Continuum Limit, Galilean Invariance, and Solitons in the Quantum Equivalent of the Noisy Burgers Equation

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A continuum limit of the non-Hermitian spin-1/2 chain, conjectured recently to belong to the universality class of the noisy Burgers or, equivalently, Kardar-Parisi-Zhang equation, is obtained and analyzed. The Galilean invariance of the Burgers equation is explicitly realized in the operator algebra. In the quasiclassical limit we find nonlinear soliton excitations exhibiting the $\omega \propto k^z$ dispersion relation with dynamical exponent $z = 3/2$.

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The Kardar-Parisi-Zhang (KPZ) equation plays an important role in modern nonequilibrium statistical mechanics as a continuum description of growing interfaces [1,2]. In a comoving frame the equation of motion for the relative height variable $h(x,t)$ has the form

$$\partial h/\partial t = \nu\nabla^2 h + (1/2)\lambda(\nabla h)^2 + \eta\,, \tag{1}$$

where ν is an effective diffusion coefficient, λ a coupling constant characterizing the slope dependence of the growth velocity, and $\eta(x,t)$ a white noise, $\langle\eta(x,t)\eta(x',t')\rangle = \Delta\delta(x-x')\delta(t-t')$, representing fluctuations either in the drive or in the environment.

The slope variable $u = \nabla h$ satisfies the noisy Burgers equation [3,4]

$$\partial u/\partial t - \lambda u \nabla u = \nu\nabla^2 u + \nabla\eta\,. \tag{2}$$

The universality class represented by (1) and (2), besides driven interface and irrotational fluid dynamics, includes such important models as a driven lattice gas [5], as well as a directed polymer [6], smectic liquid crystals [7], or a quantum particle [8] in a random environment. These models serve as paradigms in the theories of driven and disordered systems. An additional attraction of the problem is its analytical tractability. A consistent, although uncontrolled, perturbative renormalization group flow has been constructed up to the two-loop level [4,9,10] and appears to give qualitatively correct results. What emerges is a critical (massless) behavior, which at and below two spatial dimensions is always governed by a strong-coupling renormalization group fixed point. Two basic scaling dimensions describe the long-wavelength low-frequency behavior of the problem: (i) the roughness exponent ζ characterizing the height correlations in (1) and (ii) the dynamical exponent z. The velocity field u or, equivalently, slope field ∇h for the interface, has scaling dimension $1-\zeta$. The pair correlation function scales as

$$\langle u(x_0,t_0)u(x_0+x,t_0+t)\rangle = x^{-2(1-\zeta)}f(t/x^z)\,, \tag{3}$$

where the scaling function $f(x) \propto x^{-2(1-\zeta)/z}$ for large values of its argument.

The case of one spatial and one time dimension, $d = 1+1$, is of particular interest. Although the perturbative renormalization group flows into the strong-coupling regime, outside of its range of validity it was noticed in [4] that both scaling dimensions follow explicitly from two exact scaling relations. The first one follows from the Galilean invariance of the original Burgers equation (generalized to $\lambda \neq -1$), i.e., $u(x,t) \rightarrow u(x-\lambda u_0 t, t) - u_0$, leaving all averages unchanged. This property implies that λ is a structural constant of the symmetry group and thus invariant under a renormalization group transformation. Comparing the scaling properties of the two terms on the left hand side of (2), we then obtain the first scaling relation $z = 2 - \zeta$. The other scaling relation is particular to the case $d = 1+1$ and follows from the fact that the simple Gaussian *equal time* stationary probability distribution $\ln P(u) \propto -\int u^2(x)dx$ satisfies the Fokker-Planck equation for the probability distribution $P(u,t)$,

$$\frac{\partial P(u,t)}{\partial t} = -\int dx\,\frac{\delta}{\delta u}\left[\left(\lambda u\frac{\partial u}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2}\right)P(u,t)\right] + \frac{\Delta}{2}\int dx\,dx'\left[\frac{\partial^2}{\partial x\partial x'}\delta(x-x')\right]\frac{\delta^2 P(u,t)}{\delta u\,\delta u'}\,, \tag{4}$$

corresponding to (2) for *any* value of λ. This property then implies $\zeta = 1/2$ (random walk behavior) and thus $z = 3/2$.

The exact rational values of the scaling exponents found in $d = 1+1$ bear a superficial resemblance to the exponents encountered in two-dimensional classical critical problems, equivalent to $d = 1+1$ relativistic quantum field theories [11]. Note, however, that the value $z = 3/2$ is manifestly nonrelativistic, making extensions of the conformal field theory methods a highly nontrivial task. We might, however, still expect to gain new insight by studying the equivalent $(1+1)$-dimensional *nonrelativistic* quantum system by the methods developed for one-dimensional spin chains [12,13].

