

## Cosmology with Ultralight Pseudo Nambu-Goldstone Bosons

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We explore the cosmological implications of an ultralight pseudo Nambu-Goldstone boson. With global spontaneous symmetry breaking scale  $f \approx 10^{18}$  GeV and explicit breaking scale comparable to Mikheyev-Smirnov-Wolfenstein neutrino masses,  $M \sim 10^{-3}$  eV, such a field, which acquires a mass  $m_\phi \sim M^2/f \sim H_0$ , would currently dominate the energy density of the Universe. The field acts as an effective cosmological constant before relaxing into a condensate of nonrelativistic bosons. Such a model can reconcile dynamical estimates of the density parameter,  $\Omega_m \sim 0.2$ , with a spatially flat universe, yielding  $H_0 t_0 \approx 1$  consistent with limits from gravitational lens statistics.

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Recently, the cosmological constant ( $\Lambda$ ) has come back into vogue. Dynamical estimates of the mass density from galaxy clusters suggest that  $\Omega_m = 0.2 \pm 0.1$  for the matter that clusters gravitationally [where  $\Omega(t)$  is the ratio of the mean mass density of the Universe to the critical Einstein-de Sitter density] [1]. However, the inflation scenario for the early Universe suggests that  $\Omega_{\text{tot}} = 1$ . A cosmological constant is one way to resolve the discrepancy between  $\Omega_m$  and  $\Omega_{\text{tot}}$ .

The second motivation for the revival of the cosmological constant is the “age crisis” for  $\Omega_m = 1$  models. Current estimates of the Hubble parameter, most recently from Cepheid variable stars in the Virgo cluster [2], are (with notable exceptions) converging to  $H_0 \approx 80 \pm 15$  km/sec Mpc [3], while estimates of the age of the Universe from globular clusters are holding at  $t_{\text{gc}} \approx 13 - 15$  Gyr or more [4]. Thus, the product  $H_0 t_0 = 1.14 [H_0/(80 \text{ km/sec Mpc})][t_0/(14 \text{ Gyr})]$  is high compared to the Einstein-de Sitter value  $H_0 t_0 = 2/3$ . With a cosmological constant,  $H_0 t_0$  can be significantly larger: e.g.,  $H_0 t_0 = 1.076$  for  $\Omega_\Lambda \equiv \Lambda/3H_0^2 = 0.8 = 1 - \Omega_m$ .  $\Lambda$ -dominated models for a large-scale structure with cold dark matter (CDM) and a scale-invariant spectrum of primordial density perturbations (as predicted by inflation) also provide a better fit to observed galaxy clustering than the “standard”  $\Omega_m = 1$  CDM model [5].

However, models with a relic cosmological constant have problems of their own. In the context of quantum field theory, there is as yet no understanding of why the vacuum energy density  $\rho_{\text{vac}} = \Lambda/8\pi G \approx (0.003 \text{ eV})^4 \Omega_\Lambda$  is not of order  $M_{\text{Pl}}^4$  or at least of order the supersymmetry breaking scale,  $M_{\text{SUSY}}^4 \sim 1 \text{ TeV}^4$ , both many orders of magnitude larger. Classically, there is no understanding of why  $\rho_{\text{vac}}$  is not of order the scale of vacuum condensates, e.g.,  $M_{\text{GUT}}^4$ ,  $M_W^4$ , or  $f_\pi^4$ . A vacuum density of order  $(0.003 \text{ eV})^4$  appears to require cancellation between large numbers to very high precision. In

addition, it implies that we are observing the Universe just at the special epoch when  $\Omega_m \sim \Omega_\Lambda$ .

