Glueballs and Instantons

T. Schäfer and E. V. Shuryak
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 10 November 1994)

Gluonic correlation functions and Bethe-Salpeter amplitudes are calculated in an instanton-based model of the QCD vacuum. We consider both the pure gauge case and the situation for real QCD with two light quark flavors. We show that instantons lead to a strong attractive force in the $J^{PC} = 0^{++}$ channel, which results in the scalar glueball being much smaller than other glueballs. In the 0^{-+} channel the corresponding force is repulsive, and in the 2^{++} case it is absent. The resulting correlators, masses, coupling constants, and wave functions are compared to the results of lattice simulations.

PACS numbers: 12.39.Mk, 12.38.Gc, 12.38.Lg

One of the most obvious questions in QCD is why the observed hadrons are made of quarks and not of gluons. It appears that glueballs are much heavier than typical quark-model hadrons, and therefore they have large widths and/or complicated decay patterns, making them difficult to find. But why are glueballs so heavy? What are their masses, radii, and other parameters in a purely gluonic world, and how do they change if one includes light quarks?

To get a reference point, consider a conventional model of glueball states such as the bag model. The lowest fermion and (electric) gluon modes in a spherical cavity have energies $2.04/R$ and $2.7/R$. Thus, glueballs are expected to be heavier than quark states, but not much. Ignoring spin-dependent forces, one expects $m_{0^{++}} \simeq m_{2^{++}} \simeq 1$GeV and $m_{0^{-+}} \simeq 1.3$ GeV. Including these forces (and other refinements) [1] the model predicts that the low-lying glueballs have masses $m \simeq 1.0-1.8$ GeV and very similar radii $r \simeq 0.7-0.9$ fm.

A number of "glueball candidates" have been experimentally observed, but none was unambiguously identified (see, however, [2]). Lattice simulations provide important qualitative insights, and (although large-scale numerical efforts are still necessary) a few statements appear to be firmly established [3]: (i) The lightest glueball is the scalar, with a mass in the 1.6–1.8 GeV range. (ii) The tensor glueball is significantly heavier $m_{2^{++}}/m_{0^{++}} \simeq 1.4$, with the pseudoscalar one heavier still $m_{0^{-+}}/m_{0^{++}} = 1.5-1.8$ [4]. (iii) The scalar has a much smaller size than other glueballs. This is seen both from the magnitude of finite size effects [5] and directly from measurements of the wave functions [6,7]. The size of the scalar glueball (defined through the exponential decay of the wave function) is $r_{0^{++}} \simeq 0.2$ fm, while $r_{2^{++}} \simeq 0.8$ fm [7]. For comparison, a similar measurement for the π and ρ mesons gives 0.32 and 0.45 fm [7], indicating that spin-dependent forces between gluons are stronger than between quarks.

Important tools that provide information about gluonic interactions are the correlation functions of gluonic operators with the relevant quantum numbers, such as the field strength squared (0^{++}), the topological charge density (0^{-+}), and the energy density (2^{++}):

$$O_S = (gG^a_{\mu\nu})^2, \qquad O_P = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}g^2G^a_{\mu\nu}G^a_{\rho\sigma}, \qquad O_T = \tfrac{1}{4}(gG^a_{\mu\nu})^2 - g^2G^a_{0\alpha}G^a_{0\alpha}\,. \tag{1}$$

In the following, we consider correlation functions $\Pi_\Gamma(x) = \langle 0|O_\Gamma(x)O_\Gamma(0)|0\rangle$ for Euclidean separation x. An important low energy theorem was proven in [8]: The integral of the scalar correlator is determined by the gluon condensate, $\int d^4x\,\Pi_S(x) = (128\pi^2/b)\langle(gG)^2\rangle$, where b denotes the first coefficient of the beta function and the integral is regularized by subtracting the perturbative contribution. This theorem indicates the presence of rather large nonperturbative corrections in the scalar channel. On the other hand, the operator product expansion (OPE) predicts that the leading-order power correction $O(\langle G^2_{\mu\nu}\rangle/x^4)$ vanishes [8], while radiative corrections of the form $\alpha_s \ln(x^2)\langle G^2_{\mu\nu}\rangle/x^4$ or higher order power corrections like $\langle gf^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}\rangle/x^2$ are very small.

