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Quantum Phase of Induced Dipoles Moving in a Magnetic Field

Haiqing Wei* and Rushan Han
Department of Physics, Peking University, Beijing 100871, People's Republic of China

Xiuqing Wei
Clinical Department, The Sun Yat-Sen University of Medical Sciences, Guangzhou 510089, People's Republic of China
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We derive the quantum phase accompanying a neutral particle (with no permanent electric dipole moment) moving in an area where a nonuniform electric field and a uniform magnetic field are simultaneously applied. Such a quantum phase can be ascribed to the sum of the Aharonov-Bohm phases of the charges constituting the neutral particle. An experimental configuration much more practical than a previous suggestion is proposed to test for the phase. The possible nonzero superfluid velocity in the ground state of the ^{4}He condensate is theoretically demonstrated.

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Aharonov and Bohm predicted in 1959 [1] and later in 1960 Shambers proved experimentally [2] that a charged particle moving in the presence of a vector potential acquires a quantum phase (AB effect) [3]. After this, Aharonov and Casher showed the AC effect [4], which is dual to the AB effect: a particle with a magnetic moment moving in an electric field obtains the AC phase due to the spin-orbit interaction. Recently, Wilkens derived a quantum phase accompanying an electric dipole moving in a magnetic field [5]. After appreciating his elegant theory, one should notice that Wilkens's experiment is not convenient to perform for it needs a radial magnetic field.

In this Letter, first we derive the quantum phase accompanying a neutral particle (with no permanent electric dipole moment), which moves in an area where both a radial electric field and a uniform magnetic field are applied. We then propose an experimental configuration and show its feasibility by means of numerical estimates. Finally, the possible nonzero superfluid velocity in the ground state of the ^{4}He condensate is theoretically demonstrated.

In the cylindrical coordinates (r, θ, z), a cylinder electrode with total charge Q lies at $r = 0$. It yields an electric field

$$\mathbf{E} = \frac{k}{r}\,\mathbf{e}_r\,, \tag{1}$$

where $k = Q/2\pi\epsilon_0 h$, ϵ_0 is the permittivity of the vacuum, h is the height of the cylinder, $h \gg r_0$, where r_0 is the radius of the electrode. There is also a uniform magnetic field $\mathbf{B}$ along the z direction

$$\mathbf{B} = B\mathbf{e}_z\,. \tag{2}$$

A neutral particle (with no permanent electric dipole moment) of mass M moving with the velocity $\mathbf{V}$ will be polarized and hence carry an induced electric dipole

$$\mathbf{d} = \alpha(\mathbf{E} + \mathbf{V} \times \mathbf{B})\,, \tag{3}$$

where α is the polarizability. The Lagrangian of the particle may be written

$$\mathcal{L} = \tfrac{1}{2}M\mathbf{V}^2 + \tfrac{1}{2}\alpha(\mathbf{E} + \mathbf{V} \times \mathbf{B})^2. \tag{4}$$

If the motion of the particle is constrained in the (r, θ) plane, i.e., $\mathbf{V} \perp \mathbf{B}$, Eq. (4) can be rewritten as

$$\begin{aligned}\mathcal{L} &= L + l\,,\\ L &= \tfrac{1}{2}(M + \alpha\mathbf{B}^2)\mathbf{V}^2 + \tfrac{1}{2}\alpha\mathbf{E}^2\,,\\ l &= \mathbf{V}\cdot(\mathbf{B} \times \alpha\mathbf{E})\,.\end{aligned} \tag{5}$$

L can be regarded as the Lagrangian of a new particle of mass $M + \alpha\mathbf{B}^2$ and polarizability α moving in a magnetic-field-free region. For

$$\nabla \times (\mathbf{B} \times \alpha\mathbf{E}) = \nabla \times \frac{\alpha k B}{r}\,\mathbf{e}_\theta = 0\,, \tag{6}$$

