron temperature T, and a wave equation for the electric field of plasma waves E_1 excited in striations in the upper hybrid resonance region by the pump wave E_0.

This system has the form [2]

$$\begin{aligned} \nabla\hat{\mathbf{D}}\nabla N_1 + \nabla\hat{\mathbf{D}}_T\nabla T &= \gamma N_1 \,, \\ \nabla\hat{\boldsymbol{\kappa}}\nabla T + \frac{\mathbf{E}_1\hat{\boldsymbol{\sigma}}\mathbf{E}_1}{N_0} &= \delta\nu(T - T_\infty)\,, \\ \nabla(\hat{\boldsymbol{\varepsilon}}\mathbf{E}_1) &= -\nabla(\delta\hat{\boldsymbol{\varepsilon}}\mathbf{E}_0)\,. \end{aligned} \tag{1}$$

Here $\hat{\mathbf{D}}$, $\hat{\mathbf{D}}_T$, $\hat{\boldsymbol{\sigma}}$, and $\hat{\boldsymbol{\kappa}}$ are tensors of diffusion, thermal diffusion, conductivity, and heat conductivity, respectively; γ is the coefficient of recombination; δ is the average part of the electron energy loss under collision with ions and neutrals; ν is the electron collision frequency, T_∞ is the plasma temperature far from the localized striation (it is determined by the average heating of ionospheric plasma by a powerful radio wave), $\mathbf{E}_0$ is the electric field of the pump wave, $\hat{\boldsymbol{\varepsilon}}$ is the tensor of plasma dielectric permittivity, and $\delta\hat{\boldsymbol{\varepsilon}}$ is the part of this tensor which is the response to plasma density perturbation.

To begin with, let us consider the case when the functions N_1 and T depend on two variables only, along (z) and perpendicular (x) to the direction of the magnetic field line in the $(\mathbf{H}, \mathbf{E}_0)$ plane. Then the system of equations takes the form

$$\begin{aligned} \frac{\partial}{\partial x}\left(D_\perp \frac{\partial N_1}{\partial x}\right) + \frac{\partial}{\partial z}\left(D_\parallel \frac{\partial N_1}{\partial z}\right) + \frac{\partial}{\partial x}\left(D_\perp^T \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(D_\parallel^T \frac{\partial T}{\partial z}\right) &= \gamma N_1 \,, \\ \frac{\partial}{\partial x}\left(\kappa_\perp \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(\kappa_\parallel \frac{\partial T}{\partial z}\right) + \frac{\sigma_\perp E_1^2}{N_0} &= \delta\nu(T - T_\infty)\,, \\ \frac{d}{dx}\left(\varepsilon_{xx}\frac{d\phi}{dx}\right) - k_z^2\varepsilon_{zz}\phi &= -P_0\frac{\partial}{\partial x}\left(\frac{N_1}{N_0}\right), \end{aligned} \tag{2}$$

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where ϕ is the potential of the electric field, $E_1 = d\phi/dx$, ε_{zz} is the longitudinal, and ε_{xx} is the transverse component of the dielectric tensor:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 - \frac{N_1}{N_0} + \lambda^2 \frac{d^2}{dx^2}, \tag{3}$$
$$P_0 = \mathbf{E}_0 \frac{\partial \hat{\varepsilon}}{\partial(\omega_p^2)} \omega^2 \mathbf{e}_x,$$
$$\lambda^2 = 3 \frac{\lambda_D^2}{1 - 4\omega_H^2/\omega^2}.$$

Here λ_D is the Debye length, ω_H is the electron gyrofrequency, ω and k_z are the frequency and wave vector of the pump wave, ω_p is the Langmuir plasma frequency, and ε_{xx}^0 is the transverse component of the dielectric tensor of an undisturbed homogeneous plasma [18].

Usually, the transverse length scale of irregularities is much smaller than $(k_z\omega_H/\omega)^{-1}$. The equation for the electric field can then be integrated over x and rewritten in the form [12]

$$\lambda^2 \frac{d^2E_1}{dx^2} + \left(\varepsilon_{xx}^0 - \frac{N_1}{N_0}\right)E_1 = -P_0 \frac{N_1}{N_0}. \tag{4}$$

The system (2) is strongly nonlinear, in particular, due to the dependence of transport coefficients on temperature T and density N. With allowance for the fact that in the F region the collisions of electrons with ions play a leading role, we have $\kappa_\perp = k_\perp y, \kappa_\parallel = k_\parallel y^{-5}, k_\parallel = 5.93T_\infty/\nu_0, k_\perp = 1.77T_\infty\nu_0/(m\omega_H^2), \nu_0 = \nu_e(T_\infty)$, and $y = (T_\infty/T)^{1/2}$. The parameter γ at heights $z > 200$ km does not depend on the electron temperature T, and we also assume the parameter δ to be independent of T.