Moreover, such models now face strong observational constraints from gravitational lens statistics: In a spatially flat universe with nonzero  $\Lambda$ , the lensing optical depth at moderate redshift is substantially larger than in the Einstein-de Sitter model [6]. In the Hubble Space Telescope (HST) Snapshot Survey for lensed quasars, there are four lens candidates in a sample of 502 QSOs; from these data, the bound  $\Omega_\Lambda \lesssim 0.6-0.8$  has been inferred [7]. For  $\Omega_\Lambda = 1 - \Omega_0 < 0.7$ , the product  $H_0 t_0 < 0.96$ . If the age  $t_0 \geq 14$  Gyr, this implies  $H_0 < 67$  km/sec Mpc, somewhat below the recent  $H_0$  determinations.

It is conventional to assume that the fundamental vacuum energy of the Universe is zero, due to some as yet not understood mechanism, and that this new physical mechanism “commutes” with other dynamical sources of energy density. This assumption underlies the inflation scenario, which relies on a temporary nonzero vacuum energy. If this hypothesis is the case, then the effective vacuum energy at any epoch will be dominated by the heaviest fields that have not yet relaxed to their vacuum state; at late times, such fields must be very light.

Adopting this working hypothesis, in this Letter we explore the consequences of an ultralight pseudo Nambu-Goldstone boson (hereafter, PNGB) field which is *currently* (i) relaxing to its vacuum state and (ii) dominating the energy density of the Universe. PNGB models are characterized by two mass scales, a spontaneous scale and an explicit symmetry breaking scale; the two dynamical conditions above fix these two mass scales to values which are “reasonable” from the viewpoint of particle physics. Thus, we may have an explanation for the “coincidence” that the vacuum energy is dynamically important at the present epoch. In this model, the cosmological constant is evanescent, eventually converting into scalar field oscillations that redshift as nonrelativistic matter.

In particle physics, the best known example of a PNGB is the  $\pi$  meson. An example of a very light hypothetical PNGB is the axion [8], which arises when a global  $U(1)_{\text{PQ}}$  symmetry, introduced to solve the strong  $CP$  problem, is spontaneously broken by the vacuum expectation value of a complex scalar,  $\langle \Phi \rangle = f_a e^{ia/f_a}$ ; at the scale  $f_a$ , the axion  $a$  is a massless Nambu-Goldstone boson. QCD instantons explicitly break the symmetry at the scale  $f_\pi \sim 100$  MeV, generating the axion mass,  $m_a \sim O(m_\pi f_\pi/f_a)$ . Astrophysical and cosmological arguments constrain the global symmetry breaking scale to lie in a narrow window around  $f_a \sim 10^{10} - 10^{12}$  GeV.

The axion is a particular instance of a more general phenomenon that includes familons, Majorons [9], and related objects [10]. In all these models, the key ingredients are the scales of spontaneous symmetry breaking  $f$  and explicit symmetry breaking  $\mu$ ; the PNGB mass is  $m_\phi \sim \mu^2/f$ . Reference [11] introduced a class of PNGBs closely related to familons (called “schizons”),

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \sum_{j=0}^{N-1} \bar{\nu}_j i \gamma^\mu \partial_\mu \nu_j + (m_0 + \epsilon e^{i(\phi/f + 2\pi j/N)}) \bar{\nu}_{jL} \nu_{jR} + \text{H.c.}, \quad (1)$$

where  $\nu_{(R,L)}$  are, respectively, right- and left-handed projections,  $\nu_{(R,L)} = (1 \pm \gamma^5)\nu/2$ . In the limit  $m_0 \rightarrow 0$ ,  $\mathcal{L}$  is invariant under a continuous  $U(1)$  chiral symmetry, which is explicitly broken by  $m_0$  to the residual  $Z_N$  discrete symmetry:  $\nu_j \rightarrow \nu_{j+1}$ ,  $\nu_{N-1} \rightarrow \nu_0$ ,  $\phi \rightarrow \phi + 2\pi j f/N$ . The induced one-loop correction, with cutoff  $\Lambda < f$ , is

$$\mathcal{L}_{1\text{-loop}} = \sum_{j=0}^{N-1} \frac{M_j^4}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_j^2}\right), \quad (2)$$

