

Cosmology with a TeV Mass Higgs Field Breaking the Grand-Unified-Theory Gauge Symmetry

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The most natural way to break the grand-unified-theory gauge symmetry is with a Higgs field whose vacuum expectation value is of order 10^{16} GeV but whose mass is of order 10^2 to 10^3 GeV. This can lead to a cosmological history radically different from what is usually assumed to have occurred between the standard inflationary and nucleosynthesis epochs, which may solve the gravitino and Polonyi-moduli problems in a natural way.

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It is generally thought that fundamental interactions respect local supersymmetry, called supergravity [1]. The most popular implementations of supergravity generally encounter two cosmological problems. One is an overabundance of the gravitino, the spin 3/2 superpartner of the graviton [2,3]. The other is an overabundance of one or more species of spin zero particle, with mass $m_\phi \sim 10^2$ to 10^3 GeV and gravitational strength interactions [4–9]. The latter problem was first recognized [4] in an early model of supergravity involving the Polonyi field, and became known as the Polonyi problem. It has persisted in versions of supergravity derived from the superstring [5], where the troublesome particles are the moduli generic to such theories. We will use the term “moduli” to cover all cases, and for simplicity consider only one species corresponding to a real field Φ .

The observed ratios of the three gauge couplings of the standard model suggest that the correct supergravity model will contain a GUT (grand unified theory), with a unification scale $M_{\text{GUT}} \sim 10^{16}$ GeV. The GUT is broken down to the standard model when a scalar field h , charged under the GUT gauge symmetry, but neutral under the standard model gauge symmetry, and called the GUT Higgs, acquires an expectation value $|h| \equiv M_{\text{GUT}}$. This was originally supposed to be achieved by a scaled-up version of the standard model Higgs potential, $V = \lambda(|h|^2 - M_{\text{GUT}}^2)^2$ with $\lambda \sim 1$. In that case the energy scale $V_0^{1/4}$ set by the height $V_0 \equiv V(0)$ is of order M_{GUT} , and the mass of the GUT Higgs is also of order M_{GUT} . Such a potential leads to a history of the Universe that has been widely described [10,11]. But in the context of supergravity and superstrings, where one hopes to generate all energy scales dynamically in terms of the Planck mass m_{Pl} , a potential of this kind does not seem very likely. It is more natural [12] to suppose that $|h|$ corresponds to a direction in field space which is exceptionally or absolutely flat, before the nonperturbative effects that lead to supersymmetry breaking are taken into account. After supersymmetry breaking the potential is of the form

$$V = V_0 - \frac{1}{2} m^2 |h|^2 + \dots, \quad (1)$$

with $m \sim 10^2$ to 10^3 GeV (the scale of supersymmetry breaking). The higher order terms, which still correspond to an exceptionally flat direction, are negligible for $|h| \ll M_{\text{GUT}}$. The mass of the GUT Higgs is now only of order m , and the height $V_0^{1/4}$ is only of order $(mM_{\text{GUT}})^{1/2} \sim 10^9$ to $10^{9.5}$ GeV.

The purpose of this paper is to point out that such a flat GUT potential may imply a history of the early Universe very different from the usual one, in which the gravitino and moduli problems may be solved. Some aspects of this history have been considered by previous authors [13–18], but they did not consider the effect of what we shall call thermal inflation. Indeed, as far as we can discover the entire previous literature on this type of inflation consists of precisely one sentence [15].

The history is summarized in Table I. It begins as usual with an era of ordinary inflation [10] in which the energy density ρ is dominated by the potential V of the scalar fields, with one of them, termed the inflaton, slowly rolling down it. The potential at the end of ordinary inflation, V_{inf} , must satisfy $V_{\text{inf}}^{1/4} \lesssim 10^{16}$ GeV to avoid generating too much large scale cosmic microwave background anisotropy [19]. Of the many models of this era that have been proposed, the only ones that are sensible in the context of supergravity are natural inflation [20], and some versions [21,22] of hybrid inflation [23]. In none of them is the inflaton a Higgs field.