We suggest in a further analysis on the basis of previous results [2] that the density perturbation N_1/N_0 in the striations is small, while the electron temperature T may be several times greater than the background value T_∞.

Let us emphasize now that the heat energy source $\sigma_\perp E_1^2$ is concentrated in a layer of the order of L between the upper hybrid resonance level $\omega = \omega_{\rm UH} = \sqrt{\omega_p^2 + \omega_H^2}$ and the reflection point of the pump wave $\omega = \omega_p$, since only inside this region the pump wave may excite upper hybrid waves within irregularities. This length scale in the F region of the ionosphere is of the order of 1–3 km and so it is small compared to the characteristic scale of the transport processes along the magnetic field in the F region: the longitudinal diffusion length $L_N = \sqrt{D_\parallel/\gamma}$ and the longitudinal heat conductivity length $L_T = \sqrt{k_\parallel/\delta\nu}$ (see [2]).

The solution of the system (2) can be obtained in the form of expansion is small parameters L/L_T and L/L_N. We consider two separate zones: one lying in the heating layer $x \in [0, L]$ and the other outside it. In the outside region, the transport along **H** is predominant due to a large width of the heated region. The transport across **H** is predominant in the inside region together with the outflow of the particles and heat fluxes through the upper and lower boundaries of the heating region. To match the equations inside and outside the heating area, we examined the external and internal problems and matched the corresponding particle and heat fluxes. When realized consistently this procedure allows us to reduce the full system of equations (2) to an ordinary equation of one variable x within the heating layer:

$$\frac{d^2y}{dX^2} = f(y), \quad f(y) = -\frac{y^5\sqrt{\frac{4}{9}y^{-9} - \frac{4}{7}y^{-7} + \frac{8}{63}}}{2\sqrt{1 + y^{-2}}} + \frac{1}{2\sqrt{\eta}} y^{-3/2}\left(\frac{4}{9}y^{-9} - \frac{4}{7}y^{-7} + \frac{8}{63}\right)^{1/4}. \tag{5}$$

Here the dimensionless variables are

$$X = x/l, \quad l = \left(\frac{D_\perp^2 k_\parallel}{\delta\nu_0\gamma D_\parallel}\right)^{1/4},$$

and $D_\perp$, $D_\parallel$, and $k_\parallel$ are the coefficients of transport calculated for $T = T_\infty$. The characteristic length l is determined by the processes of transverse diffusion during the characteristic relaxation time of the electron temperature $(\delta\nu_0)^{-1}$ and plasma density γ^{-1}. For F-region conditions, the parameter $l \sim 5$–10 m. The effective parameter of heating is

$$\eta = \left(\frac{3}{64}\frac{\nu_0|P_0^2|L_0c}{N_0T_\infty\lambda\omega_p\sqrt{\delta\nu_0k_\parallel}}\frac{1 + (\omega_H/\omega)^2}{1 - (\omega_H/\omega)^2}\right)\frac{L_{N_0}^2}{L_{T_\infty}^2}, \tag{6}$$

where L_0 is the scale of plasma density variations in the background ionosphere in the vertical direction.

Note that y takes values in the interval $0 < y < 1$. Integrating (5) we obtain

$$\frac{1}{2}\left(\frac{dy}{dX}\right)^2 = \int_{\tilde{y}}^{y} f(y)\,dy = -\Psi(y). \tag{7}$$

The function $\Psi(y)$ reaches its local maximum value at $y = y_2$, where y_2 is the root of the equation $f(y) = 0$. It determines the localized solution

$$\int_{\tilde{y}}^{y} \frac{dy_1}{\sqrt{2\int_{\tilde{y}}^{y_1} f(\xi)\,d\xi}} = X, \tag{8}$$

where $\tilde{y} \le y \le y_2$ and the parameter $\tilde{y}$ is determined by the relation

$$\int_{\tilde{y}}^{y_2} f(y)\,dy = 0. \tag{9}$$

The solution of (8) is shown in Fig. 1. It has the form of a localized wave. The dependencies of the temperature and density perturbations at the center of a striation on the effective heating parameter $\sqrt{\eta}$ are shown in Fig. 2. As the heating parameter increases, the temperature and density perturbations grow effectively. It is easy to see from this figure that the necessary stationary solution

