CP Asymmetry Relations between $\bar{B}^0 \to \pi\pi$ and $\bar{B}^0 \to \pi K$ Rates

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(Received 30 December 1994)

We prove that the *CP* violating rate difference $\Delta(\pi^+\pi^-) = \Gamma(\bar{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^-\pi^+)$ is related to $\Delta(\pi^+K^-) = \Gamma(\pi^+K^-) - \Gamma(B^0 \to \pi^-K^+)$ in the three generation standard model. Neglecting small annihilation diagrams, and in the SU(3) symmetry limit, we show that $\Delta(\pi^+\pi^-) = -\Delta(\pi^+K^-)$. The SU(3) breaking effects are estimated using the factorization approximation, and yield $\Delta(\pi^+\pi^-) \approx -(f_\pi/f_K)^2\Delta(\pi^+K^-)$. The usefulness of this relation for determining phases in the CKM unitarity triangle is discussed.

PACS numbers: 11.30.Er, 12.15.Hh, 13.25.Hw

Detection of *CP* violation and verification of the unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is a major goal of *B* factories. In this Letter we shall prove a remarkable relationship between the rate difference $\Delta(\pi^+\pi^-(\pi^0\pi^0)) = \Gamma(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0)) - \Gamma(B^0 \to \pi^-\pi^+(\pi^0\pi^0))$ and $\Delta(\pi^+K^-(\pi^0\bar{K}^0)) = \Gamma(\bar{B}^0 \to \pi^+K^-(\pi^0\bar{K}^0)) - \Gamma(B^0 \to \pi^-K^+(\pi^0K^0))$. This relationship is shown to follow from purely SU(3) symmetry and neglecting annihilation diagrams that are estimated to make a negligible contribution. In particular, we do not rely on the spectator model, which could be unreliable for two body decays. These relations will provide a useful test of the standard model (SM) and also be of immense value in making precise measurements of the phase angles in the unitarity triangle. Another crucial observation is that many researchers have suggested looking for rate asymmetry in the $B \to \pi K$ system where there is interference between the tree and penguin contributions. On the other hand, it was thought that in the $B \to \pi\pi$ process penguins do not play an important role. Our work shows the close connection between the two systems. The presence of asymmetry in $B \to \pi K$, which can be measured at CLEO, will imply penguins in the $B \to \pi\pi$ process.

The angle $\alpha = \arg(V_{tb}V_{td}^*/V_{ub}V_{ud}^*)$ can be determined by measuring the time dependent *CP* asymmetry $a(t)_{+-(00)}$ in $\bar{B}^0(B^0) \to \pi^+\pi^-(\pi^0\pi^0)$ decays [1,2]. The coefficient of the term varying with time as $\sin(\Delta m t)$ is proportional to $\mathrm{Im}\lambda_{+-(00)}$, which is defined as

$$\mathrm{Im}\lambda_{+-(00)} = \frac{|\bar{A}(\pi^+\pi^-(\pi^0\pi^0))|}{|A(\pi^-\pi^+(\pi^0\pi^0))|}\sin(2\alpha + \theta_{+-(00)}), \tag{1}$$

