Electroweak Baryogenesis from a Classical Force

Michael Joyce, Tomislav Prokopec, and Neil Turok
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 24 August 1994)

We describe a new effect that produces baryons at a first order electroweak phase transition. It operates when there is a *CP*-violating field present on propagating bubble walls. The novel aspect is that it involves a purely classical force, which alters the motion of particles across the wall and through diffusion creates a chiral asymmetry in front of the wall. We develop a technique for computing the baryon asymmetry using the Boltzmann equation, and a fluid approximation which allows us to model strong scattering effects. The final formula for the baryon asymmetry has a remarkably simple form.

PACS numbers: 98.80.Cq, 11.15.Ex, 11.27.+d, 11.30.Er

The last ten years have seen a growing realization that the standard electroweak theory satisfies Sakharov's conditions for baryogenesis: B violation, departure from thermal equilibrium, and C and CP violation [1]. Nonperturbative B violating processes involving the electroweak $\mathrm{SU}(2)_L$ chiral anomaly appear unsuppressed at high temperatures [2]. The electroweak transition is weakly first order for light Higgs masses, $M_H < 80$ GeV [3]. It proceeds via bubble nucleation, with departures from thermal equilibrium on and around bubble walls [4]. Finally, C and CP are violated, the latter via the phase in the Kobayashi-Maskawa matrix. The CP violation in the minimal theory is very small, but there may be amplification mechanisms that enhance it, or additional Higgs fields with CP violation in the Higgs potential (for reviews see [5,6]).

In this Letter we study baryogenesis due to a CP-violating condensate on bubble walls. Several mechanisms have already been pointed out through which such a condensate can produce a baryon asymmetry. It couples via a term in the effective action to bias the winding of the gauge and Higgs fields [7]. It also acts to bias hypercharge violating particle interactions, which in a certain constrained thermal equilibrium favors baryon production—"spontaneous" baryogenesis [8–11].

In addition to these *local* effects, particle transport can carry the CP violation present on the wall into the unbroken phase, where the B violation rate is maximal [12]. Until recently, this *nonlocal* barogenesis was thought to necessarily involve quantum mechanical interference—the idea was that the condensate causes CP violation in particle-wall scattering amplitudes, leading to a chiral flux (i.e., more t_L's than $\overline{t_L}$'s) being injected into the unbroken phase where it drives baryon production. Top quarks are the obvious candidate because of their large mass and thus strong coupling to the wall. However, significant asymmetries can only be produced for very thin walls because the quantum interference effect tends to be destroyed by the strong (QCD) scattering of the quarks, and is also WKB suppressed for walls much thicker than the inverse top mass. Partly for these reasons we considered tau leptons as an alternative because they are much more weakly coupled to the plasma [13,14].

In this Letter we discuss a new, purely classical mechanism through which nonlocal baryogenesis can be driven. It does not rely on quantum mechanical interference, and thus may be calculated from a Boltzmann equation, which we shall solve analytically in a fluid approximation. The physical picture is extremely simple: the classical force drags an excess of chiral charge onto the wall, leaving a compensating deficit of chiral charge in front of the wall, which drives baryogenesis. Particle transport is the key to this mechanism—if particles are free to diffuse in the medium, they are free to respond to the chiral force on the wall. Conversely, if particles cannot move relative to the plasma (e.g., top quarks in the limit of large α_S) the whole effect goes away, and only local baryogenesis is possible.

The qualitative criterion for efficient transport is that a particle should be able to diffuse a distance $x > L$, the wall width, in the time the wall takes to pass $t = L/v_w$, with v_w the wall velocity. Setting $x^2 \sim Dt$, with D the diffusion constant, we find the following.

Condition 1: $v_w < D/L$ (for efficient transport).

Secondly we require that the phase space density approach a local thermal equilibrium (LTE) form, in which a chiral charge builds up on the wall. This requires that the equilibration time τ be smaller than the time of passage of the wall L/v_w. We will discuss below how we determine this time scale τ within our calculation and find that $\tau \sim D$. We therefore require

Condition 2: $v_w < L/D$ (for LTE to establish).