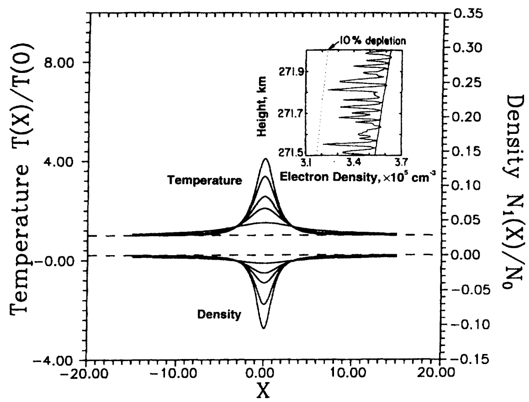

FIG. 1. The solution of Eq. (8) for different values of the effective parameter $\sqrt{\eta}$ as a function of dimensionless variable X. The upper bunch corresponds to the temperature dependencies and the lower bunch is for the density ones. Inset shows several of the plasma depletions observed in the experiment [16] (inset reprinted with permission of American Geophysical Union).

exists only for specific values of the parameter $\sqrt{\eta} > 5.8$. There is no solution below the point $\sqrt{\eta} < 5.8$. The half-width of the localized wave X_0 decreases rapidly when the heating parameter varies from 5.8 to 10, but further the fall becomes rather weak, $X_0 \simeq 1$. It should be noted that for the existence of a localized solution a small average heating is needed: $y \leq y_2 \approx 0.96$.

Thus we see that the structure of the striations is defined by the parameter

$$\frac{E_0}{E_0^*} = \frac{\sqrt{\eta}}{5.8}, \tag{10}$$

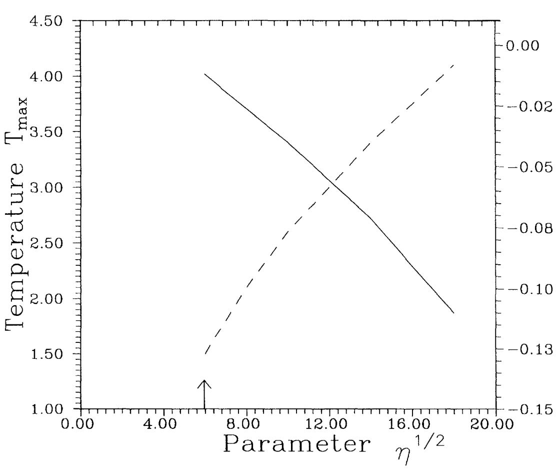

FIG. 2. The dependencies of the temperature (dotted line) and density (solid line) on the effective parameter $\sqrt{\eta}$ at the center of the striation. The arrow displays the minimum value of $\sqrt{\eta}$ for which the solution exists.

where E_0 is the amplitude of the pump wave and E_0^* is the characteristic field: a stationary solution exists only if E_0 exceeds E_0^*:

$$E_0^* = 35.3(\sin\alpha)^{-1}(T_\infty N_0)^{1/2}\sqrt{\frac{L_T^2\delta}{L_0 L_N}}\sqrt{\frac{v_{T_e}}{c}} \times \frac{[1-(\omega_H/\omega)^2]^{3/2}}{[1+(\omega_H/\omega)^2]^{1/2}[1-4(\omega_H/\omega)^2]^{1/4}}, \tag{11}$$

where v_{T_e} is the thermal velocity of electrons, and α is the angle between the pump wave electric field $\mathbf{E}_0$ and the magnetic field $\mathbf{H}$. Estimates show that E_0^* is of the order of 100 mV/m. This value is easily reached in ionospheric modification experiments.

