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Self-Organization and Chaos in a Fluidized Bed

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We present experimental evidence that a complex system of particles suspended by upward-moving gas can exhibit low-dimensional bulk behavior. Specifically, we describe large-scale collective particle motion referred to as *slugging* in an industrial device known as a *fluidized bed.* As gas flow increases from zero, the bulk motion evolves from a fixed point to periodic oscillations to oscillations intermittently punctuated by "stutters," which become more frequent as the flow increases further. At the highest flow tested, the behavior becomes extremely complex ("turbulent").

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Recently, there has been renewed interest in fundamental aspects of the dynamics of granular media (e.g., sand) [1]. It has been emphasized that when granular media are excited by mechanical vibrations they display phenomena that are, in many ways, distinct from typical behavior in both fluids and solids. Another situation in which new, interesting phenomena may be expected to arise is that in which granular particles interact strongly with a flowing gas or liquid. An issue of primary interest is the possible existence of global macroscopic behavior in the presence of inherently complex microscopic dynamics.

This Letter describes initial attempts to characterize large-scale collective motion of particles in fluidized beds. Fluidized beds are widely used in the chemical and fossil-fuel processing industries to mix particulate solids and fluids (gases or liquids) [2,3]. A typical fluidized bed consists of a vertically oriented chamber, a *bed* of particulate solids, and a fluid flow distributor at the bottom of the chamber. The fluid flows upward through the particles, creating a drag force that counteracts gravity. With sufficiently high flow, the solids are levitated and move in complex, turbulent patterns (hence the name "fluidized"). This turbulence promotes heat and mass transfer as well as chemical reactions between the fluid and the solids.

In fluidized beds, the spatial length scales of the bulk (macroscopic) motion are typically at most a factor of 10^3 larger than the length scales of individual particle (microscopic) motion. However, the qualitative behavior at each length scale is clearly different. Microscopic motion appears quite complex. The particles are suspended in a turbulent flow of gas, and they collide with each other very frequently. Macroscopic motion, in which large numbers of particles move collectively, appears more organized.

This Letter presents experimental results on the transitions between different macroscopic behaviors as the gas flow is increased. As flow increases, we observe the following sequence: (a) small-amplitude highly complex behavior, (b) large-amplitude approximately periodic behavior, (c) periodic behavior intermittently interrupted by "stutters," and (d) "turbulent" behavior. We believe that (c) may be characterized as low-dimensional chaotic behavior.

The overall magnitude and frequency of the pressure oscillations in slugging beds are of primary interest in industrial settings. Thus we believe that a description of slugging as a low-dimensional, nonlinear process—particularly one that exhibits sensitivity to initial conditions—may have important practical value for engineering process diagnostics and control.

A schematic illustration of our experimental apparatus is shown in Fig. 1. The cylindrical vessel is 10.2 cm in diameter, and the settled bed height is 23.5 cm. The particles used in the experiments described here are uniform 4.5 mm diameter steel spheres. Room-temperature air is metered at constant flow into the plenum chamber below the gas distributor.

Measurements of the bed dynamics typically consist of pressure differentials: (1) between the plenum chamber and atmosphere, and (2) between flush, wall-mounted pressure taps located 10 and 23 cm above the air distributor, respectively. (All of the measurements reported here are from the wall-mounted taps.) Analog signals from the pressure transducers are bandpass filtered (0.1–40 Hz) to remove dc bias, prevent aliasing, and remove any contamination with 60 Hz noise associated with nearby ac equipment. After filtering, the signals are digitized to

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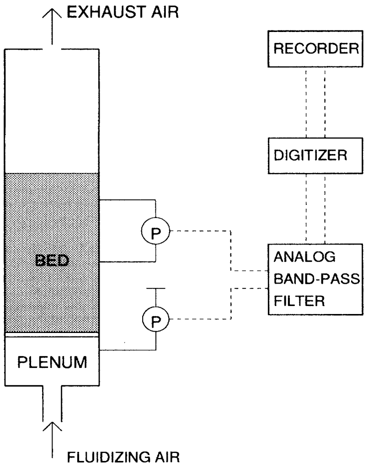

FIG. 1. Experimental fluidized bed setup.

12-bit precision at 200 samples per second. We have experimentally verified that no significant power is present in unfiltered slugging bed signals above 30 Hz. Typical digitized data sets consist of 6×10^4 sample points, representing 300 s of operation.

