Generation of Zonal Flow and Meridional Anisotropy in Two-Layer Weak Geostrophic Turbulence

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Evolution of weak anisotropic 2D turbulence in a two-layer medium is considered by way of an example of geostrophic turbulence. Numerical experiments with a kinetic (energy transfer) equation confirm that a powerful barotropic nearly zonal flow is always generated, whereas its baroclinic component is typically suppressed. For certain initial conditions a considerable large-scale baroclinic meridional anisotropy is excited.

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A wide variety of processes in plasma physics (drift waves, electromagnetic electron oscillations of an inhomogeneous magnetized plasma, trapped-ion waves in a tokamak, etc. [1]), geophysics (geostrophic turbulence, Rossby waves [2]), and astrophysics (density waves in gas disks of galaxies [3]) may be reduced to a general problem of free (quasi) 2D anisotropic turbulence. In the simplest case, the latter is described by the nondimensional Charney-Obukhov (-Hasegawa-Mima) equation:

$$\partial(\Delta\psi - a^2\psi)/\partial t + \beta\, \partial\psi/\partial x + \epsilon J(\psi, \Delta\psi) = 0, \quad (1)$$

where $\psi = \psi(x,y)$ is the stream function, $J(f,g) = f_x g_y - g_x f_y$, and the meaning of parameters a, ϵ, β depends on the physical system considered.

Evolution of motions governed by Eq. (1) has been extensively studied [1–5]. An inverse energy cascade causes increasing both the characteristic scale L and the role of the anisotropic term $\partial\psi/\partial x$ [whose presence results in the existence of wave solutions to the linearized Eq. (1)]. At some instance the motion will be gradually rearranged from a turbulencelike to a wavelike type: At $L^2 \approx L_\beta^2 = 2U/\beta$ (U is the typical velocity of fluid particles), eddies transporting the fluid are transformed into waves traveling along the fluid. Further evolution mainly takes place owing to wave interactions. Its intensity slows down essentially because a resonance condition for the frequencies of interacting waves needs to be met. The mean frequency of the wave field decreases. This together with the scale-increasing process causes intensification of flow along the x axis (zonal anisotropy). At the infinite-time limit, only stationary zonal currents (flows along the x axis) with scale at L_β are believed to exist in dissipative systems (i.e., the energy cascade stops at this scale), while methods of statistical physics predict a spectrally isotropic final state for truncated inviscid flows [5,6].

However, in the idealized case, the scale-increasing process continues. Later on, the weakly nonlinear approximation ($\epsilon = U/\beta L^2 \ll 1$) and methods of the kinetic theory [7,8] become applicable. The distinguishing feature of the latter is that it treats the flow as a system of weakly nonlinear waves with a continuous spectrum, which evolves owing to resonant interactions exclusively. It predicts that the motions in question may tend to an anisotropic equilibrium state but the energy cascade also stops at a certain wave number.

If the flow is structured in the z direction (e.g., stratified flows in geophysics), it possesses the described basic features as well. The dominant opinion in the literature is that this structure will mostly be stirred [4,5]. However, its presence actually results in an additional degree of freedom, the reasons for damping of which are not obvious. This Letter describes several features of primary interest caused by this structure in the late (weakly nonlinear) evolution of the turbulence in question. Under certain conditions, it does not vanish and enables the energy cascade to scales essentially larger than in the pure 2D case.

Energy transfer (kinetic) equation for motion components.—Since Eq. (1) has been first derived and most commonly used in Earth sciences [2,4,5], we shall follow its geophysical interpretation. It describes the evolution of geostrophic turbulence (quasi-2D flows with $L \gtrsim 20/100$ km and the time scale $\tau \gtrsim$ month/week in Earth's oceans and atmosphere). The Earth's surface is treated as an infinite even plane (β plane) in which the Coriolis parameter f (the vertical component of the background rotation) varies linearly in a North-South direction ($f = f_0 + \beta_y$, where $f_0 \sim 10^{-4}\,\mathrm{s}^{-1}$ is its typical value at midlatitudes and $\beta \sim 10^{-11}\,\mathrm{s}^{-1}\,\mathrm{m}^{-1}$). This variation is traditionally called the β effect, the typical size of vortices (synoptic rings in the ocean and cyclons or anticyclons in the atmosphere) is of the order of $a^{-1} \sim 100/2000$ km, and the wave solutions to Eq. (1) are called Rossby waves.

