

## Neurocomputation by Reaction Diffusion

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This Letter demonstrates the possible role nonsynaptic diffusion neurotransmission may play in neurocomputation using an artificial neural network model. A reaction-diffusion neural network model with field-based information-processing mechanisms is proposed. The advantages of nonsynaptic field neurotransmission from a computational viewpoint demonstrated in this Letter include long-range inhibition using only local interaction, nonhardwired and changeable (target specific) long-range communication pathways, and multiple simultaneous spatiotemporal organization processes in the same medium.

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Synaptic transmission had been considered as essentially the exclusive mechanism for neurotransmission, subject to neuromodulators, in brain functions. This is also the case in artificial neural network studies. However, there is increasing evidence of nonsynaptic diffusion neurotransmission, also referred to as nonsynaptic field neurotransmission or volume transmission, in the brain by diffusion through extracellular fluid and across membranes [1–5]. Such evidence, together with the evidence for a highly plastic brain, point to the need for a theory of neurotransmission and brain functions where nonsynaptic diffusion neurotransmission plays a fundamental role. Research into neural network models of spatiotemporal organizations in the nonsynaptic diffusion neurotransmission field is an effective way to achieve theoretical understanding of the computational roles of such neurotransmission. Nonsynaptic diffusion neurotransmission is believed to occur in a larger time scale than does communication through synapses. It distributes information to a general region of sensitive neurons. In most cases, it may be involved more in regulating and modulating the activity of a neural circuit than in determining what the activity actually is, but it may also have similar target cell specificity as do synaptic circuits [6,7]. There is evidence showing that nonsynaptic diffusion neurotransmission may be the primary information transmission mechanism in certain normal mass, sustained functions, such as vigilance, hunger, long-term potentiation, brain tone and mood, and response to certain sensory stimuli, as well as several abnormal functions such as mood disorder, spinal shock, spasticity, and drug addiction [1]. It may also play a role in functional organization following brain damage [2]. For a more recent account of details on the dynamics and role neuromodulators in the neocortex, see [8]. This Letter aims to demonstrate some advantages of nonsynaptic diffusion neurotransmission from a computational viewpoint, including long-range inhibition using only local interaction, nonhardwired and changeable long-range communication pathways, and multiple simultaneous spatiotemporal organization processes in the same medium.

It will be shown in this Letter that nonsynaptic diffusion neurotransmission can be modeled using neural network circuits with electrical connections. Furthermore, using this model, it will be shown that a neural network with diffusion transmission may compute the winner-take-all logic in a large array of neurons using only local connections, and may transmit information by propagating structured traveling waves without attenuation as in an excitable medium. The winner-take-all network is a key component in neural networks for unsupervised competitive learning and self-organizing topographic maps [9–11]. It selects from an array of neurons a single neuron, or a neighborhood of a neuron, that received the maximum stimulus. In a winner-take-all network, every unit has the same center-on surround-off connection weight pattern, and each unit must have inhibitory connections to all the other units in the network. A hardware implementation of the winner-take-all network requires the network's graph to be completely connected. Without the complete connection, multiple local maxima will be selected, instead of a single global minimum. However, the number of connections in a complete connection grows quadratically with the number of units in the network. For a large network, this high connection complexity may create a problem in hardware implementation.

Reaction-diffusion systems describe many biological processes. Examples include the Hodgkin-Huxley equations, the equations describing the propagating of nerve pulses and the potassium ion and calcium ion concentrations in cortical structures. Reaction-diffusion systems have been shown to be able to propagate structured wave patterns without attenuation [12–14], to form stable patterns, and to select the maximum stimulus using only local interaction [15–19]. A reaction-diffusion system requires the presence of a pair of antagonistic neurotransmitters. Note that a neurotransmitter may interact with more than one other neurotransmitter. Of course, the same system may also model diffusion neurotransmission of a single neurotransmitter without the antagonistic inhibitor. This can be treated as a special case in a reaction-diffusion system. However, without an antagonistic inhibitor, the spa-

