Anomalous Diffusion in Chaotic Scattering

T. Srokowski* and M. Ploszajczak
Grand Accélérateur National d'Ions Lourds (GANIL), B.P. 5027, F-14021 Caen Cedex, France
(Received 7 October 1994)

The anomalous diffusion is found for peripheral collision of atomic nuclei described in the framework of the molecular dynamics. Similarly as for chaotic billiards, the long free paths are the source of the long-time correlations and the anomalous diffusion. Consequences of this finding for the energy dissipation in deep-inelastic collisions and the dynamics of fission in hot nuclei are discussed.

PACS numbers: 24.60.Lz, 25.70.–z

Chaotic properties of atomic nuclei are well known from studies of compound nucleus resonances [1]. In application to nuclear collisions, these properties have recently been discussed in connection with chaotic scattering [2,3]. In the framework of molecular dynamics (MD), two colliding nuclei regarded as a dynamical system form a chaotic transient; i.e., they abide close together with the survival probability decaying exponentially in time. The system formed in this way can be interpreted as a compound nucleus [3]. This result comes out of the MD model provided the collision is sufficiently violent to get both the fast memory loss and the equilibration, corresponding to the excitation of many overlapping resonances (Ericson fluctuations), as well as the Lorentzian shape of the energy autocorrelation function of an S-matrix element $C_{ij}(\epsilon) = \langle S_{ij}^*(E)\, S_{ij}(E + \epsilon)\rangle_E$ [4].

A different picture is expected if decay proceeds through a small number of open channels [5]. In this case, the scattering is nonhyperbolic and the survival probability is algebraic. Such behavior results from the stickiness of KAM tori existing in this case [6]. The decay is slow because the trajectory must cross a hierarchy of cantori inhibiting the transport [7]. Both semiclassical and quantum calculations show that the energy autocorrelation function $C(\epsilon)$ has a cusp at $\epsilon = 0$, i.e., $dC(\epsilon)/d\epsilon \longrightarrow \infty$ as $\epsilon \longrightarrow 0$ [4]. This means that the fine scale fluctuations of the S matrix are greatly enhanced. The presence of cantori also affects the velocity autocorrelation function $C(t)$ which decreases at a slower rate than in the purely chaotic case [8]. Transport through cantori can be described in the language of the diffusion process. Such a diffusion is anomalously enhanced, as has been found in modeling the two-dimensional solids [9]. In this case the variance of position, $\sigma^2(t)$, rises with time faster than linearly, and the diffusion coefficient $D = \lim_{t\to\infty} \sigma^2(t)/2t$ is infinite. One also obtains a "$1/f$" power spectrum of velocity fluctuations. The anomalous diffusive behavior relates to many physical problems of general application [10,11]. In the MD approach, the anomalous diffusion could appear in peripheral collisions, because in this case the projectile and target preserve their identity for a long time; i.e., the correlations fall off slowly. This expectation is supported by the observation of the power-law decay $p(t) \sim t^{-z}$ with $z = 4.56$ for the quasielastic reaction $^{12}C + {}^{12}C$ at angular momentum $l = 16\hbar$ and energy $E_{\mathrm{c.m.}} = 20$ MeV [3]. However, this value of the exponent z is much larger than the typical value $1 < z < 2$ implied by the mechanism of transport through cantori [12]. It is also doubtful that such a chaotic and unstable system has any regular orbit to which trajectories could stick leading to the algebraic decay. The question therefore remains: What is the origin of this power-law survival probability?

In the following, we address this problem by studying the diffusive features in the MD model of nuclear reactions for peripheral collisions. We will show that the motion in phase space has indeed a diffusive component for large angular momenta and that, similarly as for chaotic billiards, there exist long-time correlations which are a source of both the algebraic decay and the anomalous diffusion. Hence, the transport features of MD for peripheral collisions do not require the existence of the KAM tori to explain the observed algebraic decay.

Let us consider the collision of atomic nuclei consisting of some elementary constituents interacting via the two-body van der Waals–type potential with the attracting component, the Coulomb tail, and the strongly repulsive core at small distances. In the following, we regard α particles as such elementary constituents and take the potential from the adiabatic time-dependent Hartree-Fock calculations [13]. The ground state of both projectile and target is defined as a statistical ensemble; i.e., the phase space coordinates of all particles are sampled uniformly, taking into account the energy, linear momentum, and angular momentum conservation [14]. Within this framework, various static properties of α-cluster nuclei [14] (density distributions, binding energies, separation energies of α particles) as well as the fusion cross section for the reaction $^{12}C + {}^{12}C$ at low bombarding energies [3] are correctly reproduced. The collision is represented by an ensemble of events with the same collective quantities, such as the relative energy and the impact parameter, but with different internal coordinates.