The above-mentioned structure is induced by vertical density alteration, which in a simple manner is captured in a model consisting of two nonmixing layers. Evolution of β-plane motions in such a model is governed by two coupled equations, both of which are similar to Eq. (1) [4,5,9]. We shall use a traditional decomposition of the flow (and wave harmonics) into the barotropic $\Psi_0 = \psi_1 + \psi_2 h_2/h_1$ and the (first) baroclinic $\Psi_1 = \psi_1 - \psi_2$ mode (ψ_1, ψ_2, h_1, h_2 are the stream functions and mean thicknesses of the upper and lower layers, respectively).

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The barotropic component roughly corresponds to the vertically averaged flow while the baroclinic component characterizes its shear.

The weakly nonlinear spectral evolution of the system is described by the kinetic (energy transfer) equation [9]:

$$\frac{\partial F_p}{\partial T} = 8\pi \sum_{m,n=0}^{1} \int C^{pmn}_{\vec{\kappa}_1\vec{\kappa}_2} K_{pmn} N^{-1}_{pmn} \times \delta(\omega^{012}_{pmn})\delta(\vec{\kappa}_{012})\, d\vec{\kappa}_{12} . \quad (2)$$

Here $F_p(\vec{\kappa}, T)$ is the energy spectrum of the barotropic ($p = 0$) or the baroclinic ($p = 1$) mode; $\vec{\kappa} = (k, l)$ the wave vector; $T = \epsilon^2 t$ the slow time; $2C^{pmn}_{\vec{\kappa}_i\vec{\kappa}_j} = \gamma^p_{mn}|\vec{\kappa}_i \times \vec{\kappa}_j|(\kappa_j^2 + a_n^2 - \kappa_i^2 - a_m^2)$ the interaction coefficients (the coefficients γ^p_{mn} depend on the vertical structure of the medium; see [9]); $K_{pmn} = C^{pmn}_{\vec{\kappa}_1\vec{\kappa}_2} F^1_m F^2_n + C^{npm}_{\vec{\kappa}\vec{\kappa}_1} F_p F^1_m + C^{mpn}_{\vec{\kappa}\vec{\kappa}_2} F_p F^2_n$; $F^q_p = F_p(\vec{\kappa}_q)$ for $q = 1, 2$; $\kappa = |\vec{\kappa}|$, $N_{pmn} = (\kappa^2 + a_p^2)(\kappa_1^2 + a_m^2)(\kappa_2^2 + a_n^2)$; $\delta(x)$, the Dirac delta function; $\omega^{012}_{pmn} \equiv \omega_p(\vec{\kappa}) + \omega_m(\vec{\kappa}_1) + \omega_n(\vec{\kappa}_2)$; $\omega_p(\vec{\kappa}) = -k/(\kappa^2 + a_p^2)$ the dispersion relation of the Rossby waves of the pth mode, $\vec{\kappa}_{012} = \vec{\kappa} + \vec{\kappa}_1 + \vec{\kappa}_2$; and $d\vec{\kappa}_{12} = dk_1 dl_1 dk_2 dl_2$, $\vec{\kappa} \in \mathbf{R}^2$. The quantities a_p^{-1} are called the barotropic ($p = 0$) and the (first) baroclinic ($p = 1$) Rossby radii and depend on the background physics. In our experiments we have used the nondimensional values $a_0 = 0$, $a_1 = 1$, and $h_1/h_2 = 0.2$.

We choose the physical background to be motions in the oceans, where the density of water masses varies insignificantly. We disregard the quantities of the order of its relative alteration, which yields $\gamma^0_{01} = \gamma^0_{10} = \gamma^1_{00} = 0$. This reflects the fact that the interactions between two barotropic and one baroclinic Rossby wave are much less intense than other resonant interactions [10]. Equation (2) is truncated at $\kappa = 4$ (the interactions involving waves with $\kappa > 4$ are ignored) and the Cauchy problem for them is solved numerically. For technical details we refer to Ref. [11]. The initial conditions have energy maximum at $\kappa \approx 0.7$, but control runs show that its disposition does not qualitatively affect the spectral behavior.

We performed simulations during 5–10 slow time units. Each unit is equivalent to several years. During this time interval, the total interaction intensity $I = \int(|\partial F_0/\partial T| + h_1/h_2|\partial F_1/\partial T|)dk\,dl$ decreases from ten to several dozens of times. Also, the system entropy $H = \int \ln(F_0 F_1)dk\,dl$ reaches a nearly constant level. Thus, the computed final states evidently reflect the main features of the equilibrium state.

