Reduction of Weak Interaction Rates in the Supernova Core

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We reinvestigate the effects of nucleon-nucleon interactions on the semileptonic weak processes that control the neutrino physics in a supernova core. We find clear evidence that any spin or isospin dependent interactions act to reduce the weak interaction rates. Taking the effects of the single pion exchange force as an example, we find large reductions over the whole range of conditions that prevail from the inner core to the neutrino photosphere.

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The theory of the supernova process depends in a number of ways on the rates of neutrino processes within the core and the hot expanding cloud. These rates are important to supernova dynamics [1–3], to setting the parameters of the neutrino pulse that escapes the star [4,5], and to the nucleosynthesis scenario [6,7]. The rates are equally important in the dense interior, where they determine the time scales for cooling and deleptonization, and at much lower densities, where they determine the location of the neutrino photosphere, and, in consequence, the effective temperature of the radiated neutrino spectrum.

Most calculations of macroscopic phenomena have used the free particle cross sections, with the required modifications for Fermi statistics, as the basis for the neutrino transport estimates. Modifications of these formulas in the region of the dense core have been proposed, using Fermi liquid theory [8], and there are some further effects of the nuclear forces that can be calculated from the bulk properties of the matter and that probably reduce the neutral current opacities by 30%–40% even at lower densities of the neutrino photosphere where the nucleons are totally nondegenerate [9]. The effects that we investigate in the present paper are different and, we believe, more important over the whole density range.

We consider a system composed of a nondegenerate gas of nucleons at temperature T, together with electrons (and positrons) in thermal equilibrium with a chemical potential, μ_e. These are the conditions that prevail, post-shock, in much of the region between the neutrinosphere and the dense interior region. We refer to the basic weak reactions, $\nu + n \leftrightarrow e + p$, etc., as quasielastic reactions. When we include interactions between the nucleons in the medium we then obtain, in addition to the quasielastic amplitudes, amplitudes in which there are changes in the state of the medium, which we shall refer to as inelastic channels. It is the inclusive rates of conversion between one lepton state and another that matter in neutrino transport calculations. The key to the reduction of rates is a close relation between the inelastic channels and a medium dependent (and momentum dependent) wave function renormalization, applied to the quasielastic amplitude of the participating nucleon in the reaction. The two contributions tend to cancel, but the negative renormalization contribution is characteristically larger in magnitude than the (positive) inelastic contribution.

This phenomenon, the cancellation of the largest part of quasi-inelastic contributions by what could be called "radiative corrections" to the quasielastic part, is a familiar effect in the perturbation theory of some vacuum rates, for example, bremsstrahlung in the collisions of charged particles, where the infrared divergence cancels when the terms are summed. In media, the calculation of the finite temperature corrections (from the photon and e^+e^- baths) to the rates of weak processes just prior to He synthesis in the early Universe [10] provides a closer analogy to the considerations of the present paper [11]. The necessity for medium dependent wave function renormalization is quite general in any perturbation calculation for processes involving continuum states in an infinitely extended medium, since the asymptotic wave functions never escape the interactions with the medium.

As a demonstration, we first consider the inclusive rate Γ for transformation of a neutrino through the process $\nu_e + n \rightarrow e + p +$ *(excitations or deexcitations of the medium),* where the neutrino has four-momentum, $p^{(\nu)}$, and where we have, for the moment and for illustration, taken the incoming neutron to be at rest and to have no interactions with the medium:

$$\Gamma = (g_w^2 n_n/2E_e)\mathrm{Tr}[\gamma_\mu(1-\gamma_5)\not{p}_e\gamma^\lambda(1-\gamma_5)H(p^{(\nu)})][\delta_0^\mu\delta_\lambda^0 + G_A^2(\delta_\lambda^\mu - \delta_0^\mu\delta_\lambda^0)]. \tag{1}$$

Here n_n is the density of neutrons, and

$$H(p_\nu) = \mathrm{Im}\left[i\int\frac{d^3p\,dE}{(2\pi)^4}(\not{p}_\nu - \not{p} - m_e + i\epsilon)^{-1}G(\boldsymbol{p},E+i\epsilon)\right], \tag{2}$$