where

$$M_j^2 = m_0^2 + \epsilon^2 + 2m_0\epsilon \cos\left(\frac{\phi}{f} + \frac{2\pi j}{N}\right), \quad (3)$$

which respects the discrete symmetry. For  $N = 2$ , the leading contribution is logarithmic divergent, and the induced PNGB mass is of order  $m_\phi \sim m_0\epsilon/f$ ; if  $\epsilon \sim m_0 \sim m_\nu$ , then  $m_\phi \sim m_\nu^2/f$ . For  $N > 2$ , the sum  $\sum_j M_j^4$  is independent of  $\phi$ ; thus, the  $\phi$  potential is explicitly calculable, and one again finds  $m_\phi \sim m_\nu^2/f$ .

We are thus led to study the cosmological evolution of a light scalar field  $\phi$  with effective Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - M^4 [\cos(\phi/f) + 1]. \quad (4)$$

The theory is determined by two mass scales,  $M$ , which from (1) is expected to be of order a light neutrino mass, and  $f$ , the global symmetry breaking scale. Since  $\phi$  will turn out to be extremely light, we assume that it is the only classical field that has not yet reached its vacuum expectation value. In accordance with our working hypothesis, the constant term in the PNGB potential has been chosen to ensure that the vacuum energy vanishes at the minimum of the  $\phi$  potential. We assume that the spatial fluctuation  $\delta\phi(\vec{x}, t)$  is small compared

with masses  $m_\phi \approx m_{\text{fermion}}^2/f$ . Models in which  $m_{\text{fermion}}$  is associated with a hypothetical neutrino mass,  $m_\nu \sim 0.001 - 0.01$  eV, and  $f \sim 10^{15} - 10^{19}$  GeV, were studied in Ref. [12] in the context of late time phase transitions [13] and form the theoretical basis for the present work.

From the viewpoint of quantum field theory, PNGBs are the only way to have naturally ultralow mass, spin-0 particles. “Technically” natural small mass scales are those that are protected by symmetries: When the small masses are set to zero, they cannot be generated in any order of perturbation theory [14]. For PNGBs, when the symmetry breaking scale  $\mu$  is set to zero, the symmetry becomes exact, and radiative corrections do not yield an explicit symmetry breaking term (they are “multiplicative” of the scale  $\mu$ ). For example, in the schizon models, the small mass  $m_\phi$  is protected by fermionic chiral symmetries (and additional discrete symmetries).

As an example, consider the  $Z_N$ -invariant low-energy effective chiral Lagrangian for  $N$  neutrinos [12],

to the spatially homogeneous mode  $\phi(t) = \langle \phi(\vec{x}, t) \rangle$ , as would be expected after inflation if the reheat temperature  $T_{\text{RH}} < f$ : In this case, aside from inflation-induced quantum fluctuations (which correspond to isocurvature density perturbations [15]),  $\phi$  will be homogeneous over many present Hubble volumes. Since we will be interested in the case  $f \sim M_{\text{Pl}}$ , this is not a significant restriction. For simplicity we assume that finite-temperature corrections to (4) are unimportant at the epochs of interest (different from axions, for which finite-temperature effects do affect the field evolution). The scalar equation of motion is then  $\ddot{\phi} + 3H\dot{\phi} - (M^4/f)\sin(\phi/f) = 0$ , where  $H^2 = (\dot{a}/a)^2 = (8\pi/3M_{\text{Pl}}^2)(\rho_m + \rho_\phi)$  for a spatially flat universe,  $\Omega_m + \Omega_\phi = 1$ , and  $a(t)$  is the cosmic scale factor.