In practice, there are two approaches to QCD sum rules for scalar glueballs. In the original work [8], the low energy theorem was enforced by introducing a subtraction constant. In this case, the subtraction constant completely dominates over ordinary power corrections, and one finds a large scalar glueball coupling. However, the vacuum picture advocated in this paper was a rather homogeneous one (with large size instantons melted with other vacuum fluctuations), so that the source of the large subtraction constant was not clear. In more recent works on the subject [9,10], the low energy theorem was not enforced and instead a number of higher order corrections in the OPE were evaluated. Although the resulting mass estimate is similar to what was obtained earlier, the resulting glueball correlation functions are very different. In particular, the low energy theorem is underestimated by about an order of magnitude and the scalar glueball coupling constant is substantially smaller.

In this paper we study gluonic correlation functions in an instanton-based model of the QCD vacuum [11]. The main assumption underlying the model is that the gauge fields in the QCD vacuum are dominated by the

0031-9007/95/75(9)/1707(4)$06.00

strong fields of small size instantons. Support for this assumption is provided by the calculation of hadronic correlators in the model [12] and the analysis of "cooled" lattice configurations [13], where nonclassical fluctuations are removed from the vacuum. Our main point is that small size instantons can lead to a strong enhancement in the scalar correlation function and give a consistent description of the low energy theorem. We believe that the smallness of the scalar glueball provides a strong argument in favor of the picture presented here.

Before we discuss quantitative predictions, let us consider qualitative features of point-to-point correlators [14,15]. Quark correlation functions roughly fall into three classes, depending on how asymptotic freedom is broken at intermediate distances. The nonperturbative corrections may either be (i) large and attractive (the corrections have the same sign as the free correlation function), as for the pseudoscalar π, K, σ mesons; (ii) large and repulsive, as for the heavy scalars η' and δ; or (iii) they can be small even at rather large distances $x \simeq 1$ fm, as is the case for the vector mesons ρ, a_1, ω, ϕ. This classification is easily understood [15] since the instanton-induced interaction between quarks found by 't Hooft [16] has precisely the required spin-isospin properties.

Instanton effects in the gluonic correlation functions can be studied by calculating the correlator in the classical field of a single instanton. Adding the short-distance contribution from the free gluon propagator, one finds

$$\Pi_{S,P}(x) = (\pm)\frac{384 g^4}{\pi^4 x^8} + n\rho^4 \Pi_{\text{inst}}(x), \qquad (2)$$

$$\Pi_T(x) = \frac{24 g^4}{\pi^4 x^8}, \qquad (3)$$

where g is the running coupling constant and $\Pi_{\text{inst}}(x)$ is the instanton contribution

$$\Pi_{\text{inst}}(x) = \frac{12\,288\pi^2\rho^{-8}}{y^6(y^2+4)^5}\left\{y^8 + 28y^6 - 94y^4 - 160y^2 - 120 + \frac{240}{y\sqrt{y^2+4}}(y^6 + 2y^4 + 3y^2 + 2)\operatorname{arcsinh}\left(\frac{y}{2}\right)\right\}, \qquad (4)$$

with $y = x/\rho$. The approximation used is that we ignore any interference between quantum and classical fields. The reason is that these contributions correspond to power corrections $O(\langle G_{\mu\nu}^2\rangle/x^4)$, which, as mentioned above, are very small in the channels considered here.

To first order in the instanton density, we find the three scenarios discussed above: *attraction* in the scalar channel, *repulsion* in the pseudoscalar, and *no* effect in the tensor channel. The last case is a consequence of the fact that the stress tensor in the self-dual field of an instanton is zero. If one compares the result (4) with a similar calculation for the pion, one finds that the instanton contribution in the glueball channel is enhanced by a factor $S_0^2 = [8\pi^2/g(\rho)^2]^2$ with respect to the result for the pion. This means that despite the fact that the scalar glueball is so much heavier than the pion, the correlation function at distances $x \simeq \rho$ is even larger.

In order to make these statements more quantitative and study their dependence on the presence of light quarks, we have calculated the correlators on the presence of light quarks, we have calculated the correlators for three different instanton ensembles. The simplest is the random instanton liquid model (RILM), which assumes that instantons and anti-instantons are distributed randomly in position and color space. Already this simple model is very successful in the description of a large number of hadronic correlation functions [12]. In this model the QCD vacuum is dominated by small-size ($\rho \simeq 0.3$ fm) instanton or anti-instanton fluctuations with a total density $n \simeq 1\ \text{fm}^{-4}$. Including the correlation between instantons, introduced by the gluonic interaction between them, gives a more complicated ensemble that we call the quenched instanton liquid model (QILM). Also taking into account the fermionic determinant one arrives at the unquenched interacting instanton liquid model (IILM). As shown in [17], this ensemble reproduces an important feature of QCD: the screening of the topological charge and a correct description of the η' channel.