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where $G(p,E)$ is the nonrelativistic Green's function for the proton. (In this example, we have, for simplicity, taken the medium to contain no electrons; otherwise an electron Fermi function would appear in the result.) Figure 1 pictures a contribution to Γ form the term in G that contains an interaction with a "spectator" particle in the medium. If we substitute the free Green's function,

$$G_0(\mathbf{p},E) = (E - \mathbf{p}^2/2M)^{-1}, \tag{3}$$

we obtain the usual form for the free reaction rate, with

$$H^{(0)}(p_\nu) = \int d^3p \frac{\gamma_0|\mathbf{p}_\nu - \mathbf{p}| - \boldsymbol{\gamma}\cdot(\mathbf{p}_\nu - \mathbf{p}) - m_e}{(2\pi)^3[(\mathbf{p}_\nu - \mathbf{p})^2 + m_e^2]^{1/2}} \delta\left[E_\nu + \Delta_{np} - [(\mathbf{p}_\nu - \mathbf{p})^2 + m_e^2]^{1/2} - \frac{\mathbf{p}^2}{2M}\right], \tag{4}$$

where Δ_{np} is the protron-neutron mass difference. The interaction with the medium, which is taken to be translationally invariant, is contained in the complete Green's function,

$$G(\mathbf{p},E) = \left[E - \frac{\mathbf{p}^2}{2M} - \Sigma(\mathbf{p},E)\right]^{-1}. \tag{5}$$

We separate the nucleon pole term in $G(\mathbf{p},E)$ as

$$\begin{aligned} G(\mathbf{p},E) &= \frac{[1 - \partial\Sigma(\mathbf{p},E)/\partial E]^{-1}|_{E=\mathbf{p}^2/2M-\delta E(\mathbf{p})}}{E - \mathbf{p}^2/2M - \delta E(\mathbf{p})} \\ &\quad + \text{ higher singularities} \\ &\equiv \frac{Z(\mathbf{p})}{E - E(\mathbf{p})} + \text{ higher singularities}. \end{aligned} \tag{6}$$

The decay function H, of (2), coming from the pole term in (6) alone, is now given by (4) with one modification; the renormalization factor $Z(\mathbf{p})$ is inserted under the integral.

Next we apply these results by calculating the rate for the reverse process, $e^- + p \to \nu + n$, where for the moment only the proton interacts with the nucleons in the medium. The complete reaction rate, averaged over a thermal distribution for the initial proton, is given by

$$\begin{aligned} \Gamma_{\rm av} &= \int d^3p_e\, d^3p_p\, n_e(\mathbf{p}_e) n_p(\mathbf{p}_p) Z(\mathbf{p}_p) \Gamma^{(0)}(\mathbf{p}_e) \\ &\quad + \text{ excitation terms}, \end{aligned} \tag{7}$$

where $\Gamma^{(0)}$ is the unperturbed rate of the reaction. The factor of $Z(p)$ in this expression is assured, by detailed balance, to be the same that we calculated above. We have here made a quasistatic approximation of the nucleons in which we have neglected nucleon motion in the basic weak interaction process. This is a valid approximation when $E_e \approx k_B T$, since the change of the nucleon energy is then of order $E_e(k_B T/M)^{1/2}$, and it is the reason that the function $\Gamma^{(0)}$ in (8) is a function only of the electron energy. But in the excitation terms the recoil in the scattering of nucleons from each other will have to be taken into account, since the energy transfers to and from particular nucleons are of order $k_B T$.

We calculate the function $\Sigma(p,E)$ for the case in which the incoming proton interacts with other nucleons in the bath through a potential, $V(r)$. To second order we obtain

$$\begin{aligned} \Sigma[\mathbf{p},E] &= \int d^3q \frac{d^3k}{(2\pi)^3} \\ &\quad \times \frac{2Mn(\mathbf{q})[V(\mathbf{k})]^2}{2ME - (\mathbf{p} - \mathbf{k})^2 - 2\mathbf{q}\cdot\mathbf{k} - k^2}, \end{aligned} \tag{8}$$

where $n(\mathbf{q})$ is the density function for the scatterers in the medium. The Z factor, to second order, is then

$$Z[\mathbf{p}] = 1 - \int d^3q \frac{d^3k}{(2\pi)^3} \frac{M^2 n(\mathbf{q})[V(\mathbf{k})]^2}{[\mathbf{p}\cdot\mathbf{k} - \mathbf{q}\cdot\mathbf{k} - \mathbf{k}^2]^2}. \tag{9}$$

