Reducibility and Thermal Scaling of Charge Distributions in Multifragmentation

L. Phair, K. Tso, R. Ghetti, G. J. Wozniak, and L. G. Moretto
Nuclear Science Division, Lawrence Berkeley Laboratory, Berkeley, California 94720

R. T. de Souza,* D. R. Bowman,† N. Carlin,‡ C. K. Gelbke, W. G. Gong,§ Y. D. Kim,‖ M. A. Lisa,§ W. G. Lynch, G. F. Peaslee,¶ M. B. Tsang, and F. Zhu**
National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824
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A strong thermal signature is found in the charge distributions associated with multifragmentation from the reaction ^{36}Ar + ^{197}Au at $E/A = 110$ MeV. The n-fold charge distributions are reducible to the onefold charge distributions through a simple scaling that is dictated by fold number and charge conservation.

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Historically the charge (mass) distribution has played and still plays a very important role in multifragmentation. Since the inception of the studies on this subject, the near power-law shape of the charge and mass distributions was considered an indication of criticality for the hot nuclear fluid produced in light ion and heavy ion collisions [1,2]. More modern studies still infer critical behavior from the moments of the charge distribution [3–8]. Furthermore, a charge distribution is readily predicted by most models and easily testable.

Recently, it has been experimentally observed in ^{36}Ar + ^{197}Au reactions that for any value of the transverse energy E_t the n-fragment emission probability P_n is reducible to the one-fragment emission probability p through a binomial distribution [9]

$$P_n^m = \frac{m!}{n!(m-n)!} p^n (1-p)^{m-n}. \qquad (1)$$

This empirical evidence indicates that multifragmentation can be thought of as a special combination of nearly independent fragment emissions. The binomial combination of the elementary probabilities points to a combinatorial structure associated with a timelike or spacelike one-dimensional sequence. It was also found that the log of such one-fragment emission probabilities ($\log p$) plotted vs $1/\sqrt{E_t}$ (Arrhenius plot) gives a remarkably straight line. This linear dependence is strongly suggestive of a thermal nature for p,

$$p = e^{-B/T}, \qquad (2)$$

under the assumption that the temperature $T \propto \sqrt{E^*}$ where E^* is the excitation energy. These observations were made with data integrated over a broad range of fragment atomic numbers ($3 \le Z \le 20$). The difficulty of a thermal interpretation of the probability p averaged over Z was tentatively resolved by observing that if B is weakly (polynomial) dependent on Z then

$$p = \int e^{-1/T(B_0 + aZ^s)} dZ = \left(\frac{T}{a}\right)^{1/s} e^{-B_0/T}, \qquad (3)$$

and therefore p retains the form of Eq. (2).

These aspects of reducibility and thermal scaling in the integrated fragment emission probabilities lead naturally to the question: Is the charge distribution itself reducible and scalable? In particular, what is the charge distribution form that satisfies the condition of reducibility and of thermal dependence? Strong hints of this are seen when the lower Z cutoff is increased in the intermediate mass fragment (IMF) definition. The resulting fragment multiplicities are still binomially distributed and the Arrhenius plots become steeper in accordance with the barrier B in Eq. (2) increasing with Z.

In what follows we will show that experimental charge distributions do, in fact, show most interesting reducibility and thermal scaling properties.

Let us first consider the aspect of reducibility as it applies to the charge distributions. In its broadest form, reducibility demands that the probability $p(Z)$, from which an event of n fragments is generated by m trials, is the same at every step of extraction. The consequence of this extreme reducibility is straightforward: the charge distribution for the onefold events is the same as that for the n-fold events and equal to the singles distributions, i.e.,

$$P_{(1)}(Z) = P_{(n)}(Z) = P_{\text{singles}}(Z) = p(Z). \qquad (4)$$

We now consider the consequences of the thermal dependence of p [9] on the charge distributions. If the onefold = n-fold = singles distributions is thermal, then

$$P(Z) \propto e^{-B(Z)/T} \qquad (5)$$

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or $T \ln P(Z) \propto -B(Z)$. This suggests that, under the usual assumption $E_t \propto E^*$ [9], the function

$$\sqrt{E_t}\,\ln P(Z) = D(Z) \tag{6}$$

should be independent of E_t.

In the ^{36}Ar + ^{197}Au reaction considered here, as in other reactions [10,11], the charge distributions are empirically found to be nearly exponential functions of Z,

$$P_n(Z) \propto e^{-\alpha_n Z}, \tag{7}$$

as shown in Fig. 1. In light of the above considerations, we would expect for α_n the simple dependence

$$\alpha_n \propto \frac{1}{T} \propto \frac{1}{\sqrt{E_t}} \tag{8}$$

for all folds n. Thus a plot of α_n vs $1/\sqrt{E_t}$ should give nearly straight lines. This is shown in Fig. 2 for ^{36}Ar + ^{197}Au at $E/A = 110$ MeV.