During ordinary inflation, noninflaton fields typically acquire masses squared at least of order H^2 [21,24], which

TABLE I. History of the early Universe. There are large uncertainties in our estimates.

$\rho^{1/4}$	T	H	
$V_{\text{inf}}^{1/4}$	0	$V_{\text{inf}}^{1/2}/m_{\text{Pl}}$	Ordinary inflation ends
T_R	T_R	T_R^2/m_{Pl}	Hot big bang begins (unbroken GUT vacuum)
10^9 GeV	10^9 GeV	100 MeV	Thermal inflation begins
10^9 GeV	10^3 GeV	100 MeV	Cold big bang begins (present day vacuum)
10 MeV	10^{-3} eV	10^{-14} eV	GUT Higgs decay starts
10 MeV	10 MeV	10^{-14} eV	Hot big bang begins (present day vacuum)

may be of either sign [16]. We make the assumption that the effective GUT Higgs mass squared is positive during inflation, so that it is trapped at $|h| = 0$. At some epoch after ordinary inflation “reheating” occurs, which means that the bulk of the energy density thermalizes at some “reheat temperature” $T_R \sim (\rho/g_*)^{1/4}$, where g_* is the effective number of massless species. For simplicity, we assume in what follows that $T_R \gtrsim V_0^{1/4}$. When the GUT Higgs field is in thermal equilibrium at a temperature in excess of some critical value $T_c \sim m$, its effective potential acquires a minimum at $|h| = 0$ [14,25]. Even if full reheating is long delayed, one expects some fraction ϵ of the energy density to thermalize promptly, leading to an initial temperature $T_{\text{inf}} \sim (\epsilon V_{\text{inf}}/g_*)^{1/4}$. Provided that ϵ exceeds $(T_c/T_R)^4 (V_{\text{inf}}^{1/4}/T_R)^{4/3} \lesssim 10^{-16}$ one will have $T > T_c$ even before full reheating, and we assume that this is so. The net effect of these conditions is to trap the GUT Higgs at $|h| = 0$ until $T = T_c$.

Thermal inflation begins at $T \sim (V_0/g_*)^{1/4}$, when the GUT potential V_0 starts to dominate the thermal energy density $\sim g_* T^4$, and it ends at $T = T_c$, after $\ln[(V_0/g_*)^{1/4}/T_c] \sim 15$ e -folds of inflation, when $|h|$ rolls away from zero. At around this same temperature the standard model Higgs also rolls away from zero. Thus the full GUT symmetry breaks more or less directly to the broken standard model symmetry $SU(3)_C \otimes U(1)_{\text{EM}}$ at $T \sim m$. [Note that the expansion of the Universe does not prevent this phase transition because the Hubble time $H^{-1} \sim (m_{\text{Pl}}/M_{\text{GUT}}) m^{-1}$ is bigger than the duration $\sim m^{-1}$ of the transition. We define the Planck mass as $m_{\text{Pl}} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV.] This is in contrast with the traditional case of a nonflat potential, where the GUT symmetry breaks to the unbroken standard model symmetry at $T \sim M_{\text{GUT}}$, leaving the electroweak phase transition to complete the breaking at $T \sim m$.

After thermal inflation ends, relic radiation from the first hot big bang plays no further role and, in particular, the quark-hadron transition is of no interest. A cold big bang now begins, with ρ dominated by the oscillation of the homogeneous GUT Higgs field, or equivalently by nonrelativistic GUT Higgs particles (matter). After a few Hubble times the amplitude of the oscillation has been reduced by the expansion of the Universe, so that the GUT Higgs field is of order M_{GUT} . This means that the GUT Higgs couples directly only to particles with mass of order M_{GUT} , so that its interaction with ordinary particles is very weak.

The cold big bang ends at a time of order the inverse GUT Higgs decay rate, M_{GUT}^2/m^3 [13] (we are not considering possible parametric resonance effects [26]). If the decay products thermalize the temperature is then $T_{\text{decay}} \sim m_{\text{Pl}}^{1/2} m^{3/2}/M_{\text{GUT}} \sim 10$ MeV to 100 keV. In order not to affect nucleosynthesis one will need T_{decay} at the upper end of this range, which among other things ensures thermalization (except for the LSP which

we will discuss later). This formally corresponds to $m \simeq 10^3$ GeV but the uncertainties in our estimates are such that a value $m \simeq 10^2$ GeV cannot be excluded.