The cosmic evolution of  $\phi$  is determined by the ratio of its mass,  $m_\phi \sim M^2/f$ , to the expansion rate,  $H(t)$ . Let  $t_x$  denote the epoch when the field becomes dynamical,  $m_\phi = 3H(t_x)$ , with corresponding redshift  $1 + z_x = a(t_0)/a(t_x) = (M^2/3H_0 f)^{2/3}$ ; for comparison, the transition from radiation to matter domination is at  $z_{\text{eq}} \approx 2.3 \times 10^4 \Omega_m h^2$  [where  $h = H_0/(100 \text{ km/sec Mpc})$ ]. The  $f$ - $M$  parameter space is shown in Fig. 1. To the left of the diagonal line  $m_\phi = 3H_0$  (the region denoted by “ $\Lambda$ ”),  $\phi$  is still frozen by Hubble damping to its initial value (in general, displaced from the minimum at  $\phi_m = \pi f$ ) and acts as a cosmological constant, with  $\rho_{\text{vac}} \sim M^4$ . To the right of this diagonal line,  $m_\phi > 3H_0$ : here,  $\phi$  becomes dynamical before the present and currently redshifts as non-relativistic matter,  $\rho_\phi \sim a^{-3}$ . In this dynamical region,  $\Omega_\phi \approx 24\pi(f/M_{\text{Pl}})^2$ , independent of  $M$  [12] [assuming the initial field value  $\phi_i = O(1)f$ ]; the horizontal line at  $f = 1.4 \times 10^{18}$  GeV indicates the limit  $\Omega_\phi = 1$ . In the

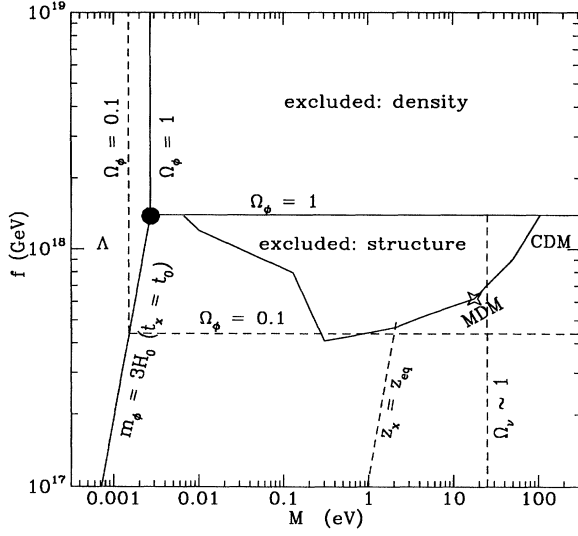


FIG. 1. The PnGB model parameter space.

frozen ( $\Lambda$ ) region, on the other hand,  $\Omega_\phi$  is determined by  $M^4$ , independent of  $f$ ; the bound  $\Omega_\phi = 1$  is indicated by the vertical line.

Focus on the dynamical region in the right-hand portion of Fig. 1. If  $\phi$  dominates the energy density of the Universe, the growth of density perturbations is suppressed for physical wave numbers larger than the ‘‘Jeans scale’’ [16]  $k_J \approx m_\phi [\phi_m(t)/M_{\text{Pl}}]^{1/2}$ , where  $\phi_m(t) \sim f[(1+z(t))/(1+z_x)]^{3/2}$  is the amplitude of the field oscillations at  $z(t) < z_x$ . We can express the resulting perturbation power spectrum in terms of the standard CDM spectrum as  $P(k) = P_{\text{CDM}}(k)F^2(k)$ ; for  $z_x > z_{\text{eq}}$ , the suppression factor due to the scalar field is [17]

$$F(k) \approx \left( \frac{1+z_{\text{eq}}}{1+z_*(k)} \right)^{(5/4)[(1-24\Omega_\phi/25)^{1/2}-1]} \\ = \left[ \left( \frac{110h \text{ eV}}{M} \right) \left( \frac{k}{1h \text{ Mpc}^{-1}} \right) \right]^{5[(1-72.4(f/M_{\text{Pl}})^2)^{1/2}-1]}.$$