spatiotemporal organizations possible in a diffusion process will be rather limited, e.g., to a monotone spatial gradient distribution in a homogeneous medium. Therefore the focus will be on using reaction-diffusion systems to model the spatiotemporal organizations in a nonsynaptic diffusion neurotransmission field. The two-dimensional reaction-diffusion system is described by the following equations:

$$\begin{aligned}\frac{\partial a}{\partial t} &= \rho_a \left( c_a + \frac{ca^2}{h} \right) - \mu_a a + D_a \left( \frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} \right), \\ \frac{\partial h}{\partial t} &= \rho_h c_h a^2 - \mu_h h + D_h \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right),\end{aligned}\quad (1)$$

where  $a(x, y, t)$  and  $h(x, y, t)$  are the concentrations of the activator and the inhibitor in a two-dimensional space. Both the activator and the inhibitor diffuse with diffusivities  $D_a$  and  $D_h$ , respectively. The inhibitor is produced at a rate of  $\rho_h c_h a^2$  where  $\rho_h$  is the density of neurons participating in the production of the inhibitor. This implies that the inhibitor is produced only when there is an activator present. The activator catalyzes its own production, represented by the term  $\rho_a c_a a^2/h$ , where  $\rho_a$  is the density of neurons participating in the production of the activator and  $c$  is a constant. The  $\rho_a c_a a^2/h$  term also shows that the inhibitor concentration  $h$  diminishes the production of the activator because  $h$  is the denominator. The constant term  $\rho_a c_a$  means that there is activator production even at very low activator concentrations. The catalytic effect is in the form of  $a^2$  to obtain the desired behavior. One principle of chemical production in a biological system is that there should not be continual accumulation. Therefore the activator and the inhibitor should be absorbed in some way. In the above equation, they decay in proportion to their concentration, represented by  $-\mu_a a$  and  $-\mu_h h$ , respectively.

Formation of stable spatial patterns has been shown by analysis and by simulation if the inhibitor diffuses much faster than the activator and has a much shorter time constant. In more detail, in order to achieve stable spatial patterns from random fluctuations, strong autocatalysis and an inhibitor range  $(D_h/\mu_h)^{1/2}$  at least 2.5 times the activator range  $(D_a/\mu_a)^{1/2}$  are required [18,19]. This is the same principle of center-on surround-off interaction pattern, i.e., short-range activation and long-range inhibition, found necessary for self-organization in neural networks and in many neural functions. Different patterns may be formed by adjusting the coefficients in Eq. (1). If the diffusion range of the inhibitor covers the entire field, then the activator distribution will converge to peak around the global maximum of the initial perturbation. When diffusion range of the inhibitor is smaller than that of the activator, more specifically, when  $D_a > D_h$  and  $\mu_a > \mu_h$ , the system becomes an excitable medium propagating structured waves without attenuation [12–14,18,19].

Therefore, if we can properly model the above reaction-diffusion system in a circuit network, it may be used to model spatiotemporal organization in a nonsynaptic diffusion neurotransmission field. How do we model diffusion

and reaction in a neural network? The reaction-diffusion equations assume a continuous medium. In a neural network, the cells are at discrete locations. Hence, the spatial Laplacian operator need be approximated by finite differences. Activator and inhibitor can be modeled by using two types of signals. Each cell in the network should be able to produce, receive, process, and pass two types of signals, one representing the activator, the other representing the inhibitor. To be able to do this while sharing the same connections, two modulated signals, e.g., amplitude modulated or frequency modulated, may be used with one carrier for each signal. The carrier for a signal is the same throughout the network. When a signal is received, a corresponding bandpass filter and demodulator will detect the next signal. After the signal is processed, it will be modulated again to be passed onto the next cell. The network shown in Fig. 1 is able to implement the above ideas. The connections are limited to only the immediate neighbors. A four-neighbor connectivity pattern is shown for clear illustration. A square eight-neighbor or a hexagonal six-neighbor connectivity pattern may be used as well. Note that the modulated signals in the network are used to model the diffusion of different chem-