Now let us ask about dangerous relics. In the usual cosmology, entropy conservation is a good approximation, and as a result the entropy density $s \sim g_* T^3$ and the number density n of any stable relic have a constant ratio after the relic stops interacting (“freezes out”). The GUT Higgs decay releases a huge amount of entropy, increasing it by a factor $\Delta \sim 3g_*^{-1} V_0 T_{\text{decay}}^{-1} T_c^{-3}$. (In this expression g_* refers to the unbroken GUT at $T \sim T_c$, and from now on we replace it by the estimate $g_*/3 \sim 10^2$.) The present number density of any species created before that time is diluted by this factor, if its initial number density depends only on the temperature. Setting $m = T_c = 10^2$ to 10^3 GeV gives $\Delta \sim 10^{29}$ to 10^{30} . Note that because of thermal inflation and the cold big bang, a given scale leaving the horizon during ordinary inflation does so $(1/3) \ln \Delta \sim 23$ e -folds later than in the usual cosmology, which could significantly affect predictions of the spectral indices of the perturbations produced during ordinary inflation.

The gravitino is harmless if $n_{3/2}/s \lesssim 10^{-12}$ to 10^{-15} at nucleosynthesis [27]. Gravitinos created during the first hot big bang have an abundance no bigger than the thermal equilibrium value $n_{3/2}/s \sim 10^{-3}$, so their present abundance is far inside the above bound. Gravitinos are not produced by the GUT Higgs decay if $m_h < m_{3/2}$. Finally, gravitinos generated during the second hot big bang are harmless, because the relevant bound $T \lesssim 10^5$ GeV is amply satisfied [3]. (Note that this is 5 orders of magnitude stronger than earlier estimates, which neglected an important mechanism for creating gravitinos).

Moduli are also harmless if $n_\Phi/s \lesssim 10^{-12}$ to 10^{-15} at nucleosynthesis [27]. We will take $\Phi = 0$ to be the vacuum value. When discussing the early time evolution of Φ various effects need to be considered [8,21,24], but the outcome [28] is that at the epoch $H \sim m_\Phi$ it starts to oscillate about $\Phi \simeq 0$ with amplitude of order m_{Pl} . The corresponding abundance $n_\Phi/s \sim (m_{\text{Pl}}/m_\Phi)^{1/2} \sim 10^8$ is cosmologically insignificant, bearing in mind the dilution factor. However, at the end of thermal inflation Φ is in general still displaced from its vacuum value by its interaction with the GUT Higgs, and by an amount which turns out to be large compared with the oscillation. To estimate this displacement [28], recall that the quantity V_0 appearing in Eq. (1) is supposed to be generated dynamically from the Planck scale m_{Pl} . In that equation $\Phi = 0$, but for fixed $|\Phi| \ll m_{\text{Pl}}$ a similar equation will hold with some $V_0(\Phi)$, whose slope $\partial V_0/\partial \Phi$ will be of order V_0/m_{Pl} . The effective potential for Φ in the regime $|\Phi| \ll m_{\text{Pl}}$ is then $m_\Phi^2 \Phi^2/2 + (\partial V_0/\partial \Phi) \Phi$, so the displacement is of order $V_0 m_\Phi^{-2} m_{\text{Pl}}^{-1} \sim (H/m_\Phi)^2 m_{\text{Pl}}$. If after thermal inflation the effective potential promptly reverted to $m_\Phi^2 \Phi^2/2$, then Φ would start to oscillate with

this amplitude corresponding to $n_\Phi \sim V_0^2 m_\Phi^{-3} m_{\text{Pl}}^{-2}$. In that case the abundance at nucleosynthesis would be

$$\frac{n_\Phi}{s} \sim 10^{-5} \left(\frac{M_{\text{GUT}}}{m_{\text{Pl}}} \right)^2 \times \left(\frac{T_{\text{decay}}}{10 \text{ MeV}} \right) \left(\frac{V_0}{m_\Phi^2 M_{\text{GUT}}^2} \right) \left(\frac{1 \text{ TeV}}{m_\Phi} \right). \quad (2)$$

The first line is of order 10^{-10} and the remaining factors are roughly of order 1. In reality the dynamics at the end of thermal inflation will be quite complicated but this estimate should still be reasonable [28]. Taking into account the considerable uncertainty, the conclusion is that the moduli problem may be solved.