L and $\mathcal{L}$ lead to the same equation of motion,

$$M\ddot{\mathbf{r}} = \nabla(\tfrac{1}{2}\alpha\mathbf{E}^2). \qquad (7)$$

The term $\alpha\mathbf{B}^2$ is omitted for it is negligibly small (which will be shown later), $\alpha\mathbf{B}^2 \ll M$. The term l yields no force on the particle, while in quantum mechanics it affects the wave function of the particles by attaching to it a nondispersive, pure topological quantum phase (geometric phase),

$$S = \hbar^{-1}\int d\mathbf{r}\cdot(\mathbf{B}\times\alpha\mathbf{E}) = \alpha kB\hbar^{-1}\int d\theta. \qquad (8)$$

It is worthwhile to mention an intuitive picture of the quantum phase acquired by the neutral particle. An induced dipole $\mathbf{d}$ can be represented by a pair of charges $+e$ and $-e$ at the distance $|\mathbf{r}_+ - \mathbf{r}_-| = \frac{1}{e}|\mathbf{d}|$, where $\mathbf{r}_+$ and $\mathbf{r}_-$ represent the positions of the two charges. We consider the simplest case in which the orbit of the particle is concentric with the cylinder electrode. When the particle completes a circular motion, the sum of the Aharonov-Bohm phases of the two charges is

$$\begin{aligned} S_{\mathrm{AB}} &= \hbar^{-1}\oint e\mathbf{A}(\mathbf{r}_+)\cdot d\mathbf{r}_+ - \hbar^{-1}\oint e\mathbf{A}(\mathbf{r}_-)\cdot d\mathbf{r}_- \\ &= \hbar^{-1}eB\pi(r_+^2 - r_-^2). \end{aligned} \qquad (9)$$

Note that $r_+ + r_- = 2r$ and $r_+ - r_- = \frac{1}{e}|\mathbf{d}| = \alpha k/er$ [we neglect the contribution of the term $\alpha(\mathbf{V}\times\mathbf{B})$ to $\mathbf{d}$; the reason will be shown below], and we get $S_{\mathrm{AB}} = 2\pi\alpha kB\hbar^{-1}$. It is clear that Eqs. (8) and (9) lead to the same result. Based on such a fact we attribute the quantum phase of the neutral particle to the sum of the Aharonov-Bohm phases. It should be pointed out that such an intuitive picture is also applicable to Wilkens's experiment [5].

To test for the quantum phase, one may employ atomic interferometers in which two coherent atomic beams pass on opposite sides of the electrode and then recombine. The estimates given below illustrate the conditions that should be fulfilled to realize the measurement. For most atoms, α is on the order of 10^{-40} F m^2. For noble gas atoms, α is small, especially for the helium atom, α is only 0.2×10^{-40} F m^2, while for alkali atoms α is larger than 10×10^{-40} F m^2 [6]. Atomic interferometers with the beam split in the millimeter range recently came into within experimental reach [7–9]. This allows a radius of the electrode $r_0 \sim 1$ mm; hence $k \sim 10^4$ V corresponds to an electric field $E \sim 10^7$ V/m, which is experimentally feasible. If one uses an alkali-atomic beam, and applies a magnetic field $B \sim 5$ T, then $\alpha kB\hbar^{-1} \sim 1/2$. It is possible to achieve a phase shift in excess of π.

Other than the quantum phase described above, the magnetic field yields no more significant effect. Generally, $B \le 10$ T, $V \le 10^3$ m/s, $E \sim 10^7$ V/m, thus $BV/E \le 10^{-3}$. Even for particles with a large polarizability $\alpha \sim 10\times10^{-40}$ F m^2, the term $\alpha B^2 \le 10^{-37}$ kg, only $1/10^{10}$ of the mass of one nucleon, and therefore it is of no significance in the equation of motion. It is also of minor importance in quantum mechanics for it attaches a very small dynamical phase shift ΔS to the particles,

$$\Delta S = \hbar^{-1}\int \alpha\mathbf{B}^2\mathbf{V}^2\,dt = \hbar^{-1}\int \alpha\mathbf{B}^2\mathbf{V}\cdot d\mathbf{r}. \qquad (10)$$

Clearly, $\Delta S/S \sim BV/E \le 10^{-3}$.