And finally we demand that the semiclassical (WKB) approximation must be accurate. For this to be true, the effect should come from particles with typical momenta p_z such that

Condition 3: $|p_z| \gg L^{-1}$ (for classicality).

Conditions 1–3 are plausibly met in the standard electroweak model, and minimal extensions. As long as they are, we shall see that the final baryon asymmetry is, to a good approximation, independent of v_w, L, and D.

One loop calculations yield $L \sim (20 - 40)T^{-1}$ [4], and in our mechanism the particles that dominate the effect have $|p_z| \sim T$, so condition 3 is easily satisfied. We have estimated [11] $D \sim 6T^{-1}$ for quarks. Calculations

0031-9007/95/75(9)/1695(4)$06.00

of the wall velocity are difficult [4], but indicate velocities in the range $v_w \sim 0.05-1$. Conditions 1 and 2 are therefore satisfied for a large part of the parameter space indicated by studies of the phase transition. However, we expect particle diffusion in front of the wall to become negligible if $v_w > v_s$, the speed of sound in the plasma, in which case *local* baryogenesis should dominate.

Recently, we pointed out in [10] that transport effects could be important in spontaneous baryogenesis [8], and raised doubts about this mechanism because transport effects spoil the constraints imposed in local equilibrium calculations [10], unless the walls are very thick. Subsequently, one of us pointed out that transport phenomena could actually enhance this mechanism [15], and this has been independently explored using a diffusion equation in [16]. We believe that our procedure provides a more complete framework within which both this and the classical force effect we focus on here can be computed and give a detailed treatment of both effects in [11].

We begin with a derivation of the chiral force. The Lagrangian describing a fermion moving in the classical background of a bubble wall with CP-violating condensate is

$$\mathcal{L}_{\rm ch} = \overline{\Psi}\gamma^\mu i(\partial_\mu - ig_A\tilde{Z}_\mu\gamma^5)\Psi - m\overline{\Psi}\Psi\,, \qquad (1)$$

where $g_A\tilde{Z}_\mu = g_A Z_\mu^{\rm GI} - \frac{1}{2}[v_2^2/(v_1^2 + v_2^2)]\partial_\mu\theta$, $g_A = \frac{1}{4}\sqrt{g_1^2 + g_2^2}$ for t quarks [14]. The two contributions are the CP-odd scalar field θ, which is the relative phase of the two Higgs fields in a two Higgs theory, $\varphi_2^\dagger\varphi_1 = Re^{i\theta}$, and the Z_μ condensate discussed in [17], which may be present even in the minimal theory. The notation GI implies that this is the gauge invariant combination of the gauge fields and Higgs phases that diagonalizes the Higgs kinetic terms. We treat the wall as planar, and assume it has reached a stationary state in which the Higgs and gauge fields are functions of $z - v_w t$. In this case, the field $\tilde{Z}_\mu$ is pure gauge.

The axial coupling in (1) leads to a classical chiral force as follows. In the rest frame of the wall $\tilde{Z}_\mu = (0,0,0,\tilde{Z}_z(z))$. From the corresponding Dirac equation, setting $\psi \propto e^{-ip.x}$, one finds the following dispersion relation [11]

$$E = [p_\perp^2 + (\sqrt{p_z^2 + m^2} \mp g_A\tilde{Z}_z)^2]^{1/2}, \qquad S_z = \pm\tfrac{1}{2}\,, \qquad (2)$$

for both particles and antiparticles. S_z is the component of the spin in the z direction, measured in the frame in which $p_\perp$ vanishes. In the WKB approximation, this dispersion relation accurately describes particles as they move across a bubble wall—the local eigenstates in (2) shall form the basis of our treatment. The particles we are most interested in for baryogenesis are left-handed particles (e.g., t_L's), and their antiparticles ($\overline{t_L}$'s, which are right handed), because these couple to the chiral anomaly. For large $|p_z|$, these are easily identifiable in terms of the eigenstates in (2). Note that they couple *oppositely* to the $\tilde{Z}$ field.