For large impact parameters we expect that the system will rotate like a molecule, slowly transferring the energy and particles (mass). A decisive factor in that

0031-9007/95/75(2)/209(4)$06.00

process is violent collisions of particles at the border between the projectile and target nuclei, due to the repulsive core in the α-α potential. The rest of the system continues its quasifree, rotational motion. We also expect that the memory of initial conditions is kept for a long time. To check this assertion we calculate the velocity autocorrelation function $C^*(t) = \langle \mathbf{v}(t_0) \cdot \mathbf{v}(t) \rangle$ for the reaction ^{12}C + ^{12}C at $l = 16\hbar$ and $E_{\text{c.m.}} = 20$ MeV, where $\mathbf{v}(t)$ is the 18-dimensional velocity vector and $\langle\ \rangle$ means from now on the average over an ensemble of events. For such high angular momenta, one expects a particularly small number of open decay channels [15] and, therefore, both the existence of algebraic survival probability [5,16] and the formation of a long-lived orbiting complex [17]. $C^*(t)$ oscillates with almost constant frequency but with diminishing amplitude [Fig. 1(a)]. The oscillating pattern reflects the rotation in the space of collective variables. Since this rotation is somewhat trivial, we get rid of it, redefining the autocorrelation function: $C(t) = C^*(t) \cos^{-1}[\omega(t)\, t]$. The value of ω, roughly equal to the collective angular velocity of the binary system, must be carefully chosen to avoid singularities. We have determined ω from extrema and zeros of $C^*(t)$, linearly extrapolating between them. Figure 1(b) shows that $C(t) \sim t^{-\gamma}$ with $\gamma = 1$. The mean square displacement is given by the Green-Kubo formula [18]:

$$\sigma^2(t) \equiv \langle [\mathbf{r}(t) - \mathbf{r}(t_0)]^2 \rangle = 2 \int_{t_0}^{t} (t - \tau)\, C(\tau)\, d\tau \propto t \ln \frac{t}{t_0} - t + t_0 . \quad (1)$$

Hence, the diffusion is anomalous and D diverges logarithmically. The same dependence holds for mean square displacement in the velocity space. The Fourier transform of $C(t)$ gives the power spectrum $S(\omega)$. In our case, $S(\omega) \sim |\ln \omega|$ and $S(\omega = 0) \equiv 2D = \infty$. In contrast to the $1/f^2$ spectrum of a random walk, this spectrum is convergent when integrated to zero frequency and is divergent when integrated to infinite frequency. There is only a finite amount of power at low frequencies, but when we look at shorter time scales the noise has no well-defined short-term mean. This resembles the features of the $1/f$ spectrum of the "flicker noise" which is also divergent when integrated to infinite frequency. The same logarithmic power spectrum has been found for the Lorentz gas of hard disks (the extended Sinai billiard) [19]. The long-time correlations in that system originate from the long free paths of particles and not from the presence of regular structures [20]. The same mechanism operates for peripheral collisions in MD. Violent hard-core collisions between α particles belonging to different nuclei are necessary to transfer the energy from one nucleus to the other one. For large impact parameters such collisions are rare, and most of the α particles are affected only by weak interactions from the side of their neighbors in the same nucleus. In the sense of memory loss, this behavior corresponds to the free motion in the Sinai billiard. During a long period of time, the collision of two nuclei resembles an integrable, two-body system. In this case the hyperbolic instabilities are not responsible for the dynamical features of the system. Hence, the Lyapunov exponents and the fractal dimensions do not determine the escape process, and the survival probability is no longer exponential. Thus, in spite of the full chaos of the system [21], the peripheral collision mimics the nonhyperbolic scattering.