Next we calculate the excitation contributions to the rate, to second order in V, and add them to the Z corrections of (9) substituted into (7),

$$\begin{aligned} \delta\Gamma_{\rm av} &= \int d^3p_e\, d^3p\, d^3q\, d^3k \frac{\Gamma^{(0)}(\mathbf{p}_e) M^2 n_e(\mathbf{p}_e) n_p(\mathbf{p}) n(\mathbf{q}) |V(\mathbf{k})|^2}{(2\pi)^3[\mathbf{p}\cdot\mathbf{k} - \mathbf{q}\cdot\mathbf{k} - \mathbf{k}^2]^2} \\ &\quad \times \{h(E_e - \Delta_{np} - M^{-1}[\mathbf{k}^2 - \mathbf{p}\cdot\mathbf{k} + \mathbf{q}\cdot\mathbf{k}])[h(E_e - \Delta_{np})]^{-1} - 1\} \end{aligned} \tag{10}$$

where we shall consider two different possibilities for the function $h(x)$, the case in which the original medium does not contain neutrinos, in which case h is just

$$h(x) = x^2\theta(x),$$

and the case in which neutrinos are trapped and are in near thermal equilibrium, with a chemical potential μ_ν,

$$h(x) = x^2\{1 + \exp[(\mu_\nu - x)/T]\}^{-1}. \tag{12}$$

In the trapped case the omission of the theta function means that we have automatically included the transition rate for the reaction $\nu + e^- + p \to n$ as well as for the reaction $e^- + p \to \nu_e + n$. Near the neutrinosphere, of course, neither (11) nor (12) holds. The results in this case should lie between the two limits.

The integrands for the renormalization correction (the -1 in the bracket) and for the excitation-deexcitation correction in (10) are almost identical, and of opposite sign. The excitation or deexcitation terms are saved from canceling the renormalization term only by the fact that the neutrino energy is altered by the amount of energy delivered to, or absorbed from, the medium. The ratio of the h functions, evaluated at different arguments, is a correction to the factor $\Gamma^{(0)}$ to take into account the new

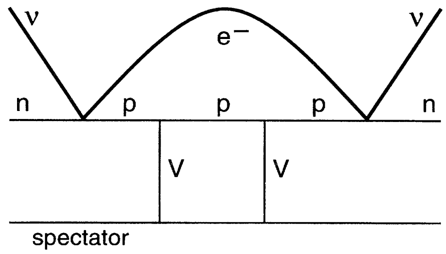

FIG. 1. A graph for $\nu + n \leftrightarrow \nu + n$, where the intermediate proton interacts with a particle in the medium, labeled "spectator." The effects of this interaction on the rate of $\nu + n \leftrightarrow e + p$ are given by (1) and (2), where G is the Green's function for the intermediate proton.

kinematics of these terms. If the greater propensity is for the reacting system to deliver energy to the medium, rather than to absorb it, as we shall see that it is, the effect of the potential, acting between the proton only and the other nucleons in the medium, is unambiguously to reduce the reaction rate.

Now we consider a situation in which the nucleons in the weak process interact with the nucleons in the medium with a spin and isospin independent potential, so that the final neutron in the decay process has the same interaction with the medium as the initial proton. The renormalization and excitation terms in the rate correction from a double interaction of the final nucleon with the medium, as shown in Figs. 2(a) and 2(b), are the same as those from the initial nucleon. But the interference terms and vertex correction, shown in Fig. 2(c), combine to cancel these terms completely.

However, the introduction of spin and isospin dependent potentials removes this cancellation by introducing differing coefficients for the individual terms. For the cases that we shall consider, the spin and isotopic spin factors favor the graphs of Figs. 1(a) and 1(b), with two interactions with the medium on either the initial or final nucleon participating in the weak interaction, over the interference terms and vertex correction. As an example we consider the spin and isospin dependence of the pion exchange force. In the graphs of Figs. 2(a) and 2(b) and considering just the axial vector nucleon current term, we encounter the spin trace $\mathrm{Tr}[(\boldsymbol{\sigma} \cdot \boldsymbol{k})(\boldsymbol{\sigma} \cdot \boldsymbol{k})\sigma_i\sigma_i] = 3k^2$, whereas for the graphs of Fig. 2(c) we obtain instead $\mathrm{Tr}[(\boldsymbol{\sigma} \cdot \boldsymbol{k})\sigma_i(\boldsymbol{\sigma} \cdot \boldsymbol{k})\sigma_i] = -k^2$. On the isospin side, we begin with an isospin density matrix for the nucleons in the medium, $\rho_T = F_p(1 + \tau_3)/2 + (1 - F_p)(1 - \tau_3)/2$, where F_p is the proton fraction. The isotopic factor of the sum of the graphs of Fig. 2(a) or 2(b) is