The classical displacement discussed in the last paragraph was not taken into account by Randall and Thomas [7] when they claimed that the moduli problem can be solved by several e -folds of inflation at the scale $V_{\text{inf}} \sim m_\Phi^2 m_{\text{Pl}}^2$. From the above discussion one, in fact, needs $V_{\text{inf}} \lesssim 10^{-7} (10^{12} A) \times [(10 \text{ MeV})/T_{\text{decay}}] [m_\Phi/(1 \text{ TeV})] m_\Phi^2 m_{\text{Pl}}^2$ to solve the moduli problem in this way, where A is the bound on n_Φ/s at nucleosynthesis.

Stable topological defects may be produced at the end of the first era of inflation, and at the GUT transition. Let us look briefly at the case of gauge monopoles produced at the GUT transition, and assume that they are not connected by strings. The strongest bound on their abundance comes from baryon decay catalysis in neutron stars, which gives $n/s \lesssim 10^{-37}$ [29]. The temperature is too low for annihilation [11], but one monopole per Hubble volume at creation gives $n/s \sim 10^{-3} (M_{\text{GUT}}/m_{\text{Pl}})^3 \sim 10^{-10}$, which requires a dilution factor $\Delta \sim 10^{27}$. Thus there may be no monopole problem.

So much for undesirable relics. What about desirable ones, in the form of matter? Hot dark matter (massive neutrinos) has the usual abundance because its freeze-out temperature is a few MeV and hence less than T_{decay} . If it is stable, the lightest supersymmetric particle (LSP) will be cold dark matter. It is not produced after GUT Higgs decay in our cosmology because its freeze-out temperature is of order 1 GeV, but it will be produced by the GUT Higgs decay unless $m_h < 2m_{\text{LSP}}$. If N LSP's are produced per GUT Higgs, then $n_{\text{LSP}}/s \sim N(T_{\text{decay}}/m)$ and $\Omega_{\text{LSP}} \sim 10^{10} N(T_{\text{decay}}/m) [m_{\text{LSP}}/(10 \text{ GeV})] \sim 10^5 N \times [m_{\text{LSP}}/(10 \text{ GeV})]$. Since $N < 10^{-5}$ seems unlikely we probably need either R -parity violation to destabilize the LSP or $m_h < 2m_{\text{LSP}}$.

For baryogenesis, the most commonly considered mechanisms in the usual cosmology are [30] nonperturbative effects at the electroweak transition, particle decay, and the Affleck-Dine mechanism [31]. In our cosmology the electroweak and GUT transitions happen at more or less the same time, but without going into detail it seems clear that the first mechanism cannot be significant because of the dilution factor. However, if R parity is violated, the

baryons might be created in the GUT Higgs decay [17,32]. The Affleck-Dine mechanism can generate both baryons and the LSP after thermal inflation [28].

The other favored cold dark matter candidate is the axion. Axion cosmology is quite subtle [10,11,33,34]. For simplicity let us ignore the saxino and axino (the axion's superpartners). Recall that the axion field is $a = f_a \theta$ where θ is the "misalignment angle" and f_a is related to the mass by $m_a/10^{-3} \text{ eV} = 0.62 \times 10^{10} \text{ GeV}/f_a$. From accelerator physics and astrophysics, $m_a \lesssim 10^{-2} \text{ eV}$. The axion mass switches on gradually as T falls towards 100 MeV.