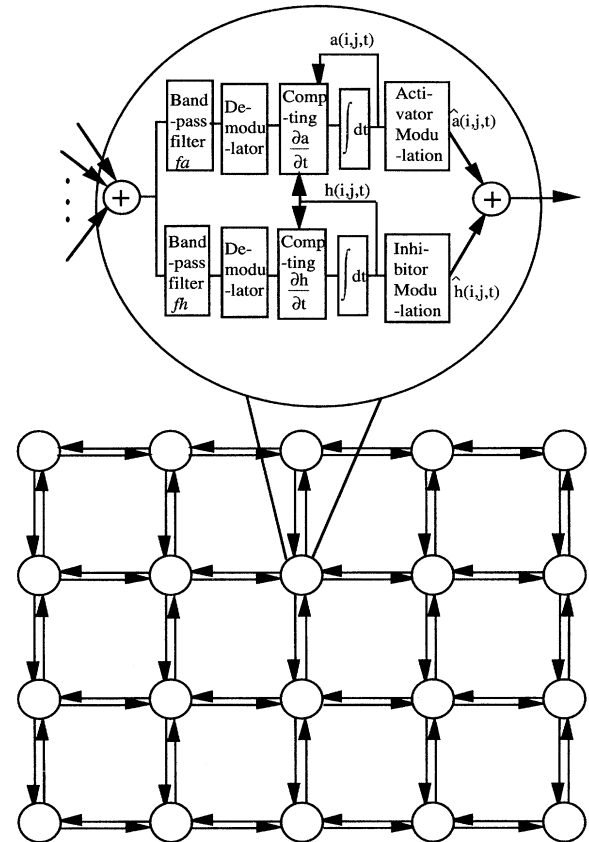
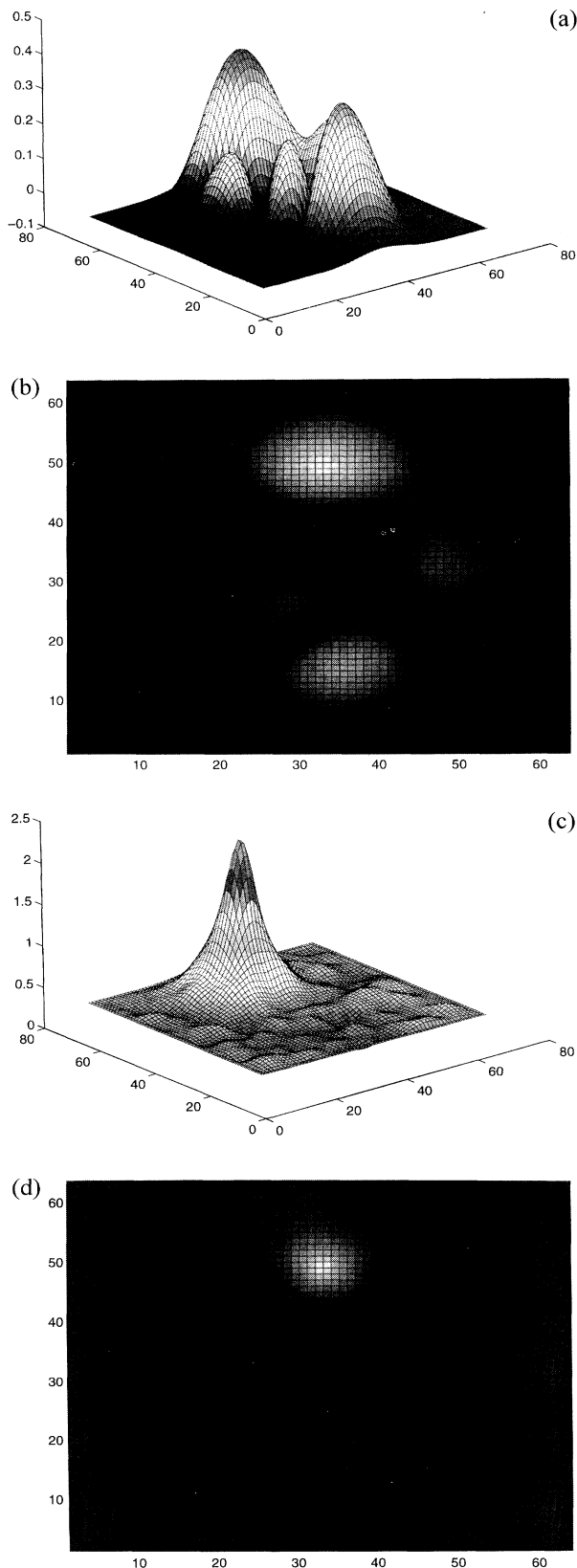