So stationary striations, according to our theory, are density depletions elongated along magnetic field lines on the scales $L_T \sim 10$–15 km (see [2]). The characteristic half-width of the striations is $l \sim 5$–10 m. The depth of the density depletions $|N_1/N_0| \sim (1$–$10)\%$, Fig. 2. The shape, depth, and width of the depletions depend on one dimensionless parameter, (6) or (11); the width is growing with diminishing depth. The considered stationary striations exist for a finite value $N_1 > N_{\min}$ only, where $N_{\min}/N_0 \simeq 0.012$ (see Fig. 2). We emphasize that the structure of the striations observed in experiments [16] (see Fig. 2 of [16]), their elongation (~10 km), the depth of density depletions (2%–10%), and their characteristic half-width scale (4–10 m), correspond well to the proposed theory. The minimum amplitude of the depletions observed (1%–2%) is also in accordance with the theory.

The main prediction of the theory is a strong enhancement of electron temperature inside striations, $T/T_\infty \sim 2$–4, which has not yet been observed (one can suppose that the optical emission which is usually observed in ionospheric modifications is connected with this temperature enhancement [19]).

We stress that the source responsible for the explosive character of resonance instability is a strong heating inside striations which is proportional to $(N_1/N_0)^2$. The stabilization comes from a nonlinear growth of the transport coefficients (mostly thermal conductivity κ) with the temperature T. That is why the stationary solution exists only for large enough values of $T/T_\infty > 1.6$ (or for $N_1/N_0 > 0.012$). This nonlinear stabilization process was not considered in previous papers [9–14].

Note that in the general case the solution of Eq. (5) has the form of a nonlinear wave, so we have a set of striations. However, in this case new macroscopic processes, which are beyond the scope of this paper, become substantial. First of all it concerns the anomalous absorption of the pump wave. This effect leads to an effective reduction of the heating zone scale L, and, as a consequence, to a slowing down of the increase of the perturbation magnitude against amplitude E_0. Another important process is self-focusing of the pump wave. The problem is that the density perturbations in striations are

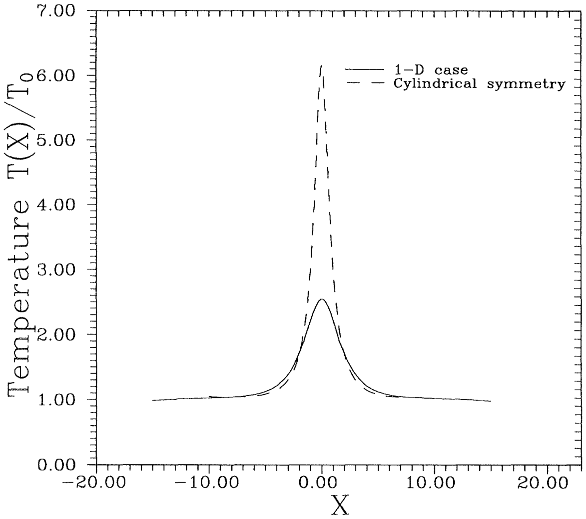

FIG. 3. Comparison of one-dimensional (solid line) versus cylindrical symmetry structure of the striation for the same value $\sqrt{\eta} = 10$.

always negative, $N_1 < 0$. Therefore the average electron density is reduced during the excitation of a large number of striations. This fact results in a self-focusing of the pump wave E_0. But the enhancement of the field E_0 in the focusing region leads in turn to increasing striations. Thus there is a close nonlinear connection between the striation formation and focusing: in the focusing zone, where the field E_0 is strong, the striations are strong as well. Otherwise outside the focusing zone the field is small (it may drop by an order of magnitude, see [2,20]). Here striations should also be small or even not excited at all. The effect of a close nonlinear connection between the striations and the self-focusing is clearly seen in the experiment [16].

In conclusion, we note that the one-dimensional structures under consideration are unstable in the plane perpendicular to the direction of the magnetic field. As a result, striations become two dimensional, forming cylindrical structures. The developed theory is easy to generalize to cylindrical symmetry. The system of Eqs. (1) for this case will be only slightly different from (2), and its further analysis is almost the same as that for the one-dimensional case. The wave equation was considered previously for cylindrical perturbations in [14] where, in particular, the heating function was obtained. Using these results and performing calculations, we arrive at the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dy}{dr}\right) = f(y), \qquad (12)$$

where $f(y)$ is given by the same relation (5).

Equation (12) has been solved numerically. The solution shown in Fig. 3 is quite analogous to the one-dimensional solution (8); the only difference is that the amplitude at the center of the perturbation for one and the same value of the effective heating parameter is approximately 2 times larger, while the half-width of the peak decreases 2 times too.

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