Here,  $1+z_*(k) = [(M/k)(3H_0/M_{\text{Pl}})]^{1/2}$  is the redshift at which the physical wave number  $k_{\text{phys}} = k(1+z)$  drops below  $k_J$ , so that scalar field perturbations on that scale can begin to grow. Thus,  $M$  sets the scale where the power spectrum turns down from the CDM spectrum, and  $f$  determines the spectral slope  $n$  of the suppression factor,  $F(k) \sim k^{-n}$  with  $-4 \leq n \leq 0$  (for  $\Omega_\phi \leq 0.2$ ,  $n \approx 12\Omega_\phi/5$ ). For galaxies and quasars to form at moderate redshift, the power at small scales should not be too strongly suppressed compared to standard CDM. We therefore impose the approximate bound  $F(k = 1.6h \text{ Mpc}^{-1}) > 0.3$ , which corresponds to the curved boundary in Fig. 1. To the right of this excluded region (the area marked CDM),  $\phi$  is an ordinary cold dark matter candidate, a lighter version of the axion. In the area marked MDM, the effects of  $\phi$  on the small-scale power spectrum are similar to those of a light neutrino in the mixed dark matter model: At the point marked by

the star, the variance of the density field smoothed with a top-hat window of radius  $R = 8h^{-1} \text{ Mpc}$  is  $\sigma_8(\phi) \approx \sigma_8^{\text{CDM}}/2$ . When the amplitude is normalized to COBE on large scales, this yields  $\sigma_8(\phi) \approx 0.6$ , as suggested by the abundance of rich clusters of galaxies and the small-scale pairwise velocity dispersion of galaxies. In this region of parameter space, the neutrinos of mass  $m_\nu \sim M \sim$  several eV could play a dynamical role in structure formation as well.

For the remainder of this Letter, we focus on the parameter region near the bullet in Fig. 1: here the field becomes dynamical recently,  $z_x \sim 0-3$ , i.e.,  $m_\phi = M^2/f \leq 3H_0 = 6.4 \times 10^{-33} \text{ eV}$ ; this has new consequences for the classical cosmological tests [18]. For the PnGB energy density to be dynamically relevant,  $\rho_\phi(t_0) \sim \rho_{\text{crit}}(t_0)$ , or  $M^4 \approx 3H_0^2 M_{\text{Pl}}^2/8\pi$ . These conditions fix the two mass scales in the theory to be  $f \geq M_{\text{Pl}}/(24\pi)^{1/2} \approx 10^{18} \text{ GeV}$  and  $M \approx 3 \times 10^{-3} h^{1/2} \text{ eV}$ . Above, we discussed models for light PnGBs with these mass scales: The scale  $M$  is comparable to that for light neutrinos in the Mikheyev-Smirnov-Wolfenstein solution to the solar neutrino problem.

Figure 2 shows several examples of the scalar field evolution (with  $\dot{\phi}_i = 0$ , since the field is Hubble damped at early times). The numerical evolution starts at  $\rho_m/M^4 \gg 1$  ( $\Omega_m \approx 1 \gg \Omega_\phi$ ) in the matter-dominated epoch. At early times, the field is frozen to its initial value, and the evolution tracks that of a cosmological constant model (labeled ‘‘vac’’ in Fig. 2). At  $t \sim t_x$ , the field begins to roll classically; on a time scale comparable to the expansion