FIG. 1. A locally connected network that models the reaction-diffusion equations.  $\hat{a}(i, j, t)$  and  $\hat{h}(i, j, t)$  are two modulated signals each of which has its own carrier.



ical neurotransmitters in the brain. The propagation and processing of each modulated signal represents the diffusion of a different kind of neurotransmitter. The network model implements the diffusion neurotransmission process by a different mechanism and should not be interpreted as a model of actual electrical neural signals in the brain. With proper choice of network parameters of diffusion ranges (diffusivities and decay rates) for the two types of signals, the network may function as a winner-take-all network or an excitable medium transmitting information by propagation of structured traveling waves.

An important feature of this network is its capability to have *multiple simultaneous spatiotemporal organization processes* in the same network. These processes may interact or may be independent. This may be advantageous in neural computation and self-organization. It can be achieved by using multiple modulation carriers and bandpass filters at each cell. Since a signal may serve as the inhibitor (activator) for more than one activator (inhibitor), the number of modulation carriers  $N_m$  satisfies  $N_p + 1 \leq N_m \leq 2N_p$ , where  $N_p$  is the number of spatiotemporal organization processes in the network.

Use of a modulated signal allows target specificity in information transmission by wide-area diffusion. This is achieved by adjusting the modulation carrier of a diffusing signal and the receptor, i.e., the central frequency of the bandpass filter. In this way, although the signal diffuses to a wide area, only those neurons in the network with the matching bandpass filter can respond to the signal. The modulation carriers and the bandpass filters may be controlled by the spatiotemporal organization pattern in the network. In nonsynaptic diffusion neurotransmission, this may correspond to neurons tuned to be sensitive only to specific neurotransmitters. The long-range communication pathways realized this way are not hardwired; they can change as the environment evolves. This shows the flexibility of diffusion neurotransmission. This flexibility cannot be achieved using hardwired connections.

Examples of computer simulation results are given below. Figures 2(a) and 2(b) show the initial input to a network with an inhibitor diffusion range covering the entire network ( $64 \times 64$  neurons) and a small (radius of  $\sim 6$  neurons) activator diffusion range. Each pixel represents a neuron. The brightness of a neuron indicates the amplitudes of the initial input to the neuron. Figures 2(c)

FIG. 2. The initial array of input to the network represented by a surface plot (a), and by a brightness image (b) where the brightness of a neuron indicates the amplitude of its initial input. (c),(d) Activator signal strength after the network stabilizes (with an inhibitor diffusion range covering the entire network,  $64 \times 64$  neurons, and a small activator diffusion range with a radius of  $\sim 6$  neurons). As illustrated, the networks picks out the global maximum in (a),(b).

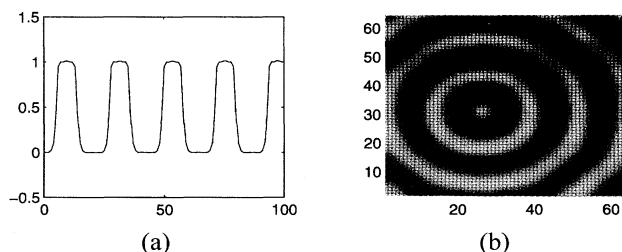


FIG. 3. Examples of a one-dimensional wave pattern (a) and a two-dimensional wave pattern (b) of the activator signal strength in a network functioning as an excitable medium.

and 2(d) show the activator signal strength after the network stabilizes. As illustrated, the network picks out the global maximum in Fig. 2(a). Figures 3(a) and 3(b) show examples of one-dimensional and two-dimensional wave patterns of the activator signal in a network functioning as an excitable medium.