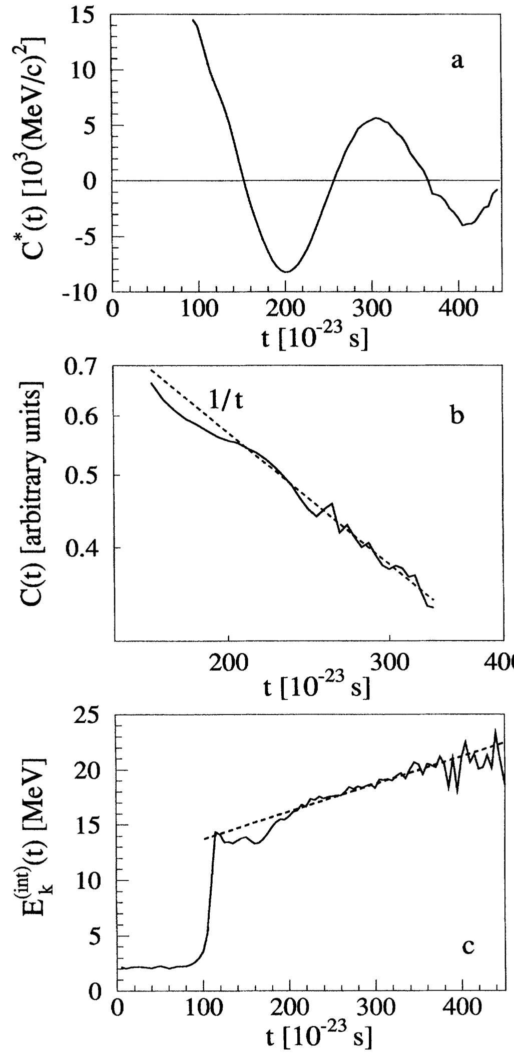

FIG. 1. Temporal characteristics for the grazing collision ^{12}C + ^{12}C at $E_{\text{c.m.}} = 20$ MeV and $l = 16\hbar$. (a) The velocity autocorrelation function $C^*(t)$. (b) The renormalized autocorrelation function $C(t) = C^*(t) \cos^{-1}[\omega(t)\, t]$. The dashed straight line denotes the function $1/t$. (c) The internal kinetic energy calculated from the velocity variance. The initial energy corresponds to the energy of an internal motion in the ground state. The dashed straight line is plotted to guide the eye.

In the momentum space, diffusion causes the kinetic energy transfer from collective to internal degrees of freedom. The internal kinetic energy can be defined as $E_k^{(\mathrm{int})} = (m/2)\langle \hat{\mathbf{v}}^2(t)\rangle$, where $\hat{\mathbf{v}}(t) = \mathbf{v}(t) - \langle \mathbf{v}(t)\rangle$ and m is the mass of the α particle. The differentiation of the Green-Kubo formula (1), applied to $\hat{\mathbf{v}}$, implies that the energy dissipation rate is closely related to the autocorrelation function $C(t)$:

$$dE_k^{(\mathrm{int})}/dt \propto \int_{t_0}^{t} C(\tau)\, d\tau\,. \tag{2}$$

The result of the evaluation of $E_k^{(\mathrm{int})}$ as the variance of $\hat{\mathbf{v}}(t)$ is presented in Fig. 1(c). Most of the energy is rapidly dissipated in the entrance channel. Let us call this dissipated energy $E^{(\mathrm{ec})}$. The distance of the closest approach is reached at about 10^{-21} s. Also the diffusion process starts at $t_0 \simeq 10^{-21}$ s [see Fig. 1(c)]. For later times, according to (1), $E_k^{(\mathrm{int})}$ increases as

$$E_k^{(\mathrm{int})} = \alpha\left(t \ln \frac{t}{t_0} - t + t_0\right), \tag{3}$$

where α is a constant. For the range of times considered, the deviation of calculated $E_k^{(\mathrm{int})}$ from the linear dependence on time is hardly visible. The finite and small available phase space cuts short the diffusion. For later times, (3) is no longer valid and $E_k^{(\mathrm{int})}$ saturates. The final collective energy for the system decaying at time t is $E(t) = E_{\mathrm{c.m.}} - E^{(\mathrm{ec})} - E_k^{(\mathrm{int})}(t)$. The decay time t, in turn, is governed by the decay probability $p(t)$. Combining $p(t)$ with $E(t)$, we get the energy spectrum in the parametric form:

$$p[E(t)] = \frac{p(t)}{dE/dt} = \frac{p(t)}{\alpha \ln(t/t_0)}\,. \tag{4}$$

It is easy to see that $dp/dE|_{E=E(t_0)} = \infty$ and, therefore, the energy spectrum has the cusp at $E = E(t_0)$. In fact, the formula (1) implies that the cusp behavior is a generic property of diffusive systems, no matter what form of $p(t)$ is actually taken and how fast the correlations fall off.

For a fixed angular momentum, the energy spectrum [(3) and (4)] has a peak at $E(t_0)$ at relatively low energy because a large amount of energy, $E^{(\mathrm{ec})}$, has been lost before in the entrance channel. The mass and atomic numbers of ejectiles are similar to those of the projectile and target. An integration over angular momentum has a smoothing effect and broadens the peak. Such a picture is typical for deep-inelastic heavy-ion collisions. For near grazing collisions in light and medium heavy-ion nuclei, one has evidence for an orbiting dinuclear system which is formed after the damping of initial energy and angular momentum and evolves through the exchange of nucleons (α particles) into different configurations of a dinuclear system [17]. The dependence of the internal energy on the average lifetime (3) can be measured directly, utilizing experimental techniques of atomic and nuclear physics [22]. This would help to verify the anomalous diffusion mechanism. Independently, the detailed analysis of the shape of experimental spectra and the comparison with (4) would give information about the decay probability $p(t)$ of the rotating system.