Such a quantum spin chain approach has been proposed in [14], and considered further in [15], on the basis of the equivalence between the master equation describing the evolution of a one-dimensional driven lattice gas or the equivalent lattice interface growth model and the non-Hermitian spin $s = 1/2$ Hamiltonian for N spins,

$$\hat{H} = -\sum_{j=1}^{N}\left[\vec{S}_j \cdot \vec{S}_{j+1} - \frac{1}{4} + i\vec{\epsilon} \cdot (\vec{S}_j \times \vec{S}_{j+1})\right]. \quad (5)$$

The mapping is achieved by identifying the eigenvalues of the z projections of the spins, S_j^z, with the slope variable u_j of the discrete interface model or the occupation numbers in the lattice gas representation. The interface dynamics is governed by two basic rates, $r_\uparrow$ and $r_\downarrow$, corresponding to flipping up or down a kink (a pair of neighboring interface segments with opposite slopes); these flips map onto the spin exchange processes in (5). The vector $\vec{\epsilon}$ is oriented along the z axis and absorbing the mean rate $r_\uparrow + r_\downarrow$ by rescaling time we have $|\vec{\epsilon}| = \epsilon \equiv (r_\uparrow - r_\downarrow)/(r_\uparrow + r_\downarrow)$, measuring the strength of the drive. For $\epsilon = 0$ up and down flips are equally probable, and we obtain the Heisenberg ferromagnet, whose spin-wave dispersion relation yields the dynamical exponent $z = 2$, corresponding to the noisy linear diffusion equation or the Edwards-Wilkinson equation for interface dynamics [1,15]. The asymmetric limit of only up or only down flips, $\epsilon = \pm 1$, has been solved exactly by the Bethe-ansatz method [14], and the finite-size gap in the spectrum has been shown to scale with the length of the chain to the power 3/2, in agreement with $z = 3/2$ for the noisy Burgers universality class.

In this Letter we obtain the continuum limit of the spin chain (5) corresponding to the noisy Burgers universality class. We show that properly defined this limit exhibits Galilean symmetry as realized by the algebra of generators of the continuous global transformations. It is known that in the Heisenberg ferromagnet [13], as well as in other XYZ spin-1/2 chains [16], quasiclassical quantization of the classical solutions of the continuum equations of motion correctly reproduces the low-energy sector of the Bethe-ansatz solution. Motivated by this fact we also study the quasiclassical limit. We find that the classical equations of motion have exact solitary wave solutions, which after quantization form a branch of elementary excitations with the $\omega \propto k^{3/2}$ dispersion relation.

As a guide to the proper continuum limit we make use of the two exact relationships employed above in the derivation of the scaling dimensions ζ and z. The implementation of the second one is readily carried out by noting that the stationary equal time probability distribution of the Burgers equation maps onto the ground state of the quantum problem. The ground state of the Heisenberg ferromagnet $|0\rangle$, for $\epsilon = 0$ is aligned with maximum total spin sN and fully degenerate spin direction. In the following we quantize along the x direction, i.e., $\langle 0|S_j^x|0\rangle = s$, $\langle 0|S_j^z|0\rangle = 0$, and $\langle 0|S_j^y|0\rangle = 0$, corresponding to a horizontal interface with zero slope. With this choice (cf. [14]) S_j^z is completely disordered and the correlation function $\langle 0|S_j^z S_{j'}^z|0\rangle = s^2\delta_{j,j'}$. In the continuum limit $\delta_{j,j'}/a \to \delta(x)$, where a is the lattice spacing, implying the rescaling $S^z(x) \sim S_j^z/a^{1-\zeta}$ with $\zeta = 1/2$. This result is also obtained by noting that a block spin $S^z(x)$ constructed from a random sequence of $S_i = \pm 1/2$ yields a spin amplitude of the order of the square root of the block size. Finally, in the mapping of a master equation onto an equivalent quantum Hamiltonian the energy levels correspond to the relaxation rates. The ground state maps onto the stationary distribution and consequently has energy zero. It is easily seen that the aligned ground state $|0\rangle$ is also an eigenstate of $\hat{H}$ in (5) with eigenvalue zero, owing to the cross product form of the last term. Consequently, the ground state structure and thus the scaling dimension of the spin density is not changed in the presence of the drive.

The excitation spectrum, however, is altered, since we expect the $\omega \propto k^2$ dispersion law of the Heisenberg ferromagnet to change to $\omega \propto k^{3/2}$ in the presence of the drive term. Clearly, the Galilean invariance which permits the evaluation of the exponent z requires the continuum limit $a \to 0$ to be taken, since discrete distances cannot be mixed with continuous time.