The group velocity of a WKB wave packet is determined from the dispersion relation by $v_z = \dot{z} = \partial E/\partial p_z$, and energy conservation $\dot{E} = 0 = \dot{z}(\partial E/\partial z) + \dot{p}_z(\partial E/\partial p_z)$ then implies that $\dot{p}_z = -\partial_z E$. These are, of course, Hamilton's equations for the motion of a particle. From these it is straightforward to calculate the acceleration

$$\frac{dv_z}{dt} = -\frac{1}{2}\frac{(m^2)'}{E^2} \pm \frac{(g_A\tilde{Z}_z m^2)'}{E^2\sqrt{E^2 - p_\perp^2}} + o(\tilde{Z}_z^2)\,, \qquad (3)$$

where E and $p_\perp$ are constants of motion.

This chiral force provided by the $\tilde{Z}_z$ field effectively produces a potential well that draws an excess chiral charge onto the wall, and leads to a compensating deficit in a "diffusion tail" in front of the wall. There is net baryon production because B violation is suppressed on the bubble wall.

We now seek to describe the particle excitations with dispersion relations (2) as classical fluids. We focus on particles with large $|p_z| \sim T \gg m$ for three reasons: they dominate phase space, the WKB approximation is valid, and the dispersion relation simplifies so one can identify approximate chiral eigenstates. The $S_z = +\frac{1}{2}$, $p_z < 0$ branch and the $S_z = -\frac{1}{2}$, $p_z > 0$ branch constitute one approximately left-handed fluid L and the other two branches an approximately right-handed fluid R.

The Boltzmann equation is

$$d_t f \equiv \partial_t f + \dot{z}\partial_z f + \dot{p}_z\partial_{p_z} f = -C(f)\,, \qquad (4)$$

where $\dot{z}$ and $\dot{p}_z$ are calculated as above, and $C(f)$ is the collision integral. This can, in principle, be solved fully. However, to make it analytically tractable we truncate it with a fluid approximation, which we now discuss. When a collision rate is large, the collision integral forces the distribution functions towards the local equilibrium form

$$f = \frac{1}{e^{\beta[\gamma(E - vp_z) - \mu]} + 1}\,, \qquad (5)$$

where $T = \beta^{-1}$, v and μ are functions of z and t, and $\gamma = 1/(1 - v^2)^{1/2}$. These parametrize the fluid velocity v, number density n, and energy density ρ. We are going to treat the approximately left-handed excitations L and their antiparticles $\overline{L}$ as two fluids, making an *ansatz* of the form (5) for each.

As mentioned, the *ansatz* (5) does allow us to describe perturbations in the energy density, number density, and velocity of each fluid, and we expect it to give a reasonable qualitative description of the true phase space density perturbations. As far as the temperature and velocity perturbations are concerned, we probably cannot expect this form to be more than qualitatively correct, because the dominant interactions that establish thermal equilibrium are those with the background plasma, which are also responsible for setting δv and δT to zero. This is not true, however, of the chemical potential perturbation, which is only attenuated by slower chirality changing processes. So as long as we can check that $\delta T/T$ and δv are small, compared to μ/T, we believe that (5) should actually provide an accurate parametrization of the phase space density.

The collision integrals are evaluated in the approximation that the particle interactions are local, by using the Dirac spinors appropriate to the local value of m and $\tilde{Z}_z$, taken to be constant. This is reasonable for the two body scattering effects we consider, because the QCD interactions are short ranged, the Debye screening length m_{gluon}^{-1} being smaller than T^{-1} and very much smaller than L.

The ansatz (5) has three arbitrary functions, which are fully determined from three independent moments of the Boltzmann equation: we take $\int d^3p$, $\int d^3p\, E$, and $\int d^3p\, p_z$. For a single interacting fluid these yield the continuity, energy, and momentum equations. We are interested in the chiral density, the difference between L and $\overline{L}$ chemical potentials, since this quantity drives the baryon asymmetry. We work to first order in $\tilde{Z}_z$ and v_w.