The long-time rotation of weakly interacting reaction constituents is the necessary condition for the anomalous diffusion to take place. This condition is not fulfilled for central collisions. The correlation function in this case drops rapidly to zero [Fig. 2(a)] and the entire energy is dissipated at the very beginning of the reaction [Fig. 2(b)]. The equilibration time is short, allowing the statistical properties of the compound nucleus to show up at the early stage of the reaction. Therefore, we predict two different mechanisms: the fusion dominated process at small angular momenta [3] with a large number of open channels and the diffusive process associated with orbiting with a small number of decay channels. The prediction of coexistence of these two mechanisms for light heavy-ion collisions is in agreement with the coupled-channel analysis [23] and supported by experiments [24].

Figure 1(c) indicates the presence of various mechanisms of excitation. The dissipation in the entrance channel could be described by a velocity-dependent friction force. However, the dissipative process at this stage is partially reversible, especially for central collisions [see Fig. 2(b)]. Such mixed elastoplastic behavior of nuclear

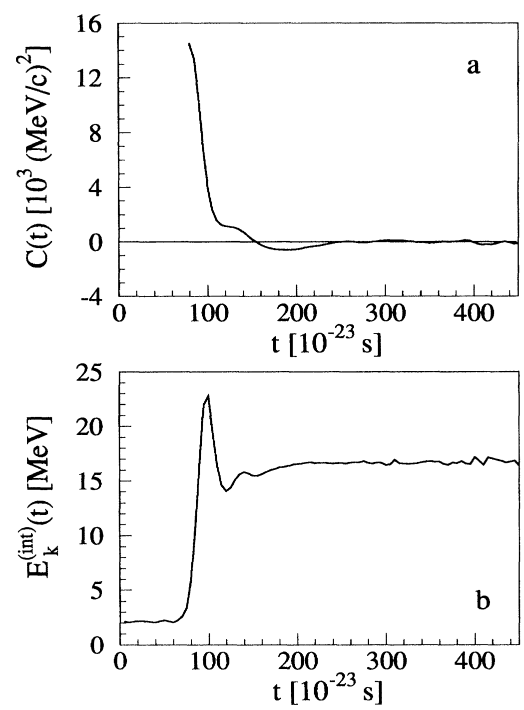

FIG. 2. Temporal characteristics for the central collision $^{12}C + {}^{12}C$ at $E_{\mathrm{c.m.}} = 20$ MeV and $l = 0$. (a) The velocity autocorrelation function. (b) The internal kinetic energy as a function of time.

matter, linking properties of elastic solids and viscous fluids, used to be attributed to the KAM tori in the phase space [25]. Since in the MD there are *no* regular structures, the present model shows that the elastoplastic behavior can be observed also in the fully chaotic systems. The dissipation due to the diffusion process, emerging at later times for peripheral collisions, is different. According to (2), its speed grows monotonically with time: The dissipation rate does not stabilize, as would be the case for the normal diffusion, but instead it diverges logarithmically.

The divergence of the dissipation rate may have important consequences for the semiphenomenological description of heavy-ion collisions and, in particular, hot fission process [22]. Going from central to peripheral collisions or from saddle to scission in the fission process, one usually employs the same model of diffusion changing only the geometrical constraints such as the size of the "window" or the deformation of the system. In view of the above results, this alone may not be sufficient as the change of the time scales involved modifies the nature of the dissipation process. Consequently, for peripheral collisions or for strongly elongated shapes from saddle to scission, it is more appropriate to use the time- or coordinate-dependent diffusion coefficient. Fröbrich, Gontchar, and Mavlitov [26] have noticed that to describe the neutron multiplicities using the overdamped Langevin equation, one has to increase the dissipation coefficient by about 1 order of magnitude while going from high fissility systems (short fission-path length) to low fissility systems (long fission-path length). This observation finds a natural explanation in the anomalous diffusion mechanism for these strongly deformed shapes.

The similarity of the diffusive behavior for systems as different as the MD and the Lorentz gas of hard disks follows from the fact that the power-law tail of the velocity autocorrelation function is due to the existence of long free paths. This behavior is insensitive both to the details of the potential, in particular, to its short distance features, and to the existence of the topological holes induced by the Pauli blocking. Hence, one expects that the above results hold for a broad class of systems, including those of the antisymmetrized molecular dynamics [27] and the quantum molecular dynamics [28]. Investigations in this direction are in progress.