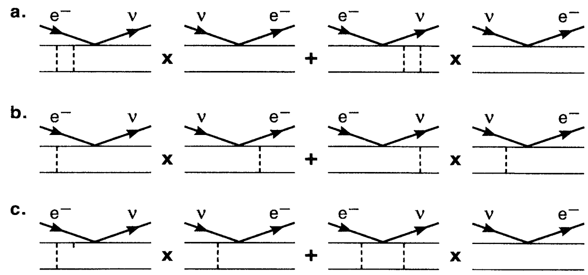

FIG. 2. Graphs for corrections to the rate of $e^- \rightarrow \nu$. The solid unlabeled lines are nucleons. The dotted lines stand for the potential. The relative spin factors are given for the axial current process. (a) Wave function renormalization terms. Isospin factor as in (14). Spin factor = 3. Multiplying function is negative. (b) Excitation terms. Isospin factor as in (14). Spin factor = 3. Multiplying function is positive. (c) An interference term and a vertex term. Isospin factor as in (15). Spin factor = −1. Multiplying functions are the negatives of those from (b) and (a), respectively.

$$(\mathrm{Tr}[\tau_i\tau_j\tau_+\tau_-\rho_T] + \mathrm{Tr}[\tau_+\tau_-\tau_i\tau_j\rho_T])\mathrm{Tr}[\tau_i\tau_j\rho_T] = 6F_p\,, \tag{13}$$

while for either of the graphs of Fig. 2(c) it is

$$(\mathrm{Tr}[\tau_i\tau_+\tau_j\tau_-\rho_T] + \mathrm{Tr}[\tau_+\tau_i\tau_-\tau_j\rho_T])\mathrm{Tr}[\tau_i\tau_j\rho_T] = -2F_p\,, \tag{14}$$

giving additional weight to the "type (a) and (b)" graphs. We define the one pion exchange potential (OPEP) as

$$V_{\mathrm{OPEP}} = \left(\frac{f}{m_\pi}\right)^2 (\boldsymbol{\sigma}^{(1)}\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\boldsymbol{k})\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)}(\boldsymbol{k}^2 + m_\pi^2)^{-1}\,, \tag{15}$$

where $f \approx 1$. In the present calculation we shall not need the usual cut off function in (15) since there turns out to be a natural cutoff at $k \approx (MT)^{1/2}$, which for temperatures of less than 50 MeV is less than any standard phenomenological cutoff. Using the spin and isospin factors calculated above, we write the sum of all the second order correction graphs, coming from the OPEP potential, to the axial vector current contributions (which contribute 3/4 of the unperturbed rate) to the weak process. In (10) we make the following modifications: (a) a multiplying factor of $8F_p/3$ from counting all of the permutations of the graphs with the appropriate isotopic and spin factors; (b) $|V|^2$ replaced by $(f/m_\pi)^4 k^4 (k^2 + m_\pi^2)^{-2}$; and (c) n_p and n are replaced by the nucleon density distribution for the medium, $n_p(p) \rightarrow n(p) = n_N(2\pi MT)^{-3/2}\exp(-p^2/2MT)$. Dividing out the unperturbed rate (itself proportional to $F_p n_N$) we obtain

$$\delta\Gamma/\Gamma^{(0)} = -\tfrac{2}{3}\pi^{-5/2}T^{-1/2}M^{3/2}m_\pi^{-4}f^4 n_N(J/J_0) = -1.2\left(\frac{\rho_{\mathrm{mass}}}{2.7 \times 10^{14}\ \mathrm{g\,cm^{-3}}}\right)\left(\frac{10\ \mathrm{MeV}}{T}\right)^{1/2}\left(\frac{J}{J_0}\right), \tag{16}$$

where

$$J = -(8\pi^2 MT^6)^{-1}\int dE_e\, p_e E_e n(E_e) \int\!\!\int d^3\zeta\, d^3k [\exp(-\boldsymbol{\zeta}^2/4MT)] \times \boldsymbol{k}^4(\boldsymbol{k}^2 + m_\pi^2)^{-2}(\boldsymbol{k}^2 - \boldsymbol{\zeta}\cdot\boldsymbol{k})^{-2}[h(E_e - \Delta_{np} - M^{-1}[\boldsymbol{k}^2 - \boldsymbol{\zeta}\cdot\boldsymbol{k}]) - h(E_e - \Delta_{np})]\,, \tag{17}$$