The fluid equations for particle minus antiparticle perturbations [$\delta T \equiv \delta T(L) - \delta T(\overline{L})$, $\mu = \mu(L) - \mu(\overline{L})$, $\delta v = \delta v(L) - \delta v(\overline{L})$] are, in the rest frame of the wall,

$$-v_w \frac{\delta T'}{T_0} + \frac{1}{3}\delta v' - a v_w \frac{\hat{\mu}'}{T_0} = -\overline{\Gamma}_\mu\left(\frac{\hat{\mu}}{T_0}\right) - \Gamma_\mu\left(\frac{\Delta}{T_0}\right), \tag{6}$$

$$-v_w \frac{\delta T'}{T_0} + \frac{1}{3}\delta v' - v_w b \frac{\hat{\mu}'}{T_0} = -\Gamma_T \frac{\delta T}{T_0} - \overline{\Gamma}^*_\mu\left(\frac{\hat{\mu}}{T_0}\right) - \Gamma^*_\mu, \tag{7}$$

$$\frac{\delta T'}{T_0} + b\frac{\hat{\mu}'}{T_0} - 2c v_w \frac{(g_A \tilde{Z}_z m^2)'}{T_0^3} = -\Gamma_v \delta v, \tag{8}$$

where the shifted chemical potential difference is $\hat{\mu} = \mu - 2v g_A \tilde{Z}_z$, $(\hat{\mu})$ denotes the signed sum of chemical potentials for particles participating in the reaction, $\Delta = (\mu) = (\hat{\mu} - 2v_w g_A \tilde{Z}_z)$ is the difference between shifted L and $\overline{L}$ potentials, prime denotes ∂_z, $a = \pi^2/27\zeta_3$, $b = n_0 T_0/\rho_0$, $c = \ln 2/14\zeta_4$, $\zeta_4 = \pi^4/90$, $n_0 = 3\zeta_3 T_0^3/4\pi^2$, $\rho_0 = 21\zeta_4 T_0^4/8\pi^2$, and ζ is the Riemann ζ function. The derivation of these equations is simplest if one shifts the canonical momentum to $k_z = p_z + g_A \tilde{Z}_z$ and the chemical potential to $\hat{\mu}$. In this way the correct massless limit emerges as one expands in powers of $\tilde{Z}_z$. The relevant collision integrals may be calculated at zero background fields [18]. Γ_v is simply related to the diffusion constant D—it is easily seen that $D = (n_0 T_0/4a\rho_0)\Gamma_v^{-1} \approx \frac{1}{4}\Gamma_v^{-1}$. In fact, we find $\Gamma_T \approx \frac{3}{2}\Gamma_v$, $\Gamma_v \approx T/24$ [11].

$\overline{\Gamma}_\mu$ and $\overline{\Gamma}^*_\mu$ are derived from hypercharge *conserving* chirality flip processes, such as those involving external Higgs particles. In this case, the $\tilde{Z}_z$ contribution to the sum of chemical potentials vanishes. Γ_μ and Γ^*_μ are the rates for hypercharge *violating* chirality flip processes, which are m^2 suppressed, and for these the $\tilde{Z}_z$ contribution does not cancel. These latter are the terms driving spontaneous baryogenesis, in its new "diffusion-enhanced" form. This enhancement was mentioned in a talk by one of us [15], and explored independently and in much greater detail by Cohen, Kaplan, and Nelson [16]. In the formalism represented by the above equations, the spontaneous baryogenesis and "classical force" driving terms are both included—this is fully discussed in Ref. [11]. Here we focus on the classical force term, which, since the equations are linear, can be considered independently of the spontaneous baryogenesis source terms.

We proceed to solve Eq. (8) to find the perturbations produced by the force term. We simplify by setting Γ_μ, Γ^*_μ, $\overline{\Gamma}_\mu$, and $\overline{\Gamma}^*_\mu$ equal to zero. We show in [11] that the suppression they produce of the classical force effect is, for a large range of parameters, a factor between $\frac{1}{2}$ and 1. With this simplification we can derive our result in a few lines. First, from (7) we see that if $v_w D/L < 1$ the temperature fluctuation is smaller than the velocity or chemical potentials by this factor (using $\Gamma_T \approx 1/3D$). This explains how we arrived at our condition 2 above. Then from (6) we find a simple relation $\delta v \sim v_w \mu$. So we do indeed find that the temperature and velocity perturbations are small. The *reason* the velocity perturbation is small is quite general—from the continuity equation it follows that the velocity perturbation required to create a given chiral excess in the wall frame is proportional to the wall velocity. This should be true independently of the detailed form of the phase space density, which as mentioned before we cannot expect to be exact. Since δv is small, we can drop the right-hand side (rhs) of (8) because on the wall it is of order $v_w L/D$ compared to the $\hat{\mu}$ term, which is small by condition 1.