Generation of a barotropic nearly zonal flow as a typical scenario.—The temporal evolution of the barotropic spectrum F_0 is always similar to that of the one-layer case [7,12] and shows a coexistence of two tendencies. First, a portion of the energy is concentrated near the l axis to form a well-defined spectral peak, corresponding to an intensive nearly zonal flow. (When speaking about zonal flow, we actually mean nearly zonal motions: Generation of flow in exactly the x direction is

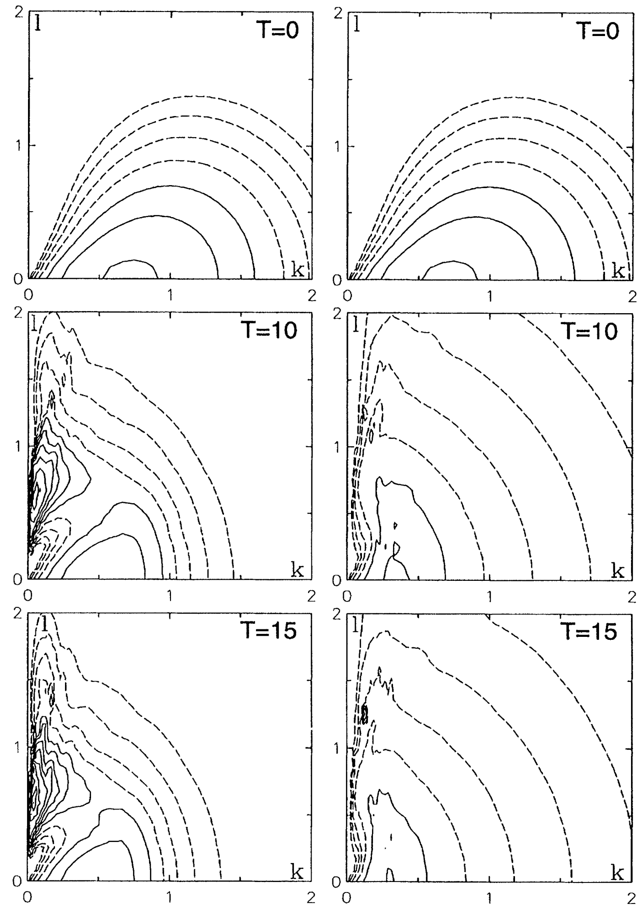

FIG. 1. *Typical spectral evolution.* Spectral density of energy $F_p(k, l, T)$ of the barotropic (left column) and the baroclinic mode (right column) is contoured at several time moments in the main energy-containing area $0 \leq k, l \leq 2$. The sign of the k component is reversed. Both the initial functions are proportional to $\kappa \exp(-\kappa^2)\cos^4\varphi$ (φ is the polar angle of the wave vector) and correspond to a mainly meridional flow. Contour intervals are logarithmic (4 lines per decade) starting from 0.01. The dashed lines are used for $F_p < 0.1$.

not possible in the system in question; see, e.g., [7]). Second, remote from this axis the spectrum tends to become isotropic (Fig. 1). The energetic maximum is located at $l \approx 0.65$ while the peak (below called the zonal peak) is located between $l \approx 0.3$ and $l \approx 1.0$. It turns out to be higher, narrower, and located closer to the l axis as compared to the one-layer flow. This difference partially results from the fact that in the one-layer experiments [7,12] typically the value $a_0 = 1$ was used instead of $a_0 = 0$ in the current study. However, a detailed analysis shows that interactions between flow components usually tend to amplify the peak in question.

As a rule (if $F_0 \gtrsim F_1$ at $k \lesssim 1$ and $T = 0$), the baroclinic spectrum F_1 reaches a practically isotropic shape relatively fast. This behavior is somewhat unexpected because self-interaction of the baroclinic mode (equivalent to that of the barotropic mode with $a_0 = 1$) should always create a zonal anisotropy. We continued computation of several cases during additional 5 time units (Fig 1). The barotropic zonal peak follows up to increase and concentrates with time in a slightly closer vicinity to the l axis, but no sign of anisotropy of the F_1 field was detected.