Other neural networks simulating diffusion processes exist. The cellular network by Chua's group [20,21] can simulate a diffusion process modeling an excitable medium. Cellular neural networks limit the synaptic connections to local neighborhoods. They have been shown to be as universal as the Turing machine [20]. The resistive network in the silicon retina by Mead's group [22,23] is also a diffusion network. The space constant in the resistive network determines the diffusion range. An antagonistic center-surround response pattern within a small immediate neighborhood is achieved using the interactions of the resistive network and the photoreceptor and bipolar cell at each node. A common characteristic between the reaction-diffusion network and resistive network is the high degree of shared connections. Many areas of the brain are known to minimize wiring by sharing connections. This is in the same spirit as nonsynaptic diffusion neurotransmission which reduces hardwiring by using wide-area diffusion neurotransmission. The reaction-diffusion network presented in this Letter distinguishes from the cellular neural network and the resistive network in its use of modulated signals, representing antagonistic signals for spatiotemporal organizations. This allows center-on surround-off response patterns of any size using only immediate local connections, multiple simultaneous spatiotemporal organization processes, and target specificity in diffusion transmission.

- [1] P. Bach-y-Rita, *Neuroreport* **4**, 343–350 (1993).
- [2] P. Bach-y-Rita, *Neurochemistry Intl.* **23**, 297–318 (1993).
- [3] K. Fuxe and L.F. Agnati, *Volume Transmission in the Brain: Novel Mechanisms for Neurotransmission* (Raven Press, New York, 1991).
- [4] L.F. Agnati, B. Bjelke, and K. Fuxe, *Am. Sci.* **80**, 362–373 (1992).
- [5] E.S. Vizi, *Nonsynaptic Transmission between Neurons: Modulation of Neurochemical Transmission* (John Wiley, New York, 1984).
- [6] F.O. Schmitt, *Neuroscience* **13**, 991–1001 (1984).
- [7] F.O. Schmitt, in *Fast and Slow Chemical Signalling in the Nervous System*, edited by L.L. Iverson and E. Goodman (Oxford University Press, Oxford, 1986), pp. 239–243.
- [8] R. Silberstein, in *Neocortical Dynamics and Human EEG Rhythms*, edited by P.L. Nunez (Oxford University Press, New York, 1995), pp. 628–681.
- [9] T. Kohonen, *Bio. Cybernet.* **43**, 59–69 (1982).
- [10] G.A. Carpenter and S. Grossberg, *IEEE Computer.* **21**, 77–88 (1988).
- [11] D.J. Willshaw and C. von der Malsburg, *Proc. R. Soc. London B* **194**, 431–445 (1976).
- [12] V.S. Markin, V.F. Pastushenko, and Y.A. Chizmadzhev, *Theory of Excitable Media* (Wiley, New York, 1987).
- [13] *Nonlinear Wave Processes in Excitable Media*, edited by A.V. Holden, H. Markus, and H.G. Othmer (Plenum Press, New York, 1991).
- [14] E. Meron, *Phys. Lett. (Review Section)* **218**, 1–66 (1992).
- [15] A.M. Turing, *Philos. Trans. R. Soc. London B* **237**, 37–72 (1952).
- [16] A. Gierer and H. Meinhardt, *Kybernetik* **12**, 30–39 (1972).
- [17] H. Meinhardt and A. Gierer, *J. Neuron Sci.* **15**, 321–346 (1974).
- [18] H. Meinhardt, *Rep. Prog. Phys.* **55**, 797–849 (1992).
- [19] L.A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (Cambridge University Press, London, 1984).
- [20] L.O. Chua, T. Roska, and P.L. Venetianer, *IEEE Trans. Circuits Syst. I* **40**, 289–291 (1993).
- [21] A. Perez-Munuzuri, V. Perez-Munuzuri, V. Perez-Villar, and L.O. Chua, *IEEE Trans. Circuits Syst. I* **40**, 872–877 (1993).
- [22] C. Mead, *Analog VLSI and Neural Systems* (Addison Wesley, Reading, MA, 1989).
- [23] C. Koch, in *Applications of Neural Networks*, edited by H.G. Schuster (VCH, Weinheim, Germany, 1992).