We are grateful to Y. Abe, G. Do Dang, A. Lépine-Szily, and J. Okolowicz for useful discussions and suggestions. This work was partly supported by KBN Grant No. 2 P302 157 04.

*On leave from Institute of Nuclear Physics, ul. Radzikowskiego 152, PL-31-342 Kraków, Poland.

[1] T. Brody, J. Flores, J. B. French, P. A. Mello, A. Panday, and S. S. M. Wong, Rev. Mod. Phys. **53**, 385 (1981).
[2] M. Baldo, E. G. Lanza, and A. Rapisarda, Chaos **3**, 691 (1993); C. H. Dasso, M. Gallardo, and M. Saraceno, Nucl. Phys. **A549**, 265 (1992).
[3] T. Srokowski, J. Okołowicz, S. Drożdż, and A. Budzanowski, Phys. Rev. Lett. **71**, 2867 (1993); (to be published).
[4] Y.-C. Lai, R. Blümel, E. Ott, and C. Grebogi, Phys. Rev. Lett. **68**, 3491 (1992).
[5] H. L. Harney, F.-M. Dittes, and A. Müller, Ann. Phys. (N.Y.) **220**, 159 (1992).
[6] S. Drożdż, J. Okołowicz, and T. Srokowski, Phys. Rev. E **48**, 4851 (1993).
[7] J. D. Meiss and E. Ott, Phys. Rev. Lett. **55**, 2741 (1985).
[8] C. F. F. Karney, Physica (Amsterdam) **8D**, 360 (1983).
[9] T. Geisel, A. Zacherl, and G. Radons, Phys. Rev. Lett. **59**, 2503 (1987); Z. Phys. B **71**, 117 (1988).
[10] J.-P. Bouchaud and A. Georges, Phys. Rep. **195**, 127 (1990).
[11] D. Kusnezov, Phys. Rev. Lett. **72**, 1990 (1994).
[12] J. D. Meiss, Rev. Mod. Phys. **64**, 795 (1992).
[13] D. Provoost, F. Grümmer, K. Goeke, and P. G. Reinhardt, Nucl. Phys. **A431**, 139 (1984).
[14] K. Möhring, T. Srokowski, and D. H. E. Gross, Nucl. Phys. **A533**, 333 (1991).
[15] R. M. Anjos, N. Added, N. Carlin, L. Fante, Jr., M. C. S. Figueira, R. Matheus, E. M. Szanto, C. Tereiro, and Szanto de Toledo, Phys. Rev. C **49**, 2018 (1994).
[16] C. H. Lewenkopf and H. A. Weidenmüller, Ann. Phys. **212**, 53 (1991); F. M. Izrailev, D. Saher, and V. V. Sokolov, Phys. Rev. E **48**, 130 (1993).
[17] D. Shapira, R. Novotny, Y. D. Chan, K. A. Erb, J. L. C. Ford, Jr., J. C. Peng, and J. D. Moses, Phys. Lett. **114B**, 111 (1982).
[18] D. A. McQuarrie, *Statistical Mechanics* (Harper & Row, New York, 1976).
[19] A. Zacherl, T. Geisel, J. Nierwetberg, and G. Radons, Phys. Lett. **114A**, 317 (1986).
[20] In fact, the KAM theorem does not apply to billiards which are fully chaotic.
[21] The system considered in this paper has the full spectrum of positive Lyapunov exponents.
[22] D. Hilscher and H. Rossner, Ann. Phys. (Paris) **17**, 471 (1992).
[23] C. Beck, Y. Abe, N. Aissaoui, B. Djerroud, and F. Haas, Phys. Rev. C **49**, 2618 (1994).
[24] A. Ray, D. Shapira, J. Gomez del Campo, H. J. Kim, C. Beck, B. Djerroud, B. Heusch, D. Blumenthal, and B. Shivakumar, Phys. Rev. C **44**, 514 (1991).
[25] W. J. Swiatecki, Nucl. Phys. **A488**, 375c (1988).
[26] P. Fröbrich, I. I. Gontchar, and N. D. Mavlitov, Nucl. Phys. **A556**, 281 (1993).
[27] W. Bauhoff, E. Caurier, B. Grammaticos, and M. Płoszajczak, Phys. Rev. C **32**, 1915 (1985); A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, Phys. Rev. Lett. **68**, 2898 (1992).
[28] J. Aichelin and H. Stöcker, Phys. Lett. B **176**, 14 (1986).