where $\bar{A}(\pi\pi)$ and $A(\pi\pi)$ are the $\bar{B}^0 \to \pi\pi$ and $B^0 \to \pi\pi$ decay amplitudes, respectively. We will use similar notations for $B(\bar{B}) \to \pi K$ amplitudes. If penguin contributions are ignored, one finds $\bar{A}(\pi^+\pi^-(\pi^0\pi^0))/A(\pi^-\pi^+(\pi^0\pi^0)) = V_{ub}V_{ud}^*/V_{ub}^*V_{ud}$, $\theta_{+-(00)} = 0$, and $\mathrm{Im}\lambda_{+-(00)} = \sin(2\alpha)$. However, the penguin contributions may be large [3] and should not be ignored. $\theta_{+-(00)}$ may substantially deviate from zero and cause uncertainties in the determination of α. A method has been suggested to remove the uncertainties due to penguin effects by measuring the magnitudes of individual decay for $B^0 \to \pi^+\pi^-(\pi^0\pi^0)$, $B^+ \to \pi^+\pi^0$, and the corresponding anti-*B* decays, and using these amplitudes to construct isospin triangle relations for the three *B* decay modes and the three anti-*B* decay modes. Information from the differences of these two triangles will allow one to determine $\theta_{+-(00)}$ [2]. For decay modes of the charged *B*, the amplitudes are easy to measure. For decay modes of the neutral *B*, the measurements will be more difficult. For example, it is easy to measure the average decay amplitude for $\Gamma(B^0(\bar{B}^0) \to \pi^+\pi^-)$. However, to separately determine the amplitudes for $\Gamma(B^0 \to \pi^+\pi^-)$ and $\Gamma(\bar{B}^0 \to \pi^+\pi^-)$, one needs tagging and therefore a loss of event numbers. If $\Delta(\pi^+\pi^-)$ were known, then it would be quite easy to deduce the individual rates. The rate difference $\Delta(\pi^+K^-)$, on the other hand, is much easier to measure because it is a self-tagging mode. Similarly, we can get information for $\Delta(\pi^0\pi^0)$ from the measurement of $\Delta(\pi^0\bar{K}^0)$. In this case the rate difference $\Delta(\pi^0\bar{K}^0)$ is also a difficult quantity to measure because it also needs tagging. However, it might be easier to measure compared with $\Delta(\pi^0\pi^0)$ since $\bar{B}^0 \to \pi^0\pi^0$ is expected to be highly suppressed. We now proceed to prove the relation.

In the SM the most general effective Hamiltonian for $B \to \pi\pi$ and $B \to \pi K$ decays can be written as

$$H_{\mathrm{eff}}^q = \frac{G_F}{\sqrt{2}}\Bigg[V_{ub}V_{uq}^*(c_1O_1^q + c_2O_2^q) - \sum_{i=3}^{10}(V_{ub}V_{uq}^*c_i^u + V_{cb}V_{cq}^*c_i^c + V_{tb}V_{tq}^*c_i^t)O_i^q\Bigg] + \mathrm{H.c.}, \tag{2}$$

where the superscript f indicates the loop contribution from f quarks, and O_i^q are defined as

$$O_1^q = \bar{q}_\alpha \gamma_\mu L u_\beta \bar{u}_\beta \gamma^\mu L b_\alpha, \qquad O_2^q = \bar{q} \gamma_\mu L u \bar{u} \gamma^\mu L b,$$
$$O_{3,5}^q = \bar{q} \gamma_\mu L b \bar{q}' \gamma_\mu L(R) q', \qquad O_{4,6}^q = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}'_\beta \gamma_\mu L(R) q'_\alpha,$$
$$O_{7,9}^q = \tfrac{3}{2} \bar{q} \gamma_\mu L b e_{q'} \bar{q}' \gamma^\mu R(L) q', \qquad O_{8,10}^q = \tfrac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta e_{q'} \bar{q}'_\beta \gamma_\mu R(L) q'_\alpha, \tag{3}$$

where $R(L) = 1 + (-)\gamma_5$, and q' is summed over u, d, and s. For $\Delta S = 0$ processes, $q = d$, and for $\Delta S = 1$ processes, $q = s$. O_2 and O_1 are the tree level and QCD corrected operators. O_{3-6} are the strong gluon induced penguin operators, and operators O_{7-10} are due to γ and Z exchange, and "box" diagrams at the loop level. The Wilson coefficients c_i^f are defined at the scale of $\mu \approx m_b$ and have been evaluated to the next-to-leading order in QCD [4]. Our results are, however, independent of the detailed numerical values of the Wilson coefficients.