In our simulations we calculate the correlators as in (2), but the classical part is now evaluated for multi-instanton configurations. The scalar correlator has an x-independent contribution from the gluon condensate, which must be subtracted. The running coupling constant g is calculated from the perturbative beta function at short distances, but frozen at a value of $\alpha_s/\pi = 0.3$.

The resulting correlation functions are shown in Fig. 1. The correlators are normalized to the free ones, so that all curves approach 1 at short distances. Deviations from 1 at intermediate distances are very different in different channels. Up to $x \simeq 0.25$ fm, these deviations are consistent with the single-instanton correction (4), but at larger x multi-instanton contributions become important. Note that for the pseudoscalar correlator in the interacting ensemble the correction even changes sign. This is a result of correlations between instantons and anti-instantons that lead to the screening of the topological charge. In the ensemble with light quarks a new state, the η' appears. Apart from this, one observes that the results for the three ensembles considered are rather close, suggesting that single-instanton effects (rather than correlations among them) are dominant.

One consistency check is provided by the low energy theorem. We find that the integral of the scalar correlation function (integrated up to $x = 0.7$ fm) is $97 \pm 6\ \text{GeV}^4$ for the random and $66 \pm 7\ \text{GeV}^4$ for the interacting ensembles, to be compared with the low energy theorem

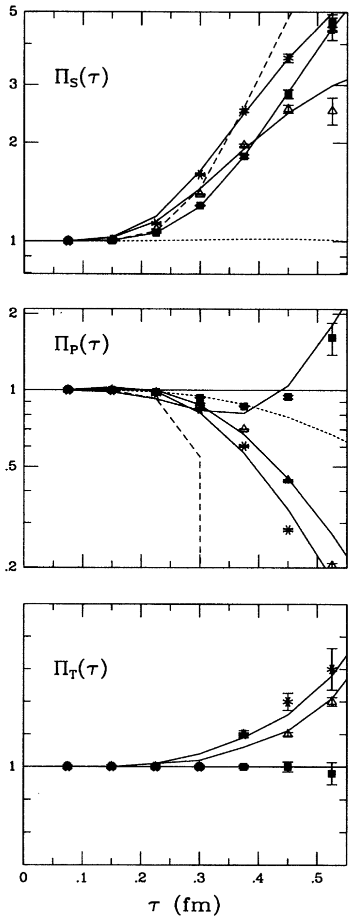

FIG. 1. Scalar, pseudoscalar, and tensor glueball correlation functions normalized to the corresponding free correlators. The results in the random, quenched, and full ensembles are denoted by stars, open triangles, and solid squares, respectively. The solid lines show the parametrization described in the text, the dashed line the dilute instanton gas approximation, and the dotted line the QCD sum rule calculation [10]. The horizontal line in the second figure was added to guide the eye, the vertical scale in the third figure is 10^{-4}.

value 62 GeV^4. In Fig. 1 we also compare our results with predictions from QCD sum rules. The dotted lines correspond to the glueball parameters obtained in [10] (the results from [9] are very similar). We clearly observe that QCD sum rules do *not* predict a substantial enhancement of the scalar correlator, the value of the sum rule is only 13 GeV^4. Unfortunately, most lattice simulations use very nonlocal operators (in order to increase the ground state signal), and their correlators cannot be compared directly to our results. We strongly encourage direct measurements of point-to-point correlation functions and the corresponding coupling constants, similar to the results for quark correlators reported in [14].

We have fitted the glueball masses and coupling constants using a parametrization of the spectral function that consists of a zero-width pole and a continuum starting at a threshold s_0. For $\Gamma = S, P$ the correlator reads

$$\Pi_\Gamma(x) = \lambda_\Gamma^2 D(m_\Gamma, x) + \frac{2g^4}{\pi^2}\int_{s_0} ds\, s^2 D(\sqrt{s}, x), \quad (5)$$