We are left with a relation between the chemical potential $\hat{\mu}$ and the force term on the wall, from (8)

$$\hat{\mu} = -\frac{2\ln 2}{3\zeta_3} v_w \frac{g_A \tilde{Z}_z m^2}{T_0^2} \quad \text{on the wall.} \tag{9}$$

We can now determine $\hat{\mu}$ in front of the wall as follows. Integrating (7) and (8) (with all Γ_μ's zero) gives $\int_{-\infty}^{\infty} \delta T \approx 0$ and $\int_{-\infty}^{\infty} \delta v = 0$. Then integrating (6) twice we find $\int_{-\infty}^{\infty} \mu = 0$, i.e., no net integrated chemical potential perturbation is generated. This means that the chemical potential generated on the wall is compensated by an opposite chemical potential off it. As mentioned above, off the wall the equations for μ reduce to the diffusion equation, and it is straightforward to see that in the absence of particle number violation the only nontrivial solution for μ is a diffusion tail *in front* of the wall. This is where the chiral charge deficit occurs, which drives baryogenesis.

Hence the integral of the chemical potential in front of the wall ($z > 0$) equals

$$\int_0^\infty dz\, \mu = \frac{2\ln 2}{3\zeta_3}\, v_w \int_{\rm wall} dz\, g_A \tilde{Z}_z m^2. \qquad (10)$$

Now, using the standard formula for baryon number violation

$$\dot{n}_B = -v_w n_B' = -\frac{3}{2} N_C \frac{\bar{\Gamma}_s}{T^3} (\mu_{t_L} - \mu_{\bar{t}_L}), \qquad (11)$$

where $\bar{\Gamma}_s = \kappa(\alpha_w T)^4$ is the weak sphaleron rate in the unbroken phase, N_c is the number of colors, $\kappa \in [0.1, 1]$ [2]. We have reexpressed μ in terms of top quark and antiquark chemical potentials. We arrive at a formula for the baryon to entropy ratio,

$$\frac{n_B}{s} = \frac{135 \ln 2}{2\pi^2 \zeta_3} \frac{\kappa \alpha_w^4}{g_*} \int dz \frac{m^2 g_A \tilde{Z}_z}{T^2}$$
$$\approx 4 \frac{\kappa \alpha_w^4}{g_*} \int dz \frac{m_t^2 g_A \tilde{Z}_z}{T^2}, \qquad (12)$$

where $s = (2\pi^2/45) g_* T^3$ is the entropy density, with $g_* \approx 100$ the effective number of degrees of freedom.

This result is remarkably simple—all dependence on the wall velocity, thickness, and the diffusion constant drops out, provided conditions 1 and 2 are satisfied. It is also quite large: $(m_t/T)^2 \sim 1$, so $n_B/s \sim 4 \times 10^{-8} \kappa \theta_{CP}$, where θ_{CP} characterizes the strength of the CP violation. In a longer paper [11] we give a more detailed derivation of (12) with a full discussion of parameter dependences, including the effect of the Γ_μ terms we have neglected here.

The m^2 dependence in (12) means that, at least with standard model-like Yukawa couplings, the top quark dominates the effect. The mass-over-temperature suppression can be significant, if the $\tilde{Z}_z$ field is localized on the front of the wall where the Higgs vacuum expectation value is small.