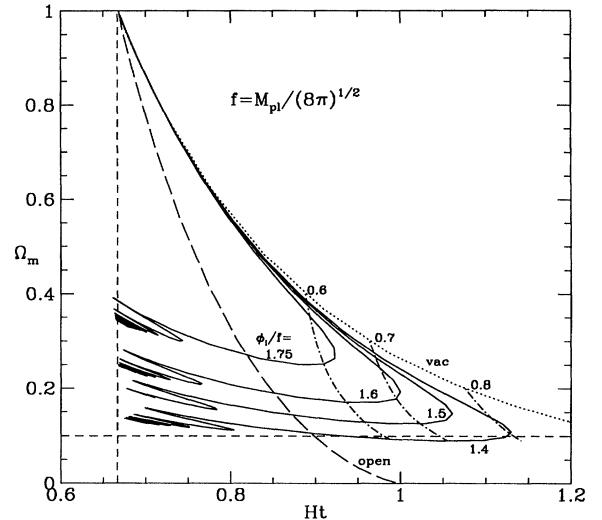


FIG. 2.  $\Omega_m = 1 - \Omega_\phi$  vs  $Ht$ , for  $f = M_{\text{Pl}}/\sqrt{8\pi}$ . Solid curves correspond to initial field values  $\phi_i/f = 1.4, 1.5, 1.6$ , and  $1.75$ . The vertical dashed line denotes the Einstein–de Sitter value  $Ht = 2/3$ , the horizontal dashed line the lower bound  $\Omega_m = 0.1$  from mass estimates; the dotted curve (labeled ‘‘vac’’) shows the evolution for a  $\Lambda$  model, and the long-dashed curve corresponds to an open model with  $\Omega_\phi = 0$ . Dot-dashed curves (0.6, 0.7, and 0.8) bracket the constraints from lensed QSOs.

time,  $Ht$  reaches a maximum and subsequently falls toward  $2/3$  as  $\phi$  undergoes damped oscillations about  $\phi_m$ . The evolutionary tracks are universal: A shift in the scale  $f$  accompanied by an appropriate rescaling of  $\phi_i$  leads to essentially identical tracks.

The observational consequences of this model follow when one identifies the present epoch  $t_0$  on an evolutionary track—this corresponds to fixing the mass  $M$ . Given the dynamical lower bound  $\Omega_m \geq 0.1$ , the lower “branch” of the evolution is excluded if the initial value of the field is below some value, e.g.,  $\phi_i/f \approx 1.3$  for  $f = M_{\text{Pl}}/\sqrt{8\pi}$ . Physically, for small  $\phi_i/f$ , the Universe undergoes several  $e$ -foldings of inflation before  $\phi$  begins to oscillate, diluting the density of nonrelativistic matter. Consequently, to achieve  $H_0 t_0 \sim 1$ , the present epoch must be in the vicinity of the “nose” of the evolutionary track, which corresponds to  $t_x \sim t_0$ .

As with  $\Lambda$  models, the scalar field models are constrained by the statistics of gravitationally lensed quasars. We computed the number of lensed QSOs expected in the HST Snapshot Survey [19] for  $\Lambda$  models with  $\Omega_\Lambda = 0.6, 0.7,$  and  $0.8$ ; along the three dot-dashed curves in Fig. 2, the number of expected lensed QSOs in the PNGB models are equal to these three values. Since different assumptions about galaxy models yield different lensing bounds, we show the limits corresponding to these three cases to cover the spread in the literature [7]. The lensing upper bound on  $H_0 t_0$  is increased in the PNGB models compared to the  $\Lambda$  models; imposing  $\Omega_m > 0.1$ , the bound on  $H_0 t_0$  can be relaxed by 7%–10%.

We have presented a class of models that gives rise to natural ultralight pseudo Nambu-Goldstone bosons. With spontaneous and explicit symmetry breaking scales comparable to those plausibly expected in particle physics models, the resulting PNGB becomes dynamical at recent epochs and currently dominates the energy density of the Universe. Such a field acts as a form of smoothly distributed dark matter, with a stress tensor at the current epoch intermediate between that of the vacuum and nonrelativistic matter. Such a model “explains” the coincidence between matter and vacuum energy density in terms of particle physics mass scales, reconciles low dynamical mass estimates,  $\Omega_m \sim 0.2$ , with a spatially flat universe, and does somewhat better than a cosmological constant at alleviating the age crisis for spatially flat cosmologies while remaining within the observational bounds imposed by gravitational lens statistics.

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*Note added.*—After this work was completed, we became aware of related work by Fukugita and Yanagida [20] that considers an axion model for the nondynamical ( $\Lambda$ ) region of Fig. 1.

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