Using the unitarity property of the CKM matrix, we can eliminate the term proportional to $V_{cb}V_{cq}^*$ in the effective Hamiltonian. The B decay amplitude due to the complex effective Hamiltonian displayed above can be parametrized, without loss of generality, as

$$\langle \text{final state} | H_{\text{eff}}^q | B \rangle = V_{ub} V_{uq}^* T_q + V_{tb} V_{tq}^* P_q, \tag{4}$$

where T_q contains the *tree contributions* and *penguin contributions* due to u and c internal quarks, while P_q only contains *penguin contributions* from internal c and t quarks.

Since the effective Hamiltonian H_{eff}^d responsible for $\Delta S = 0$ B decays is related to H_{eff}^s for $\Delta S = 1$ B decays by just changing d quark to s quark, one expects certain relations between T_d, P_d and T_s, P_s in the SU(3) limit. Let us consider the two pseudoscalar meson decays of B mesons in a general framework.

SU(3) relations for B decays have been studied by several authors [5–7]. We will follow the notation used in Ref. [6]. The operators $Q_{1,2}$, O_{3-6}, and O_{7-10} transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. In general, we can write the SU(3) invariant amplitude for B to two octet pseudoscalar mesons for T_q in the form

$$\begin{aligned} T = {} & A_{(\bar{3})}^T B_i H(\bar{3})^i (M_l^k M_k^l) + C_{(\bar{3})}^T B_i M_k^i M_j^k H(\bar{3})^j \\ & + A_{(6)}^T B_i H(6)_k^{ij} M_j^l M_l^k + C_{(6)}^T B_i M_j^i H(6)_l^{jk} M_k^l \\ & + A_{(\overline{15})}^T B_i H(\overline{15})_k^{ij} M_j^l M_l^k + C_{(\overline{15})}^T B_i M_j^i H(\overline{15})_l^{jk} M_k^l, \end{aligned} \tag{5}$$

where $B_i = (B^-, \bar{B}^0, \bar{B}_s^0)$ is an SU(3) triplet, M_{ij} is the SU(3) pseudoscalar octet, and the matrices H represent the transformation properties of the operators O_{1-10}. $H(6)$ is a traceless tensor that is antisymmetric on its upper indices, and $H(\overline{15})$ is also a traceless tensor but is symmetric on its upper indices. For $q = d$, the nonzero entries of the H matrices are given by

$$\begin{gathered} H(\bar{3})^2 = 1, \qquad H(6)_1^{12} = H(6)_3^{23} = 1, \qquad H(6)_1^{21} = H(6)_3^{32} = -1, \\ H(\overline{15})_1^{12} = H(\overline{15})_1^{21} = 3, \qquad H(\overline{15})_2^{22} = -2, \qquad H(\overline{15})_3^{32} = H(\overline{15})_3^{23} = -1. \end{gathered} \tag{6}$$

For $q = s$, the nonzero entries are

$$\begin{gathered} H(\bar{3})^3 = 1, \qquad H(6)_1^{13} = H(6)_2^{32} = 1, \qquad H(6)_1^{31} = H(6)_2^{23} = -1, \\ H(\overline{15})_1^{13} = H(\overline{15})_1^{31} = 3, \qquad H(\overline{15})_3^{33} = -2, \qquad H(\overline{15})_2^{32} = H(\overline{15})_2^{23} = -1. \end{gathered} \tag{7}$$

We obtain the amplitudes $T_d(\pi\pi)$, $T_s(\pi K)$ for $\bar{B}^0 \to \pi\pi$, $\bar{B}^0 \to \pi K$ as

$$\begin{aligned} T_d(\pi^+\pi^-) &= 2A_{(\bar{3})}^T + C_{(\bar{3})}^T - A_{(6)}^T + C_{(6)}^T + A_{(\overline{15})}^T + 3C_{(\overline{15})}^T, \\ T_d(\pi^0\pi^0) &= \frac{1}{\sqrt{2}} (2A_{(\bar{3})}^T + C_{(\bar{3})}^T - A_{(6)}^T + C_{(6)}^T + A_{(\overline{15})}^T - 5C_{(\overline{15})}^T), \\ T_d(\pi^-\pi^0) &= \frac{8}{\sqrt{2}} C_{(\overline{15})}^T, \\ T_s(\pi^+K^-) &= C_{(\bar{3})}^T - A_{(6)}^T + C_{(6)}^T - A_{(\overline{15})}^T + 3C_{(\overline{15})}^T, \\ T_s(\pi^0\bar{K}^0) &= -\frac{1}{\sqrt{2}} (C_{(\bar{3})}^T - A_{(6)}^T + C_{(6)}^T - A_{(\overline{15})}^T - 5C_{(\overline{15})}^T). \end{aligned} \tag{8}$$