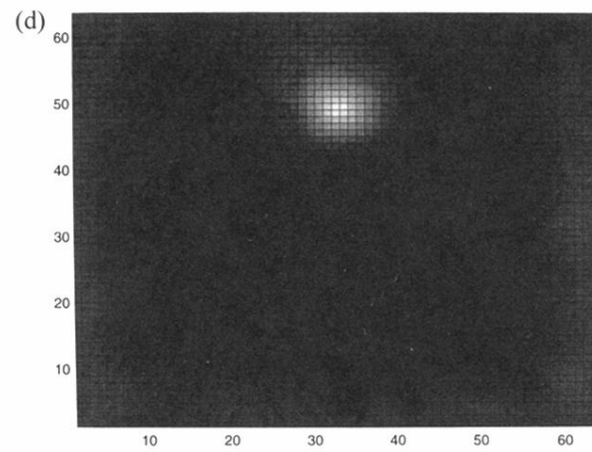
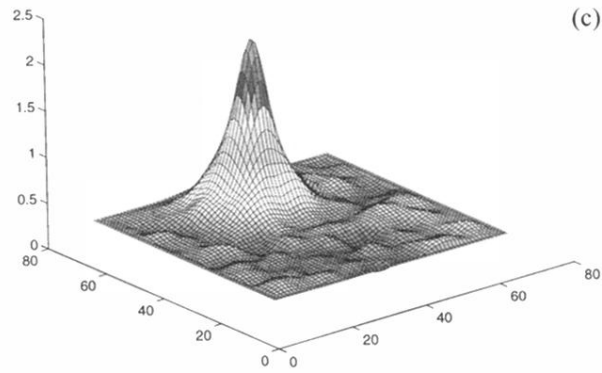
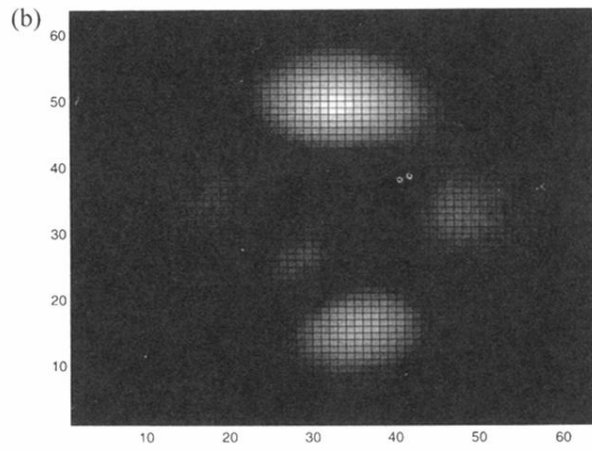
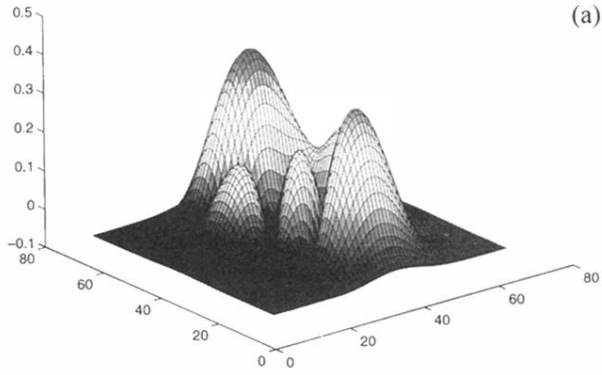


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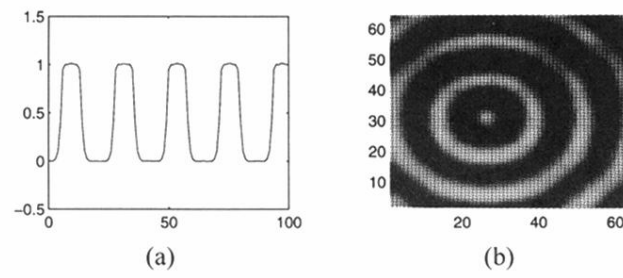


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