The expectation of thermal scaling appears to be met quite satisfactorily. For each value of n the exponent α_n shows the linear dependence on $1/\sqrt{E_t}$ anticipated in Eq. (8). On the other hand, the extreme reducibility condition demanded by Eq. (4), namely, that $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, is not met. Rather than collapsing on a single straight line, the values of α_n for the different fragment multiplicities are offset one with respect to another by what appears to be a constant quantity.

In fact, one can fit all of the data remarkably well, assuming for α_n the form

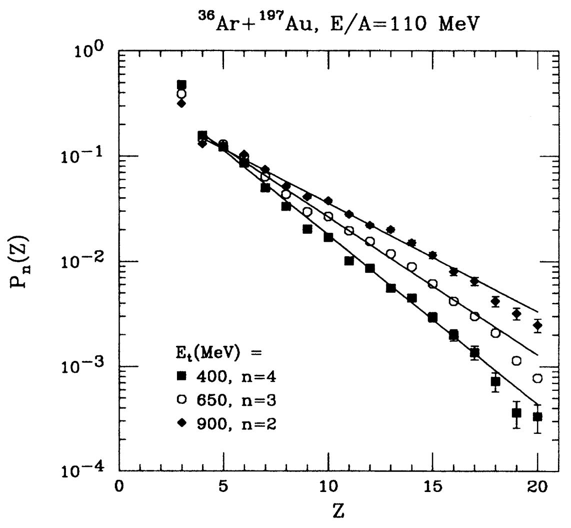

FIG. 1. The n-fold charge distributions $P_n(Z)$ for intermediate mass fragments (IMF: $3 \le Z \le 20$) are plotted for the indicated cuts on transverse energy E_t and IMF multiplicity n. The width of the cuts ΔE_t is 37.5 MeV. The solid lines are exponential fits over the range $Z = 4-20$.

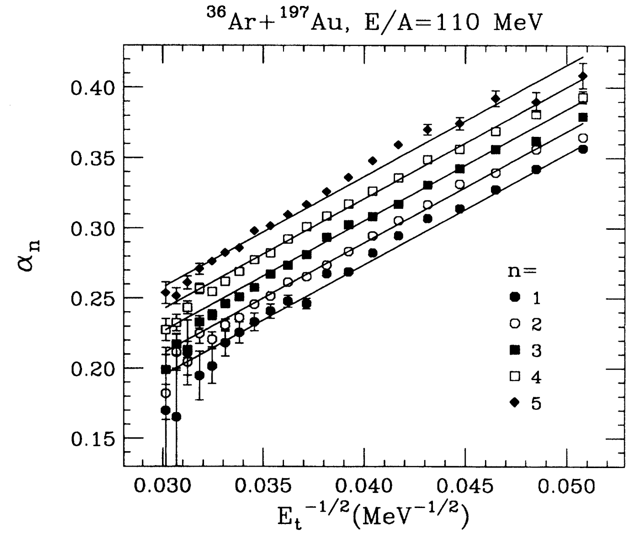

FIG. 2. The exponential fit parameter α_n [from fits to the charge distributions, see Eq. (7)] is plotted as a function of $1/\sqrt{E_t}$. The solid lines are a fit to the values of α_n using Eq. (9).

$$\alpha_n = \frac{K'}{\sqrt{E_t}} + nc\,, \tag{9}$$

which implies

$$\alpha_n = \frac{K}{T} + nc\,, \tag{10}$$

or, more generally, for the Z distribution

$$P_n(Z) \propto e^{-[B(Z)/T]-ncZ}. \tag{11}$$

Thus, we expect a more general reducibility expression for the charge distribution of any form to be

$$[\ln P_n(Z) + ncZ]\sqrt{E_t} = F(Z) \tag{12}$$

for all values of n and E_t. This equation indicates that it should be possible to reduce the charge distributions associated with any intermediate mass fragment multiplicity to the charge distribution of the singles. As a demonstration of this reducibility, we have compared $P_n(Z)$ and $F(Z)$ in Figs. 1 and 3. Figure 1 compares three charge distributions for different cuts on E_t and n; their slopes are clearly different. The reduced quantity $F(Z)$, on the other hand, collapses to a single line in Fig. 3.

We stress that the reduced quantity in Eq. (12) is *independent* of the functional form of the charge distribution. However, we have used the fact that the charge distributions are well described by exponential fits in the ^{36}Ar + ^{197}Au reaction to summarize the reducibility of an enormous amount of data. Nearly one hundred different charge distributions are represented in Fig. 2. We feel this

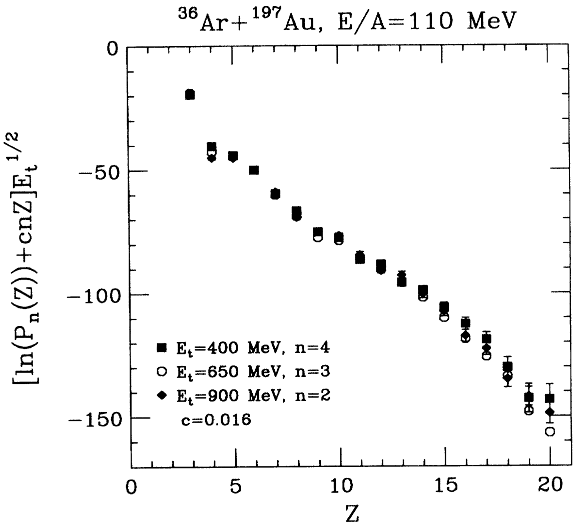

FIG. 3. The "reduced" charge distributions [see Eq. (12)] are plotted for the same cuts on E_t and n as Fig. 1. The different data sets are normalized at $Z = 6$. The value of $c = 0.016$ is the spacing between the curves shown in Fig. 2.