We also have similar relations for the amplitude P_q. The corresponding amplitudes will be denoted by A_i^P and C_i^P. We clearly see that the triangle relation (which follows from isospin) holds: $\bar{A}(\pi^0\pi^0) + \bar{A}(\pi^-\pi^0) = \bar{A}(\pi^+\pi^-)/\sqrt{2}$. A similar relation for the charge conjugate decay modes also holds.

The amplitudes $A_{(\bar{3}),(6),(\overline{15})}$ all correspond to annihilation contributions. This can be verified because the light quark index in the B meson is contracted with the Hamiltonian. It has been argued that the annihilation contributions are small [7]. In the factorization approximation, these amplitudes correspond to a matrix element of the form, for example, for

$\bar{B}^0 \to \pi^+\pi^-$ decay,

$$M = \langle 0|\bar{d}\Gamma^1 b|\bar{B}^0\rangle\langle\pi^+\pi^-|\bar{q}\Gamma_2 q|0\rangle\,, \tag{9}$$

where q can be u or d quarks. If $\Gamma^1 = \gamma_\mu(1-\gamma_5)$ and $\Gamma_2 = \gamma^\mu(1\pm\gamma_5)$, this matrix element is equal to zero due to vector current conservation. The only exception is when the operators are Fierz transformed, one also obtains a contribution of the type $\Gamma^1 = 1-\gamma_5$ and $\Gamma_2 = 1+\gamma_5$. However, this contribution is suppressed compared with other contributions. In the factorization approximation, for $q = d$, this contribution is given by

$$M = if_B m_B^2 \frac{m_\pi^2}{m_u + m_d}\frac{1}{m_b + m_d}F^{\pi\pi}(m_B^2)\,, \tag{10}$$

where we have used $\langle 0|\bar{d}(1-\gamma_5)b|\bar{B}^0\rangle = if_B m_B^2/(m_b + m_d)$ and $\langle\pi^+\pi^-|\bar{d}d|0\rangle = F^{\pi\pi}(q^2)m_\pi^2/(m_u+m_d)$ [8]. Assuming the single pole model for the form factor, $F^{\pi\pi}(q^2) = 1/(1-q^2/m_\sigma^2)$ with $m_\sigma = 700$ MeV, $F^{\pi\pi}(m_B^2) \approx -0.02$. For $\bar{B}^0 \to \pi^+\pi^-$ we find that the annihilation contribution to $P_d(\pi^+\pi^-)$ is only about 4%, and the contribution to $T_d(\pi^+\pi^-)$ is much smaller. To a good approximation all annihilation amplitudes $A_{(\bar{3}),(6),(\overline{15})}$ can be neglected. From now on we will work in this approximation. We obtain

$$\begin{aligned}T(P)_{+-} &\equiv T(P)_d(\pi^+\pi^-) = T(P)_s(\pi^+K^-)\,,\\ T(P)_{(00)} &\equiv T(P)_d(\pi^0\pi^0) = -T(P)_s(\pi^0\bar{K}^0)\,,\end{aligned} \tag{11}$$

and

$$\begin{aligned}\bar{A}(\pi^+\pi^-(\pi^0\pi^0)) &= V_{ub}V_{ud}^*T_{+-(00)} + V_{tb}V_{td}^*P_{+-(00)}\,,\\ \bar{A}(\pi^+K^-(\pi^0\bar{K}^0)) &= (-)V_{ub}V_{us}^*T_{+-(00)}\\ &\quad + (-)V_{tb}V_{ts}^*P_{+-(00)}\,.\end{aligned} \tag{12}$$