Fluidized beds are more prone to slugging when a gas (rather than a liquid) is the operating fluid and when the particles are relatively large (e.g., greater than 800 μm in diameter). Here we are concerned with air fluidization of particles that are classified as Geldart group D according to the fluidized-bed literature [4].

The fluidization behavior of group D particles progresses through macroscopically distinct regimes as gas flow increases from zero. These transitions are illustrated in Fig. 2, which depicts the general trend in the standard deviation of a differential pressure measurement over a bed with increasing gas flow. The particles remain still until the flow exceeds the minimum required to overcome gravity (referred to as the *minimum fluidization flow*, U_{mf}). As the gas flow increases, the particles at the top of the bed begin to vibrate but experience no net translation. The layer of vibrating particles deepens with an increasing flow rate, but the movement still occurs at spatial scales on the order of 1–2 particle diameters. Pressure differential measurements across the layer of vibrating particles suggest a complex dynamics involving a large number of interparticle collisions, but these fluctuations are very small and effectively invisible at the scale of observation in Fig. 2.

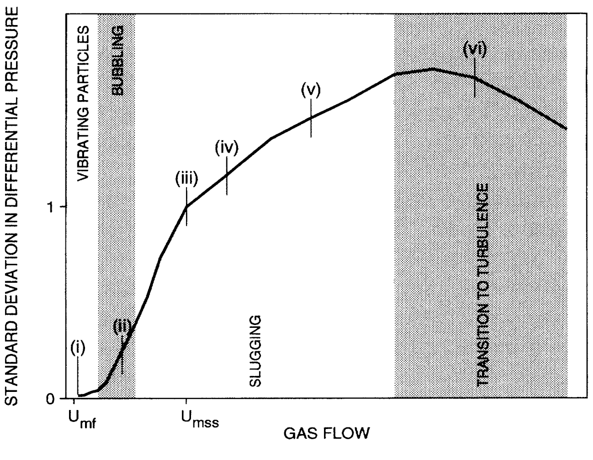

FIG. 2. Variation of differential pressure oscillations with fluidizing gas flow. Qualitatively distinct regimes are indicated along with the location of the example time series shown in Fig. 3.

With further increase in gas flow, macroscopic motion begins in the form of pockets of gas (resembling bubbles in a liquid) traveling upward through the particles, creating the appearance of a boiling liquid. The bubbling regime for group D particles occurs only over a narrow flow range and changes abruptly to slugging at higher flow (see Fig. 2). In slugging, each rising bubble spans nearly the entire cross section of the bed and pushes a large mass of particles in front of it. (We refer to each set of upward-moving particles as a *slug*.) Particles move downward through and around the rising bubble until it reaches the top of the bed, whereupon a settled bed is reestablished, and the cycle repeats.

When the flow is within about 2% of a critical value, the slugging evolves to a limit cycle that is stable to large momentary perturbations. We refer to this condition as *maximum stable slugging* and to the critical gas flow as the *maximum stability flow*, U_{mss}. As flow increases from U_{mss}, the slugging amplitude increases and the time interval between slugs becomes more irregular. We describe the onset of this irregularity as occurring in the form of intermittent stutters in the otherwise periodic slugging pattern. Eventually, as the gas flow reaches a significant fraction of the terminal flow (where particles are blown out of the apparatus), no slugs are visible. Instead, the dominant large-scale structures are strands of particles that form and disappear rapidly. This latter state is called the *turbulent regime* in the fluidized-bed literature.

Figure 3 illustrates a sequence of time series plots and Fourier power spectra for the wall-mounted pressure taps at six different flow conditions: (i) vibrating particles, well below U_{mss} (thin lines), (ii) bubbling, below U_{mss} (dashed lines), (iii) maximum stable slugging, at U_{mss} (thick lines), (iv) occasional stuttering slugs, slightly above U_{mss} (thin lines), (v) frequent stuttering slugs, well above U_{mss} (dashed lines), and (vi) near-turbulent fluidization (thick lines).