The calculation of the classical force effect above uses the opposite (WKB) approximation to those employed in quantum mechanical reflection calculations (thin walls) [12,13]. The classical force calculation is in some respects "cleaner," because the production of chiral charge and its diffusion are treated together. The classical force affects particles from all parts of the spectrum, mostly with typical energies $E \sim T$, and with no preferential direction, while the quantum mechanical effect comes mainly from particles with a very definite ingoing momentum perpendicular to the wall: $p_z \approx m_H$ (Higgs mass). The quantum result falls off strongly with L (at least as L^{-2}) as the WKB approximation becomes good. The quantum result also has a v_w^{-1} dependence coming from the diffusion time in the medium, which the classical result loses because the force term is proportional to v_w.

Finally, we mention possible extensions of these methods. In the above treatment we have completely ignored collective plasma effects (Debye screening, Landau damping, etc.) and merely treated *local* particle interactions. The Boltzmann equation is easily modified to include these effects, with force terms due to the electric (and magnetic) fields, which are solved for self-consistency. The fluid truncation may be a useful way to compute the bubble wall velocity (at least the friction due to top quarks), and we shall return to this in future work. We intend also to extend these methods to study Z condensation in the standard model [17] in the presence of strong interactions.

M. J. is supported by a Charlotte Elizabeth Procter Fellowship, and the work of T. P. and N. T. is partially supported by NSF Contract No. PHY90-21984, and the David and Lucile Packard Foundation.

[1] A. Sakharov, JETP Lett. **5**, 17 (1967); V. Kuzmin, JETP Lett. **12**, 335 (1970).

[2] J. Ambjorn, T. Askgaard, H. Porter, and M. Shaposhnikov, Phys. Lett. B **244**, 497 (1990); Nucl. Phys. **B353**, 346 (1991).

[3] K. Farakos, K. Kajantie, K. Rummukainen, and M. Shaposhnikov, Nucl. Phys. **B407**, 356 (1993); Report No. CERN-TH.7244/94.

[4] M. Dine, R. Leigh, P. Huet, A. Linde, and D. Linde, Phys. Rev. D **46**, 550 (1992); B-H. Liu, L. Mclerran, and N. Turok, Phys. Rev. D **46**, 2668 (1992).

[5] N. Turok, in *Perspectives in Higgs Physics,* edited by G. Kane (World Scientific, Singapore, 1992), p. 300.

[6] A. Cohen, D. Kaplan, and A. Nelson, Annu. Rev. Nucl. Part. Phys. **43**, 27–70 (1993).

[7] N. Turok and J. Zadrozny, Phys. Rev. Lett. **65**, 2331 (1990); Nucl. Phys. **B358**, 471 (1991); L. Mclerran, M. Shaposhnikov, N. Turok, and M. Voloshin, Phys. Lett. B **256**, 451 (1991).

[8] A. Cohen, D. Kaplan, and A. Nelson, Phys. Lett. B **263**, 86 (1991).

[9] M. Dine and S. Thomas, Santa Cruz Report No. SCIPP-94-01, 1994.

[10] M. Joyce, T. Prokopec, and N. Turok, Princeton Report No. PUPT-1436, 1994 (to be published).

[11] M. Joyce, T. Prokopec, and N. Turok, Princeton Report No. PUPT-1496, 1994.

[12] A. Cohen, D. Kaplan, and A. Nelson, Nucl. Phys. **B373**, 453 (1992); Phys. Lett. B **294**, 57 (1992).

[13] M. Joyce, T. Prokopec, and N. Turok, Princeton Report No. PUPT-1437, 1994 (to be published).

[14] M. Joyce, T. Prokopec, and N. Turok, Princeton Report No. PUPT-1495, 1994.

[15] M. Joyce, Princeton Report No. PUPT-1475 (to be published).

[16] A. Cohen, D. Kaplan, and A. Nelson, Report No. NSF-ITP-94-67; Report No. hep-ph-9406345.

[17] S. Nasser and N. Turok, Princeton Report No. PUPT-1456, 1994.

[18] The collision integrals vanish identically for arbitrary background fields at zero δT and δv, respectively, so they take the form $(\delta T)\Gamma(Z_z = 0, m^2 = 0)[1 + o(Z_z) + o(m^2) + \cdots]$. But any rate on the rhs multiplies a perturbation, of $o(m^2 Z_z)$ so that the linear correction to the rate contributes at second order in Z_z and can be ignored.