With the ground state aligned in the x direction the excitations above the ground state correspond to spin fluctuations in the y and z directions. Noting that a block spin pointing in the x direction has a size $s \sim a^{-1}$ for $a \to 0$, while the spins in the y and z directions scale as $a^{-1/2}$, we infer that the continuum limit is equivalent to the large spin limit in the block spin representation. Using the quasiclassical harmonic oscillator representation for the spin operators, see, e.g., [12], we have

$$\begin{aligned} S_j^x &= s - (1/2)(\hat{u}_j^2 + \hat{\varphi}_j^2), \\ S_j^y &= \hat{\varphi}_j s^{1/2}, \quad S_j^z = \hat{u}_j s^{1/2}. \end{aligned} \quad (6)$$

The oscillator position and momentum operators $\hat{\varphi}_j$ and $\hat{u}_j$ satisfy the commutator relation $[\hat{\varphi}_j, \hat{u}_k] = i\delta_{jk}$ and correspond to the polar angle and the z component of the spin, i.e., the action-angle representation of the Heisenberg spin [13]. In the continuum limit the field operators $\hat{u}(x)$ and $\hat{\varphi}(x)$ absorb the scale factor $a^{1/2}$, $\hat{u}(x) = \hat{u}_j a^{-1/2}$, and $\hat{\varphi}(x) = \hat{\varphi}_j a^{-1/2}$ and obey the canonical commutation relation $[\hat{\varphi}(x), \hat{u}(x')] = i\delta(x - x')$. Introducing the spin-wave stiffness $J = (sa)^{1/2}$, using periodic boundary conditions, omitting constant terms, and rescaling time by $s^{1/2}a^{3/2}$, the Hamiltonian finally reads

$$\hat{H} = \int dx \left\{\frac{J}{2}\left[\left(\frac{\partial \hat{u}}{\partial x}\right)^2 + \left(\frac{\partial \hat{\varphi}}{\partial x}\right)^2\right] + i\epsilon \hat{u}^2 \frac{\partial \hat{\varphi}}{\partial x}\right\}; \quad (7)$$

we note that the overall factor $a^{3/2}$ that went into the rescaling of time is consistent with the dynamical exponent $z = 3/2$. It is easily seen that the first term in (7) corresponds to the spin-wave approximation for the Heisenberg ferromagnet [the first term in (5)]; the second term representing the drive then corresponds to a spin-wave interaction.

We can now show that the Hamiltonian (7) is equivalent to the Fokker-Planck equation (4) for the noisy Burgers equation (2). Consider u and $i\delta/\delta u$ in (4) as canonical operators, $[i\delta/\delta u, u'] = i\delta(x - x')$. The canonical transformation $i\delta/\delta u = (\nu/\Delta)^{1/2}[\hat{\varphi}(x) - i\hat{u}(x)]$ and $u = (\Delta/\nu)^{1/2}\hat{u}(x)$ takes $\mathcal{L}$ in $dP(u,t)/dt = \mathcal{L}(u)P(u,t)$, Eq. (4) into a form $\tilde{\mathcal{L}}$ similar to (7), but with other coefficients. With the identification $J = \nu$ and $\epsilon = \frac{1}{2}\lambda(\Delta/\nu)^{1/2}$, we obtain $-\hat{H} = \tilde{\mathcal{L}}$, and the equivalence follows, since the state $|\Psi\rangle$ representing $P(u,t)$ evolves according to $d/dt|\Psi\rangle = -\hat{H}|\Psi\rangle$.

Noting that the time evolution operator for the master equation is $\exp(-t\hat{H})$ [14], the equations of motion for $\hat{\varphi}$ and $\hat{u}$ follow from $d\hat{\varphi}/dt = [\hat{H}, \hat{\varphi}]$ and $d\hat{u}/dt = [\hat{H}, \hat{u}]$, i.e.,

$$\frac{\partial \hat{\varphi}}{\partial t} = iJ\frac{\partial^2 \hat{u}}{\partial x^2} + 2\epsilon\hat{u}\frac{\partial\hat{\varphi}}{\partial x}, \tag{8}$$

$$\frac{\partial \hat{u}}{\partial t} = -iJ\frac{\partial^2 \hat{\varphi}}{\partial x^2} + 2\epsilon\hat{u}\frac{\partial\hat{u}}{\partial x}. \tag{9}$$