Let us first suppose that there are no axionic strings. Then θ is typically homogeneous with some initial value $\tilde{\theta}$, and in the standard cosmology it starts to oscillate when $m_a(T) \sim H$, leading to $\Omega_a \sim \tilde{\theta}^2 (10^{-5} \text{ eV}/m_a)^{1.2}$. In our cosmology oscillation starts when $m_a \sim H$, the temperature being negligible, and this leads to $\Omega_a \sim \tilde{\theta}^2 [(10^{-8} \text{ eV})/m_a]^2 [T_{\text{decay}}/(10 \text{ MeV})]$.

Now suppose that there are strings. In the standard cosmology there is roughly one string per Hubble volume, until $m_a(T) \sim H$ when domain walls form between the strings and the wall-string network annihilates, and axions radiated from strings prior to this epoch give [34] $\Omega_a \sim [(10^{-3} \text{ eV})/m_a]$. In our cosmology the string spacing leaves the horizon at the beginning of thermal inflation, and the axion field then freezes until $H \sim m_a$. At that epoch domain walls form, and between them the almost homogeneous axion field oscillates to give a contribution $\Omega_a \sim [(10^{-8} \text{ eV})/m_a]^2 [T_{\text{decay}}/(10 \text{ MeV})]$. The string-wall network reenters the horizon at the epoch $\rho_{\text{entry}}^{1/4} \sim 10m(m/M_{\text{GUT}})^{1/4}$ corresponding to $\rho_{\text{entry}}/T_{\text{decay}} \sim 10^3 [M_{\text{GUT}}/(10^{16} \text{ GeV})]^{3/4}$, when it decays into marginally relativistic axions giving a contribution $\Omega_a \sim [(10^{-8} \text{ eV})/m_a] [(10^{16} \text{ GeV})/M_{\text{GUT}}]^{1/2}$. The overall conclusion is that the axions can provide cold dark matter in our cosmology, provided that $m_a \lesssim 10^{-8} \text{ eV}$ corresponding to $f_a \gtrsim 10^{15} \text{ GeV}$.

Except in the last paragraph we have ignored the inhomogeneity of the Universe. One might wonder if GUT Higgs "stars" could form during the cold big bang (cf. [9]). If they form sufficiently early they might be dense enough to briefly thermalize the GUT Higgs decay products. During ordinary inflation the quantum fluctuation effectively generates a classical curvature perturbation as each scale leaves the horizon, which remains constant until horizon reentry and has a roughly scale-independent magnitude $\lesssim 10^{-5}$. One does not expect a significant quantum fluctuation during thermal inflation because $H \ll m$. If the gap between ordinary and thermal inflation is negligible, this information allows one to estimate the density perturbation. On the scale leaving the horizon at the beginning of thermal inflation it is $\lesssim 10^{-5}$ at the epoch of reentry, and it then grows like $\rho^{-1/3}$ to become $\lesssim 10^{-1}$ at GUT Higgs decay. On bigger scales it is smaller, and on smaller scales

it vanishes, so there is no significant structure formation. To extend this analysis to the case where there is a gap one would have to consider the evolution of the curvature perturbation inside the horizon during thermal inflation.

Assuming a flat GUT Higgs potential, the main alternative to our cosmology would be to have $|h|$ initially displaced from 0, so that at the epoch $H \sim m$ it starts to oscillate about its vacuum value [16]. The GUT Higgs particles produced in this way must still decay before nucleosynthesis, and we do not now solve the moduli problem. The other alternative, which we have not considered, would be to retain the initial value $|h| = 0$, but to relax the assumption that $T_R \gtrsim V_0^{1/4}$.

In this Letter we have taken the scale of symmetry breaking to be 10^{16} GeV, because this is what experiment indicates for a gauge symmetry. From the point of view of cosmology an attractive scale is $\sim 10^{13}$ GeV, because it minimizes the moduli abundance making the nucleosynthesis constraint easier to satisfy, and it also gives $T_{\text{decay}} \sim 1$ GeV, which might be high enough for the LSP to thermalize. Such a scale might be associated with the breaking of a global symmetry, and our cosmology could work equally well in that case.

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