In contrast to Wilkens's suggestion, the advantages of our proposal for the experimental configuration are clear. In Wilkens's suggestion a radial magnetic field is required, which obstructs the experimental realization, while in our proposal a uniform magnetic field and a radial electric field are applied, which makes the experiment more practical. Wilkens's suggestion is suitable for polar molecules with a large permanent dipole moment when a moderate magnetic field is sufficient, while for single atoms and nonpolar molecules the induced dipoles are rather small even if a very strong electric field is applied. A strong magnetic field is definitely necessary. In our configuration, it is feasible for both $\mathbf{E}$ and $\mathbf{B}$ to be strong. At present, to test for the quantum phase, one prefers atomic interferometers to molecular ones until techniques relevant to the former are better established.

It was previously predicted [4] that the AC phase will act on a superfluid of bosons that have a magnetic moment in the same way that the AB phase acts on an electric superconductor. The corresponding experiment remains uncompleted because presently there is no suitable superfluid of bosons with a magnetic moment available. The most abundant superfluid is made of ^{4}He atoms, which have neither magnetic moment nor permanent electric dipole moment, while it will carry an induced dipole when subjected to a strong electric field. We predict that the quantum phase accompanying such an induced dipole will act on the superfluid. In particular, if a superfluid ring of radius $R > r_0$ encircles the cylinder electrode of radius r_0, Eq. (8) indicates that one ^{4}He atom collects a total geometric phase $S = 2\pi n_0$ when it completes a circle motion; the parameter n_0 is defined as

$$n_0 = \alpha kB\hbar^{-1}. \qquad (11)$$

For the helium atom, $\alpha \sim 0.2\times10^{-40}$ F m^2, which is rather small. However, the macroscopic quantum coherence in a superfluid system allows a large radius of the electrode. If $r_0 \sim 1$ cm, $E \sim 10^7$ V/m, then $k \sim 10^5$ V; and if $B \sim 10$ T, then $n_0 \sim 1/4$.

One employs the macroscopic wave function (usually known as the Ginzburg-Landau order parameter) $\Psi(\theta) = \sqrt{\rho/2\pi}\exp\{iS(\theta)\}$, which satisfies the Schrödinger equation

$$\mathcal{H}\Psi(\theta) = E\Psi(\theta), \qquad (12)$$

to describe the bound state of the ring of superfluid [10,11]. The Hamiltonian $\mathcal{H}$ is deduced from the Lagrangian given in Eq. (4) as

$$\mathcal{H} = \frac{1}{2m}(\mathbf{P} + \alpha \mathbf{E} \times \mathbf{B})^2 = \frac{1}{2mR^2}(L_z - n_0 \hbar)^2, \quad (13)$$

where L_z is the canonical angular momentum. Very similar to the case of the bound state Aharonov-Bohm effect of electrons [12], the bound state solutions for the ring of superfluid are

$$\Psi_n(\theta) = \left(\frac{\rho}{2\pi}\right)^{1/2} \exp\{in\theta\}$$

$$E_n = \frac{\hbar^2}{2mR^2}(n - n_0)^2, \quad (14)$$

where n should be integers in order that the functions $\Psi_n(\theta)$ are single valued. The ground state is the one in which $(n - n_0)^2$ is the minimum. The superfluid velocity is

$$\mathbf{V}_n = \frac{1}{m}\langle \Psi_n | \mathbf{P} | \Psi_n \rangle + \frac{1}{m}\langle \Psi_n | \alpha \mathbf{E} \times \mathbf{B} | \Psi_n \rangle = \frac{(n - n_0)\hbar}{mR}\mathbf{e}_\theta . \quad (15)$$

Provided $0 < n_0 < 1/2$, $\Psi_0(\theta)$ is the ground state with the nonzero superfluid velocity

$$\mathbf{V}_0 = -\frac{n_0 \hbar}{mR}\mathbf{e}_\theta . \quad (16)$$

Finally, we remark that our discussion above about the quantum phase of an induced dipole in a magnetic field is phenomenological. An open theoretical issue is how to provide a microscopic description of the quantum phase in terms of the Hamiltonian for the atom, which we expect will confirm the conclusion of the phenomenological discussion. We are presently working on such a microscopic theory. It will be reported elsewhere.

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*Electronic address: rshan@svr.bimp.pku.edu.cn

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