The strict and long-time suppression of the baroclinic zonal flow appears to be an essential feature of stratified quasi-2D flows. It indicates a possible tendency towards equilibrium state, the zonal component of which is vertically homogeneous and endorses the opinion that zonal barotropic currents should be the likely end state of the system in question in the viscid case [5,6,13]. It also reveals the fundamental difference between the isotropic and zonal parts of equilibrium solutions to Eq. (2): The vertical structure of equilibrated zonal flows remains undefined in the theory [9] and may depend on initial conditions.

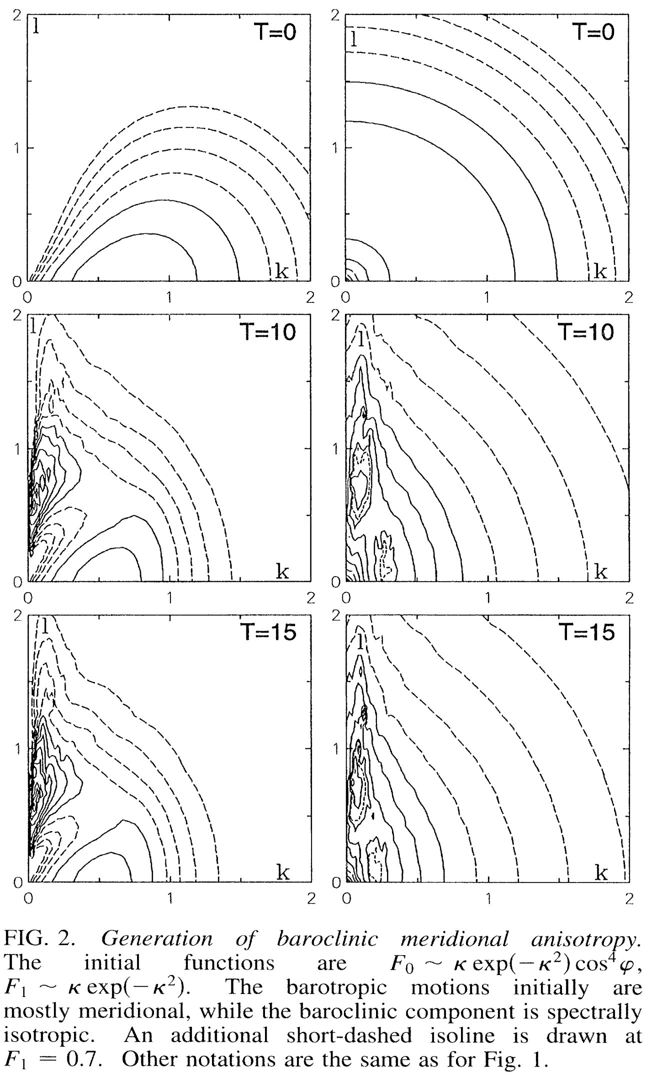

FIG. 2. *Generation of baroclinic meridional anisotropy.* The initial functions are $F_0 \sim \kappa \exp(-\kappa^2) \cos^4 \varphi$, $F_1 \sim \kappa \exp(-\kappa^2)$. The barotropic motions initially are mostly meridional, while the baroclinic component is spectrally isotropic. An additional short-dashed isoline is drawn at $F_1 = 0.7$. Other notations are the same as for Fig. 1.

Generation of baroclinic nearly zonal flow.—The total spectral isotropization of baroclinic flow components cannot be an overall feature of motions in question. Starting from a pure baroclinic flow $[F_0(\vec{\kappa}, 0) = 0]$, the baroclinic zonal anisotropy should be generated, at least, for a limited time interval. This is demonstrated by integrating several initial states with $F_0(\vec{\kappa}, 0) \ll F_1(\vec{\kappa}, 0)$ in the vicinity of the l axis (Fig. 2).

In these case, both the initial and the further evolution of the F_1 field are similar to those of the F_0 field. With time, a zonal peak of F_1 emerges near the l axis and continues to grow during all the computation time. Its disposition and geometrical features are similar to those of the barotropic zonal peak, but it is lower and wider than the barotropic one, and placed at a greater distance from the l axis. The latter features can be explained by the difference of Rossby radii for the modes.

Therefore, the final vertical structure of the (nearly) zonal flow turns out to be strongly dependent on the initial state. Typically, a barotropic zonal flow is excited relatively fast and, after some time, completely suppresses generation of a respective baroclinic motion. However, if an essential shear of the zonal flow is excited for some reason, it will not be damped. For forced dissipative flows this feature is intuitively obvious, but for the system in question (which evolves towards a thermal equilibrium) it is deeply nontrivial.