Analogous relations have been discussed in the context of obtaining information about penguin contributions to B decays and to determine the unitarity triangle of the CKM matrix [7,9]. Some of these studies suffer from uncertainties in the strong rescattering phases in the amplitudes. We shall use Eq. (12) to derive relations between the decay rate differences that do not have uncertainties associated with lack of knowledge of the strong rescattering phases. We have

$$\begin{aligned}\Delta(\pi^+\pi^-(\pi^0\pi^0)) &= -\,\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td})\,\mathrm{Im}(T_{+-(00)}P_{+-(00)}^*)\frac{m_B\lambda_{\pi\pi}}{4\pi}\,,\\ \Delta(\pi^+K^-(\pi^0\bar{K}^0)) &= -\,\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts})\,\mathrm{Im}(T_{+-(00)}P_{+-(00)}^*)\frac{m_B\lambda_{\pi K}}{4\pi}\,,\end{aligned} \tag{13}$$

where $\lambda_{ab} = \sqrt{1-2(m_a^2+m_b^2)/m_B^2 + (m_a^2-m_b^2)^2/m_B^4}$. In the SU(3) symmetry limit, $\lambda_{\pi\pi} = \lambda_{\pi K}$. Because of the unitarity property of the CKM matrix, for three generations of quarks, $\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts}) = -\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td})$ [10]. We then find

$$\Delta(\pi^+\pi^-(\pi^0\pi^0)) = -\Delta(\pi^+K^-(\pi^0\bar{K}^0))\,. \tag{14}$$

These nontrivial equality relations do not depend on the numerical values of the final state rescattering phases. Of course, these relations are true only for the three generation model. Therefore they also provide tests for the three generation model.

The relations obtained above will be modified by SU(3) breaking effects [7,11]. Since no reliable calculational tool exists for two body modes, we shall estimate these effects in the factorization approximation to get an idea of their importance. We have

$$\begin{aligned}T_d(\pi^-\pi^+) &= i\frac{G_F}{\sqrt{2}}f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2-m_\pi^2)\\ &\quad\times\left[\xi c_1 + c_2 + \xi c_3^{cu} + c_4^{cu} + \xi c_9^{cu} + c_{10}^{cu} + \frac{2m_\pi^2}{(m_b-m_u)(m_u+m_d)}(\xi c_5^{cu} + c_6^{cu} + \xi c_7^{cu} + c_8^{cu})\right],\\ T_s(\pi^+K^-) &= i\frac{G_F}{\sqrt{2}}f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\\ &\quad\times\left[\xi c_1 + c_2 + \xi c_3^{cu} + c_4^{cu} + \xi c_9^{cu} + c_{10}^{cu} + \frac{2m_K^2}{(m_b-m_u)(m_u+m_s)}(\xi c_5^{cu} + c_6^{cu} + \xi c_7^{cu} + c_8^{cu})\right],\\ T_d(\pi^0\pi^0) &= i\frac{G_F}{\sqrt{2}}f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2-m_\pi^2)\Big[-c_1 - \xi c_2 + \xi c_3^{cu} + c_4^{cu} + \tfrac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu})\\ &\quad - \tfrac{1}{2}(\xi c_9^{cu} + c_{10}^{cu}) + \frac{2m_\pi^2}{(m_b-m_d)2m_d}[\xi c_5^{cu} + c_6^{cu} - \tfrac{1}{2}(\xi c_7^{cu} + c_8^{cu})]\Big],\\ T_s(\pi^0\bar{K}^0) &= i\frac{G_F}{\sqrt{2}}\Big\{f_\pi F_0^{BK}(m_\pi^2)(m_B^2-m_K^2)[c_1 + \xi c_2 - \tfrac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu})] - if_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\\ &\quad\times\Big[\xi c_3^{cu} + c_4^{cu} - \tfrac{1}{2}(\xi c_9^{cu} + c_{10}^{cu}) - \frac{2m_K^2}{(m_b-m_d)(m_d+m_s)}[\xi c_5^{cu} + c_6^{cu} - \tfrac{1}{2}(\xi c_7^{cu} + c_8^{cu})]\Big]\Big\},\end{aligned} \tag{15}$$