$$J_0 = T^{-5} \int dE_e\, p_e E_e n(E_e) h(E_e - \Delta_{np}). \qquad (18)$$

We have carried out the integrations in (17) for two cases: (a) The case in which there is no initial neutrino occupation, with the function h given by (11). We consider the separate dependence of J/J_0 on T and μ_e, but it appears that at fixed T the dependence on μ_e is absolutely negligible, with only a 5% difference between the value for $\mu_e = 0$ and for $\mu_e = 5T$ for any of the values of temperature in our range. (b) The case in which the ν_e are trapped, in near equilibrium, with a chemical potential $\mu_e \gg T$. In this limit all of the action is near the Fermi surface for the electron and neutrino. To leading order in T/μ_e, the ratio J/J_0 is independent of μ_e. Thus we can plot the results in either case as a function of temperature alone, as displayed in Fig. 3.

We see that for a value of density near nuclear density, the correction would be more than 100%; clearly a nonperturbative approach is required. But more surprising is the fact that at a density of 10% nuclear density, and, for example, a temperature of 10 MeV, the correction is approximately a 50% reduction, in either of our cases. At the neutrinosphere, at about $\rho = 7 \times 10^{12}$ g cm^{-3} and $T = 5$ MeV, the correction is still at the 15% level. For the vector nucleon current part of the charged current reaction we get a result for $\delta\Gamma/\Gamma$ of the same form, but increased by a factor of 3/2.

We have, in all of the above, omitted the exchange terms that are present when the two initial nucleons or

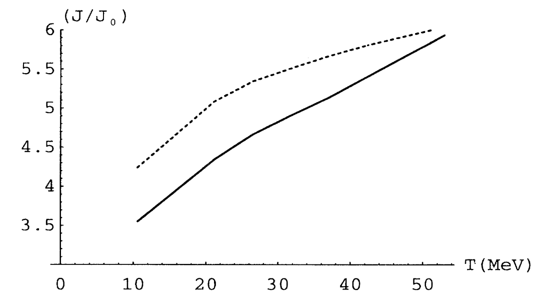

FIG. 3. The values of J/J_0 versus temperature as calculated in two cases. Dotted curve is the case of no neutrino occupancy of the medium. Solid curve is the case of trapped neutrinos, with $\mu_\nu \gg T$.

two final nucleons are in the same spin and isospin state. Roughly speaking, we would expect a reduction of a factor of 4, compared with the direct terms, from the spin and isotopic part of the calculation, and some further reduction that comes from replacing the factor $[V(\boldsymbol{k})]^2$ in (11) by $V(\boldsymbol{k})V(\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{k})$. Calculations confirm that the corrections due to the exchange terms are about 10% of the direct terms. (They are of miscellaneous sign and depend somewhat differently on the temperature.)

All of the above considerations are applicable to the neutral current neutrino cross sections in a medium with modification only of the spin and isospin factors, For example, (13), for the isospin factor, is replaced in Figs. 2(a) and 2(b) by

$$\mathrm{Tr}\{\tau_i\tau_j[\tau_3(1 - \sin^2\theta_W) - \sin^2\theta_W]^2\rho_T\}\mathrm{Tr}[\tau_i\tau_j\rho_T] + \mathrm{Tr}\{[\tau_3(1 - \sin^2\theta_W) - \sin^2\theta_W]^2\tau_i\tau_j\rho_T\}\mathrm{Tr}[\tau_i\tau_j\rho_T]. \qquad (19)$$

We find reductions about 50% as large as in the case of the charged current reactions, for the dominant axial vector nucleon current (A) term.

It seems likely that the single pion exchange force will make the greatest modification in the weak rates. Other spin or isospin dependent forces between the nucleons will also give rise to a decrease in rates, as is possible to see by looking at the internal quantum number traces in the graphs of Fig. 2 for all possibilities; the weights of graphs of Figs. 2(a) and 2(b) are always greater than those for Fig. 2(c). Therefore it seems unlikely that there would exist a mechanism that would undo or greatly mitigate the substantial decreases that we find from the mechanism of the present paper.

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