where $D(m,x) = (m/4\pi^2 x)K_1(mx)$ is the Euclidean scalar propagator and $\lambda_\Gamma = \langle 0|O_\Gamma|0^{PC}\rangle$ is the coupling of the resonance to the gluonic current. Statistical fluctuations at large distances limit the accuracy of our mass determination, but the coupling constants are determined rather well. Fitting the parametrization (5) to the measured correlator for scalar gluonium in the random model we find a mass $m_{0^{++}} = 1.4 \pm 0.2$ GeV with the coupling strength $\lambda_{0^{++}} = 17.2 \pm 0.5\ \mathrm{GeV}^3$. In the quenched and the full ensemble the correlation function is somewhat smaller at intermediate distances, a consequence of the low energy theorem discussed above. At distances $x > 0.5$ fm there are large uncertainties due to the subtraction. The mass and coupling constant are $m_{0^{++}} = 1.25$ GeV and $\lambda_{0^{++}} = 15.6\ \mathrm{GeV}^3$ in the full theory and $m_{0^{++}} = 1.75$ GeV and $\lambda_{0^{++}} = 16.5\ \mathrm{GeV}^3$ in the quenched case. These values are about twice as big as the value obtained in the only lattice measurement of this quantity, $\lambda_{0^{++}} = 7.8\ \mathrm{GeV}^3$ [18].

In the pseudoscalar case the classical and one-loop contributions have opposite signs. At distances where they tend to cancel each other our approximation (which neglects the interference between the two) becomes questionable. However, the rapid downturn directly translates into the position of the perturbative threshold, for which we find $\sqrt{s_0} \simeq 3.0$ GeV in the random model and $\sqrt{s_0} \simeq 2.4$ GeV in the quenched theory. We see no clear evidence for a pseudoscalar glueball state below the continuum threshold. In the unquenched theory we observe the η' signal with $m_{\eta'} \lesssim 800$ MeV and $\lambda_{\eta'} \simeq 7.0\ \mathrm{GeV}^3$. Using the anomaly equation, this corresponds to $f_{\eta'} = 200$ MeV.

The tensor channel has no large nonperturbative corrections, because isolated instantons and anti-instantons have a vanishing energy-momentum tensor. Thus the classical contribution is entirely due to the interaction between instantons. One may therefore question the importance of instantons in this channel. On the other hand, the OPE also does not predict any power corrections at $O(G^2)$ and $O(G^3)$, and in a self-dual background field all power corrections vanish [8]: the smallness of the nonperturbative corrections may therefore survive. We find a small classical contribution to the tensor correlator (see Fig. 1), but have not made an attempt to determine the corresponding mass value. The data can be used to put an upper limit $\lambda_T \lesssim 0.6\ \mathrm{GeV}^3$ on the tensor coupling.

Since the scalar correlator is so much bigger than the other ones, one may speculate that the scalar glueball should also be much more compact. We have checked this statement by calculating the Bethe-Salpeter amplitudes (or "wave functions"), defined as

$$\psi_\Gamma(y) = \lim_{x\to\infty} \frac{1}{\Pi_\Gamma(x)} \langle 0|O_\Gamma(x)O_\Gamma^y(0)|0\rangle, \quad (6)$$

where $O_S^y(0) = g^2 G^a_{\mu\nu}(-y/2) G^a_{\mu\nu}(y/2)$, etc. are point-split operators and y is orthogonal to x. By definition, $\psi(0) = 1$. At small separation x the wave function of the scalar glueball essentially measures the size of the instanton, $\psi_S(y) = 1 - (2y/3\rho)^2 + O(y^4)$.

Our results for the random ensemble are shown in Fig. 2 (those for other ensembles are very similar). The scalar wave function is indeed found to be very compact. It is not exponential at short distances (presumably due to the lack of short range perturbative interactions), but the overall shape can be described by an exponential decay $\psi(y) = \exp(-y/R)$, with a fitted radius $R = 0.21$ fm. This value is in good agreement with the lattice result $R \simeq 0.2$ fm [7]. The tensor wave function is much larger in size, $R = 0.61$ fm, to be compared with the lattice result $R \simeq 0.8$ fm. Together with our earlier work [12], in which we determined the sizes of the pion $r = 0.56$ fm and rho meson $r = 0.70$ fm, this shows that the instanton model leads to significantly larger spin splittings for the glueball radii. For the pseudoscalar glueball the interaction is repulsive and we do not find a localized wave function in our model. We therefore conjecture that lattice measurements should find a dip in the wave function at small distances. Let us conclude by noting that the observed hierarchy of sizes, from a very small scalar to a large tensor and a presumably large pseudoscalar, is of great significance for phenomenological searches, since it may affect the branching ratios into different final states (see [2] and the discussion in [7]).

In summary, we have shown that instanton-induced forces between gluons lead to strong attraction between gluons in the 0^{++} channel, strong repulsion in the 0^{-+} channel, and no short-distance effects in the 2^{++} channel.