where $c_i^{cu} = c_i^c - c_i^u$, $c_i^{ct} = c_i^c - c_i^t$, and $\xi = 1/N_c$ with N_c being the number of color. The amplitudes $P_{d,s}$ are obtained by setting $c_{1,2} = 0$ and changing c_i^{cu} to c_i^{ct}. We have used the following decompositions for the form factors:

$$\langle \pi^+(q)|\bar{d}\gamma_\mu(1-\gamma_5)u|0\rangle = if_\pi q_\mu, \qquad \langle K^+(q)|\bar{d}\gamma_\mu(1-\gamma_5)u|0\rangle = if_K q_\mu,$$

$$\langle \pi^-(k)|\bar{u}\gamma_\mu b|\bar{B}^0(p)\rangle = (k+p)_\mu F_1^{B\pi} + (m_\pi^2 - m_B^2)\frac{q_\mu}{q^2}[F_1^{B\pi}(q^2) - F_0^{B\pi}(q^2)],$$

$$\langle K^-(k)|\bar{u}\gamma_\mu b|\bar{B}^0(p)\rangle = (k+p)_\mu F_1^{BK} + (m_\pi^2 - m_B^2)\frac{q_\mu}{q^2}[F_1^{BK}(q^2) - F_0^{BK}(q^2)]. \quad (16)$$

Using the fact that $m_\pi^2/(m_u + m_d) = m_K^2/(m_u + m_s)$, we obtain in place of Eq. (14)

$$\Delta(\pi^+\pi^-) = -\frac{[f_\pi F_0^{B\pi}(m_\pi^2)]^2}{[f_K F_0^{B\pi}(m_K^2)]^2}\frac{\lambda_{\pi\pi}}{\lambda_{\pi K}}\Delta(\pi^+K^-). \quad (17)$$

The above expression neglects possible SU(3) breaking corrections to the strong rescattering phases, which cannot be calculated using the simple minded factorization approximation. We expect these corrections to be small. Assuming a single pole for the form factor $F_0^{B\pi}(q^2)$, the form factor has the form $F_0^{B\pi}(q^2) = 1/(1 - q^2/m_{0^+}^2)$ with $m_{0^+} = 5.78$ GeV. To a good approximation, we have $(\lambda_{\pi\pi}/\lambda_{\pi K})[F_0^{B\pi}(m_\pi^2)/F_0^{B\pi}(m_K^2)]^2 \approx 1$. We finally obtain

$$\Delta(\pi^+\pi^-) \approx -\frac{f_\pi^2}{f_K^2}\Delta(\pi^+K^-). \quad (18)$$

The result can be understood by noting that in the spectator approximation the main SU(3) breaking effect arises from the difference in the $W^+ \to \pi^+$ compared to the $W^+ \to K^+$ transition. Although we expect this to be a reasonable estimate of the SU(3) breaking effects, if nonspectator diagrams play a more important role than the factorization approximation suggests, the result could be seriously contaminated.

For $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \pi^0\bar{K}^0$, the correction is more complicated for two reasons: (i) in general $f_\pi F_0^{BK}(m_\pi^2)$ is not equal to $f_K F_0^{B\pi}(m_K^2)$, and (ii) the u and d quark masses are not equal. These cause the amplitudes $T(P)_d(\pi^0\pi^0)$ and $T(P)_s(\pi^0\bar{K}^0)$ for $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \pi^0\bar{K}^0$ to be different not simply by an overall factor as in the case for $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^+K^-$. However, we estimate that the SU(3) breaking effect is about 30%.

This work was supported in part by Department of Energy Grant No. DE-FG06-85ER40224.

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