These field equations are invariant under the Galilean transformation

$$\begin{aligned}\hat{u}(x,t) &\longrightarrow \hat{u}(x - 2\epsilon u_0 t, t) - u_0, \\ \hat{\varphi}(x,t) &\longrightarrow \hat{\varphi}(x - 2\epsilon u_0 t, t),\end{aligned} \tag{10}$$

in addition to being invariant under an arbitrary shift in $\hat{\varphi}$. These two continuous symmetry relations correspond to the two generators of the full group of rotations of the Heisenberg spin in (5) for $\epsilon = 0$. The oscillator representation corresponds to a local Abelian limit of the non-Abelian group $O(3)$; the spin-wave Hamiltonian is invariant under arbitrary shifts in both canonical fields $\hat{\varphi}$ and $\hat{u}$. In the presence of the spin-wave interaction, the invariance under shifts in $\hat{\varphi}$ is preserved, while the other symmetry is replaced by the Galilean space-time mixing (10).

These symmetries have a compact representation in terms of the algebra of the operators $\hat{M}^z = \int dx\, \hat{u}$ and $\hat{\Phi} = \int dx\, \hat{\varphi}$, generating rotations about the y and the z axes, or, equivalently, shifts in $\hat{\varphi}$ and $\hat{u}$, respectively. Adding the momentum operator $\hat{P} = +\int dx\, \hat{u}\, \partial\hat{\varphi}/\partial x$ and $\hat{H}$, generating space and time translations, respectively, we arrive at the operator algebra: $[\hat{H}, \hat{M}^z] = [\hat{H}, \hat{P}] = [\hat{P}, \hat{\Phi}] = [\hat{P}, \hat{M}^z] = 0$, and $[\hat{\Phi}, \hat{M}^z] = iL$, where L is the length of the interface, while the Galilean invariance yields

$$[\hat{H}, \hat{\Phi}] = +2\epsilon\hat{P}. \tag{11}$$

Thus ϵ is indeed a structural constant of the algebra of symmetries justifying the renormalization leading to (7), which was chosen so that ϵ is invariant under changes in the microscopic scale a. The above commutation relation has a simple interpretation in the case of the elementary excitations: For the lowest energy states carrying momentum k (11) implies rotation of the global polar angle $\hat{\Phi}$ with constant angular velocity $d\hat{\Phi}/dt = 2\epsilon k$.

We now attempt to find an elementary excitation with the $\omega \propto k^{3/2}$ dispersion relation by a direct analysis of the nonlinear equations of motion (8) and (9). Such an analysis is most easily carried out in the classical limit followed by quasiclassical quantization (cf. [13]). We therefore first replace the operators $\hat{\varphi}$ and $\hat{u}$ in (7) by the field variables φ and u satisfying the Poisson bracket $\{u(x), \varphi(y)\} = \delta(x - y)$. The classical equations of motion $\partial u/\partial t = -i\{H, u\}$ and $\partial\varphi/\partial t = -i\{H, \varphi\}$ then have the same form as (8) and (9) with $\hat{u}$ and $\hat{\varphi}$ replaced by u and φ. For $\epsilon = 0$ we find small amplitude spin waves about the ground state with quadratic dispersion leading to $z = 2$. However, for $\epsilon \neq 0$ we identify nonlinear nonperturbative soliton solutions propagating with velocity v, i.e., replacing $\partial/\partial t$ by $-v\partial/\partial x$ we obtain $-v\,\partial u/\partial x = -iJ\partial^2\varphi/\partial x^2 + 2\epsilon u\,\partial u/\partial x$ and $-v\,\partial\varphi/\partial x = iJ\partial^2 u/\partial x^2 + 2\epsilon u\,\partial\varphi/\partial x$, which are easily solved by quadrature.

We note that by forming the quotient of the two equations and integrating once we obtain $(\partial u/\partial x)^2 + (\partial\varphi/\partial x)^2 = C$. For solutions with vanishing derivatives at infinity, $C = 0$, and we have

$$\partial u/\partial x = \pm i\partial\varphi/\partial x. \tag{12}$$

By insertion we obtain one equation for the slope u, which by quadrature yields $\partial u/\partial x = \pm(\epsilon/J)(u - u_+)(u - u_-)$, where we have related the constant of integration to the boundary values $u_\pm = u(\pm\infty)$ at infinity, and the "mean amplitude-velocity" condition