The vertical axes in the time series have been scaled to the standard deviation at U_{mss}, and the vertical axes in the power spectra have been normalized to the dominant peak at U_{mss}. At flows below slugging, particle vibrations or small bubbles produce small, complex pressure

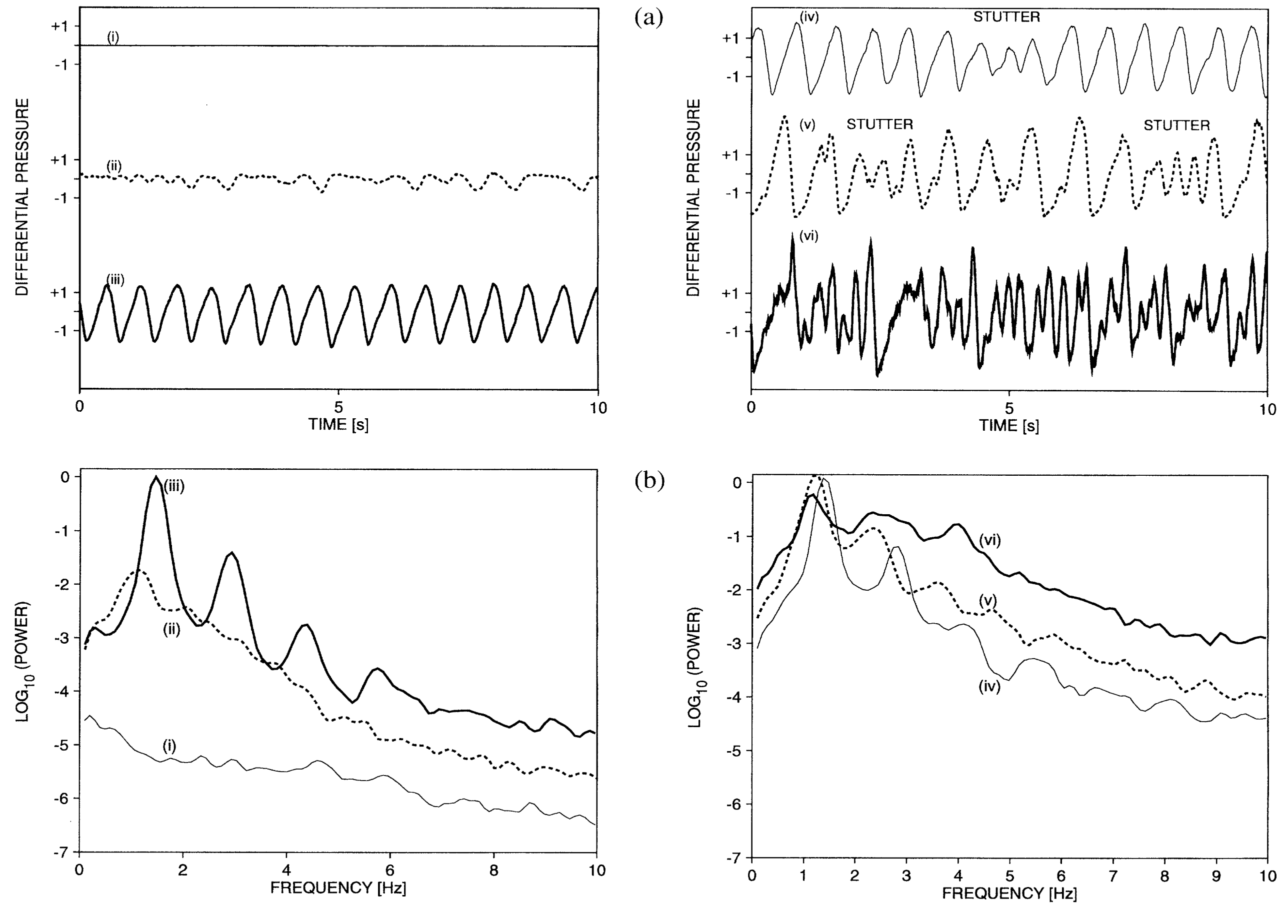

FIG. 3. Time series plots (a) and Fourier power spectra (b) for six flow conditions: (i) vibrating particles, (ii) bubbling, (iii) maximum stable slugging, (iv) occasional stuttering slugs, (v) frequent stuttering slugs, and (vi) near turbulent fluidization.

fluctuations, and the corresponding differential pressure signals (i) and (ii) in Fig. 3(a) have small magnitudes and broadband power spectra.

As gas flow increases toward U_{mss}, larger bubbles form, and the pressure oscillations and resulting power spectrum become simpler and less broadband, respectively. Near U_{mss}, the fluctuating pressure differentials shown in series (iii) in Fig. 3(a) are large in magnitude and almost periodic, resulting in a large spectral peak near 1.4 Hz with distinct harmonics.