Large-scale meridional anisotropy.—In the one-layer simulations, the spectral energy exchange is mostly local (i.e., takes place between waves of comparable length) [7,12]. Energy transfer to meridional (directed along the y axis) motion components appears seldom and is identified as the tendency toward an isotropic final state. As striking contrast to these features, a nonlocal energy redistribution to a mostly meridional flow is detected in the later phases of several current experiments. This phenomenon only appears in cases when both the barotropic and the baroclinic nearly zonal flow have gained a certain intensity (Fig. 2).

A spectral peak of some extension along the l axis arises in the vicinity of the k axis in the baroclinic spectrum F_1. Its maximum is located at $\vec{\kappa} \approx (0.3; 0.1)$. It thus represents a mainly meridional flow with the horizontal scale exceeding that of the initial motion several times. It usually remains, at most, less than half the height of the zonal peak, but increases during all the later phases of computations. The peak is evidently unstable, but its smoothing is likely to take a long time and will result in the creation of extreme slowly changing motion components.

The interactions with zonal flow play a crucial role in the formation of this peak. An arbitrary triad $\vec{\kappa} = (k, 0), \vec{\kappa}_1 = (0, 1), \vec{\kappa}_2 = (-k, -1)$ satisfies the resonance conditions $\omega_{110}^{012} = 0$; $\vec{\kappa}_{012} = \vec{0}$ provided the waves with $\vec{\kappa}, \vec{\kappa}_1$ represent the baroclinic and $\vec{\kappa}_2$ the barotropic modes. If both the modes have a significant zonal anisotropy at $k \ll 1$, $l \approx 1$, the main contribution to $\partial F_1(\vec{\kappa})/\partial T$ is proportional to $F_1(\vec{\kappa}_1)[F_0(\vec{\kappa}_2) - F_1(\vec{\kappa})]$ and results in energy transfer from the barotropic nearly zonal flow $F_0(\vec{\kappa}_2)$ to baroclinic large-scale disturbances $F_1(\vec{\kappa})$. Obviously, this mechanism is also active for other triads unless there exist appropriate interacting waves with vectors located close to either spectral peak.

An analogical mechanism (interaction between two harmonics of a fixed mode with the zonal flow) exhibits a tendency of spectral symmetrization [7,12] and appears as a flag of impossibility of nonsymmetric equilibrium distributions. The meridional anisotropy is created by intermodal interactions exclusively. In contrast with the spectral symmetrization, energy may now be redistributed between waves of drastically different wavelengths.

In conclusion, the reinforcement of the zonal component of motion is typical for large-scale flows. The basic effect of barotropization of geophysical flows has been detected from both experimental data and numerical simulations [2,4,7,13]. A deeply interesting peculiarity of the described spectral evolution consists of the suppression of the baroclinic zonal anisotropy (damping of the shear of the nearly zonal flow) by already existing strong barotropic zonal flow. Hence, barotropization of realistic flows may occur selectively, i.e., mainly for their zonal components.

A basically new phenomenon is the possibility to excite intense large-scale mainly meridional motion components. It is directly related to the vertical structure of flows and enables energy cascade to scales much greater than the baroclinic Rossby radius $L_1 = a_1^{-1}$. Thus, for vertically structured flows, energy condensation at $L \approx L_\beta$ is unlikely. Another principal point is that these disturbances gather energy from the barotropic nearly zonal flow while all the other processes seem to support it.

In the dynamics of the oceans, the described phenomenon may become evident in the form of annual or interannual oscillations. A mechanism of creating large-scale nonzonal structures is apparently active quite often in planetary atmospheres (although the two-layer model poorly represents their dynamics). The baroclinic meridional peak can be thought of as a spectral evidence of blocking phenomena or large-scale hot or cold waves (responsible for abnormally warm summers or cold winters). The fact that this peak becomes evident only after the excitation of a multimodal zonal flow probably can be used for prediction of these phenomena.

Interesting in itself is the cascadelike approach to the final state, with alternating generation of slowly changing barotropic nearly zonal flow, then the baroclinic meridional motions and, at the very end, creation of a new phase of extreme slowly changing flow. This peculiarity may be one of the reasons why in direct simulations the two-layer β-plane turbulence needs an enormous time interval to become statistically steady [13].

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