is equally as impressive as the reducibility demonstrated directly in Figs. 1 and 3, where for practical purposes we are only able to demonstrate reducibility for a few different charge distributions.

What is the origin of the regular offset that separates the curves in Fig. 2? The general form of Eq. (11) suggests the presence of an entropy term that does not depend explicitly on temperature. The general expression for the free energy (in terms of enthalpy H, temperature T, and entropy S)

$$\Delta G = \Delta H(Z) - T\Delta S(Z) \tag{13}$$

leads to the distribution

$$P(Z) \propto e^{-[\Delta H(Z)/T]+\Delta S}. \tag{14}$$

Typically, ΔS is of topological or combinatorial origin. For instance, a factor of this sort would appear in the isomerization of a molecule involving a change of symmetry. In our specific case ΔS may point to an asymptotic combinatorial structure of the multifragmentation process in the high temperature limit. As an example, we consider the Euler problem of an integer to be written as the sum of smaller integers, and calculate the resulting integer distribution. Specifically, let us consider an integer Z_0 to be broken into n pieces. Let n_Z be the number of pieces of size Z. The most likely value of n_Z can be obtained by extremization of the function [12]

$$e^{-I} = e^{\sum[n_Z \ln n_Z - n_Z]+K\sum n_Z Z+\gamma\sum n_Z}, \tag{15}$$

where the Lagrange multipliers K and γ are associated with the constraints

$$\sum n_Z Z = Z_0, \quad \sum n_Z = n. \tag{16}$$

From the extremization we obtain

$$\frac{\partial I}{\partial n_Z} = \ln n_Z + KZ + \gamma = 0 \tag{17}$$

or

$$n_Z = e^{-KZ-\gamma}. \tag{18}$$

The constraints now read

$$Z_0 = \sum Z e^{-KZ-\gamma} \sim \frac{e^{-\gamma}}{K^2}, \tag{19}$$

$$n = \sum e^{-KZ-\gamma} \sim \frac{e^{-\gamma}}{K}, \tag{20}$$

from which

$$n_Z = \frac{n^2}{Z_0}\, e^{-nZ/Z_0}. \tag{21}$$

This expression has the correct asymptotic structure for $T \longrightarrow \infty$ required by Eq. (11). The significance of this form is transparent. First, the overall scale for the fragment size is set by the total charge Z_0. Second, for a specific multiplicity n, the scale is reduced by a factor n to the value Z_0/n.

Thus the offset introduced in Eq. (11) with increasing the multiplicity n may just be due to this scale reduction. If this is so, the quantity c in Eq. (11) takes the meaning $c = 1/Z_0$. The empirical value from Fig. 2 is $c \approx 0.016$, which corresponds to a value of $Z_0 \approx 60$ which is quite reasonable for the source size.

The implications of the experimental evidence presented above are far reaching. On the one hand, the thermal features observed generally in multifragmentation (thermal population of bound and unbound [13,14] excited states and slope parameters of Maxwell-Boltzmann velocity spectra [15]; for a review see [16]) and specifically in the n-fragment emission probabilities for the ^{36}Ar + ^{197}Au reaction [9] extend consistently to the charge distributions and strengthen the hypothesis of phase space dominance in multifragmentation. On the other hand, the reducibility of the n-fold-event charge distributions to that of the singles distribution highlights the near independence of individual fragment emission, limited only by the constraint of charge conservation.

In summary, we have found for multifragmentation produced in the ^{36}Ar + ^{197}Au reaction at $E/A = 110$ MeV (1) strong evidence for a thermal scaling of the Z distributions; (2) reducibility of the n-fold distributions to the onefold distributions through Eq. (11); and that (3) the structure of the reducibility equation is essentially given by a simple rescaling associated with the multiplicity and the source size.

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*Present address: Department of Chemistry, Indiana University, Bloomington, IN 47405.

†Present address: Chalk River Laboratories, Chalk River, Ontario, Canada K0J 1J0.

‡Present address: Instituto de Fisica, Universidade de Sao Paulo, C.P. 20516, CEP 01498, Sao Paulo, Brazil.

§Present address: Lawrence Berkeley Laboratory, Berkeley, CA 94720.

‖Present address: National Laboratory for High Energy Physics, 1-1 Oho, Tsukuba, Ibaraki 305, Japan.

¶Present address: Physics Department, Hope College, Holland, MI 49423.

**Present address: Brookhaven National Laboratory, Upton, NY 19973.

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