$$u_+ + u_- = -v/\epsilon, \tag{13}$$

defining permanent profile kink or soliton solutions. Requiring u to be finite for all x the soliton shape is given by $(u - u_+)/(u_- - u) = \exp\{[-(|\epsilon|/J)|u_+ - u_-|(x - x_0)]\}$ with "center of mass" position x_0. For $\partial u/\partial x = \pm i\,\partial\varphi/\partial x$ we have $\mathrm{sgn}[\epsilon(u_+ - u_-)] = \mp 1$, respectively, i.e., kinks with opposite slopes. In order to interpret these solutions as excitations above the ground state, which classically is the uniform solution $u = 0$, we must satisfy the boundary conditions $u_\pm = 0$. We achieve this by grouping kinks in pairs, i.e., the kink $(u_-, u_+) = (0, -v/\epsilon)$ at smaller x with the kink $(u_-, u_+) = (-v/\epsilon, 0)$ at larger x. The distance between the kinks in a pair is assumed to be much larger than the intrinsic width $J/|v|$ but much smaller than the length of the system.

The energy momentum relationship for a pair is obtained by noting that the energy E is given by (7), while the momentum, the generator of translation, is $P = +\int dx\, u\, \partial\varphi/\partial x$. Inserting (12) we obtain for a single kink $E = -(1/3)|\epsilon(u_+^3 - u_-^3)|$ and $P = -(i/2)|u_+^2 - u_-^2|\text{sgn}(v)$. For a pair of kinks moving with a constant velocity v, both energy and momentum are doubled and we obtain

$$E = -(2/3)|v^3|/\epsilon^2,\ P = -iv^2/\epsilon^2\text{sgn}(v). \qquad (14)$$

We note that the momentum is purely imaginary; this is a feature of the complex field equations (8) and (9). Eliminating v we obtain the dispersion law

$$E = -\frac{2}{3}|\epsilon|[iP\text{sgn}(v)]^{3/2}. \qquad (15)$$

In order to revert to a discussion of the quasiclassical limit of the spin chain, the final step in our analysis amounts to a quasiclassical quantization of the above pair solutions. In the spirit of the Landau quasiparticle picture we envisage that we can label the low-lying quantum states in terms of a dilute gas of pairs of nonlinear kinks with dispersion law (15). Noting that the quantum state propagates according to $\exp(-Et - iPx)$ the group velocity $|dE/dP|$ of a two-kink wave packet is v.

A few comments are in order: (a) The amplitude of a pair, v/ϵ, diverges for $\epsilon \to 0$ implying that these are indeed nonperturbative solutions of the field equations, characterizing the strong-coupling renormalization group fixed point. (b) The kink solutions characterized by (13) are manifestly invariant under the Galilei transformation (10). (c) Inserting $\partial\varphi/\partial x = i\partial u/\partial x$ into the field equations (8) and (9), we obtain the deterministic noiseless Burgers equation. We conclude that the above kink solutions correspond to the well-known shock wave solutions [3]. In the interface representation a pair of kinks forms a *step*: a localized slope fluctuation propagating in the lateral direction.

In conclusion, two main results for the theory of the noisy Burgers-KPZ universality class have been obtained in this Letter. First, although supported by an exact Bethe-ansatz solution for a special value of the parameter ϵ and by numerical results, the equivalence between the lattice model (5) and the continuum Langevin equations (1) and (2) remained a conjecture [14,15]. Here we have shown, using well-known methods in the quantum theory of spin chains [12], how to implement the continuum limit of the lattice model (5). The rescaling leads to a continuum Hamiltonian (7), which by a canonical transformation is reduced to the Liouville operator (4) of the Fokker-Planck equation corresponding to the noisy Burgers-KPZ equations. The field operators appearing in the continuum Hamiltonian absorb the powers of the lattice constants, making their dimensions equal to the scaling dimensions of the corresponding quantities of the noisy Burgers-KPZ universality class. The choice of the scaling dimensions is governed by the (generalized) Galilean invariance of the theory (10), which is explicitly realized in the commutation relation (11) of the symmetry algebra.

Second, the space-time scaling $t \propto x^z$ with the exponent $z = 3/2$ appears in the theory of the $d = 1 + 1$ noisy Burgers equation as the only one consistent with the Galilean symmetry of the problem [4]. However, little physical intuition exists concerning the physical realization of this dynamical scaling. Here we have made a step towards a better understanding of the physical nature of the problem by identifying explicit nonlinear soliton solutions to the classical limit of the nonrelativistic field theory. These solitons correspond to the shock wave solutions of the *deterministic* Burgers equation. Grouping shock waves of the opposite sign moving with the same velocity, we obtain a localized excitation with respect to a uniform solution, which in the KPZ representation corresponds to a step propagating along a growing interface. In the quasiclassical limit these soliton pairs form a branch of quantum excitations, exhibiting the nontrivial $\omega \propto k^{3/2}$ dispersion relation.

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