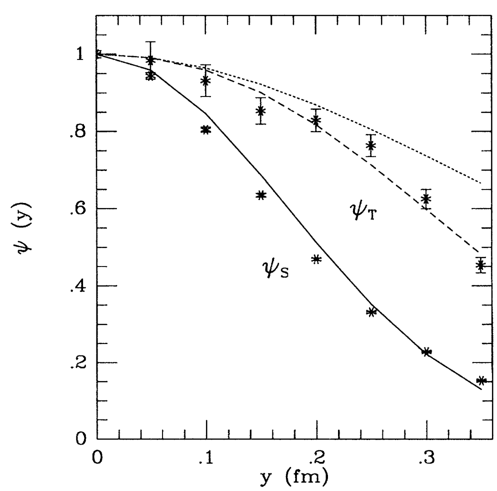

FIG. 2. Scalar and tensor glueball Bethe-Salpeter wave functions in the random instanton ensemble. All wave functions are normalized to 1 at the origin. The solid and dashed lines show a parametrization of the data used to extract the mean square radii while the dashed curve shows the scalar wave function in the dilute instanton gas.

We have calculated point-to-point correlation functions using the "instanton liquid" model (with and without quarks). The fitted masses are compatible with lattice results. More importantly, our large scalar coupling constants are in agreement with the low energy theorem, and the scalar gluonium size is as small as 0.2 fm, as was observed in [7]. We have argued that measurements of gluonic correlators and wave functions provide important insights into the structure of the QCD vacuum, and lattice measurements of the correlators predicted in this paper would be very desirable.

We would like to thank P. van Baal for a useful discussion that triggered this paper. This work was supported in part by U.S. DOE Grant No. DE-FG02-88ER40388 and the A. v. Humboldt Foundation.

[1] C. E. Carlson, T. H. Hansson, and C. Peterson, Phys. Rev. D **27**, 1556 (1983); **28**, 2895(E) (1983); M. Chanowitz and S. Sharpe, Nucl. Phys. **B222**, 211 (1983).
[2] According to D. Weingarten in a talk presented at Lattice-94 (Bielefeld), a lattice calculation of the partial decay widths of the scalar gluonium into $\pi\pi$, $\overline{K}K$, $\eta\eta$ agrees well with the observed branching ratios of the $f_0(1750)$.
[3] D. Weingarten, Nucl. Phys. (Proc. Suppl.) **34**, 29 (1994).
[4] G. S. Bali, K. Schilling, A. Hulsebos, A. C. Irving, C. Michael, and P. W. Stephenson, Phys. Lett. B **309**, 378 (1993).
[5] P. Van Baal and A. S. Kronfeld, Nucl. Phys. B (Proc. Suppl.) **9**, 227 (1989).
[6] K. Ishikawa, G. Schierholz, H. Schneider, and M. Tepper, Nucl. Phys. **B227**, 221 (1983); T. A. DeGrand, Phys. Rev. D **36**, 176 (1987); R. Gupta, A. Patel, C. F. Baillie, G. W. Kilcup, and S. R. Sharpe, Phys. Rev. D **43**, 2301 (1991).
[7] F. de Forcrand and K.-F. Liu, Phys. Rev. Lett. **69**, 245 (1992).
[8] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B165**, 67 (1979); Nucl. Phys. **B174**, 378 (1980); Nucl. Phys. **B191**, 301 (1981).
[9] S. Narison, Z. Phys. C **26**, 209 (1984).
[10] E. Bagan and S. Steele, Phys. Lett. B **243**, 413 (1990).
[11] E. V. Shuryak, Nucl. Phys. **B203**, 93 (1982); **B203**, 116 (1982).
[12] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. **B410**, 55 (1993); T. Schäfer, E. V. Shuryak, and J. J. M. Verbaarschot, Nucl. Phys. **B412**, 143 (1994); T. Schäfer and E. V. Shuryak, Phys. Rev. D **50**, 478 (1994).
[13] M.-C. Chu, J. M. Grandy, S. Huang, and J. Negele, Phys. Rev. D **49**, 6039 (1994).
[14] M.-C. Chu, J. M. Grandy, S. Huang, and J. Negele, Phys. Rev. Lett. **70**, 255 (1993).
[15] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
[16] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[17] E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **52**, 295 (1995).
[18] Y. Liang, K. F. Liu, B. A. Li, S. J. Dong, and K. Ishikawa, Phys. Lett. B **307**, 375 (1993).