For flow rates above U_{mss}, the pressure fluctuations shown in (iv) and (v) in Fig. 3(a) are large but noticeably less regular, and the corresponding power spectra become more broadband. A stutter is clearly seen in the middle of time series (iv) in Fig. 3(a), where there is a sudden reduction in amplitude followed by growing oscillations that return the system to more regular, large-amplitude cycles. Series (v) in Fig. 3(a) includes multiple stutter occurrences typical of higher flow.

Series (vi) depicts the even higher complexity reached as the system approaches the so-called turbulent regime. In this last condition, slugs are indistinct, and the stuttering condition is replaced with a qualitatively different pattern. Groups of particles are instead organized into strands or clusters that rapidly break apart and re-form. This process produces more complicated pressure oscillations at smaller time scales. Figure 3(b) shows that the total power associated with frequencies above 3 Hz increases by at least a factor of 10 as the flow rate increases past U_{mss} to the highest values considered in this experiment.

Stutters are characteristic of gas flows between U_{mss} and the turbulent regime and follow a predictable pattern. Our experimental observations indicate that for some significant distance above U_{mss}, the complexity of the dynamics increases steadily with gas flow and is characterized by more frequent stutters. Figure 4 shows a log-log plot of the mean time interval $\langle T \rangle$ between stutters as a function of ΔU, the excess gas flow above U_{mss}. When $U = U_{\mathrm{mss}}$, there are no stutters. As U increases past U_{mss}, the mean time between stutters decreases as a power of the flow rate $\langle T \rangle \propto \Delta U^{-\gamma}$, with $\gamma \approx 0.73$. These results suggest that there is a transition to a possibly chaotic state by some type of intermittency.

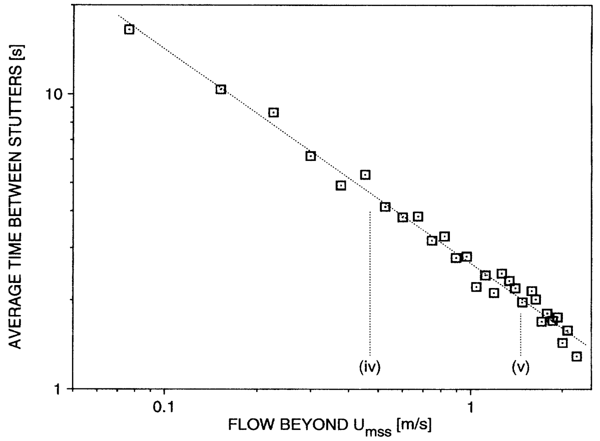

FIG. 4. Variation in average time between stutters as gas flow is increased beyond U_{mss}. The labels indicate the two stuttering conditions depicted in Fig. 3.

We also observe that stutters exhibit sensitivity to initial conditions in the sense that they can be induced or inhibited by making small time-dependent perturbations in the air flow (e.g., by injecting small air pulses at the wall near the bottom of the bed) [5].

In conclusion, the principal finding in this work is the existence of an apparent intermittent transition from periodic to possibly low-dimensional chaotic behavior in slugging regimes in fluidized beds. Although our pressure measurements are "blurred" by microscopic dynamics, we believe that the behavior associated with macroscopic motion such as slugging can be described as having low-dimensional features in the sense that there is a clear transition from no bulk motion to periodic oscillations to apparently chaotic, intermittent bursts as the gas flow is increased [6].

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[3] D. Geldart, *Gas Fluidization Technology* (Wiley, New York, 1986).

[4] D. Geldart, Powder Tech. **7**, 285 (1973).

[5] M. Vasudevan *et al.,* in Proceedings of the 13th International Conference on Fluidized Bed Combustion, Orlando, 1995.

[6] If we regard our pressure measurements as composites of a "low-dimensional" chaotic or periodic process (slugging) with a "high-dimensional" (microscopic) process, it is problematic to estimate quantities such as dimension and Lyapunov spectra, because the underlying dynamics are inherently high-dimensional. It may be possible to circumvent this difficulty by using suitable filtering or embedding techniques. Examples include finite impulse response filters (e.g., simple moving averages), integrated peak sizes (e.g., the interspike interval method [7]), measurement over large spatial dimensions, and choosing the embedding dimension m and the time delay τ so that $(m - 1)\tau$ is approximately equal to the average slugging period. In addition, the data may be "coarse grained" to restrict the resolution of the dynamics to be larger than the underlying microscopic scales.

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