Spectral Statistics of Acoustic Resonances in Aluminum Blocks

C. Ellegaard, T. Guhr, K. Lindemann, H. Q. Lorensen, J. Nygård, and M. Oxborrow

Center for Chaos and Turbulence Studies, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark

(Received 26 September 1994; revised manuscript received 22 February 1995)

We measure several hundred acoustic resonances in aluminum blocks. The statistical properties of these spectra are analyzed and compared to predictions from random matrix theory. A Poisson behavior is found for a rectangular block. The transition from Poisson to Gaussian orthogonal ensemble statistics is studied by measuring aluminum blocks manufactured in the shape of three-dimensional Sinai billiards.

PACS numbers: 62.30.+d, 03.40.Kf, 03.65.Sq, 05.45.+b

Studying the transition from regularity to chaos is a topic of great interest in classical and quantum systems. However, little experimental work has been done in quantum systems. One of the best studied systems is the hydrogen atom in a strong magnetic field [1]. For this reason, it is useful to have an experimental "toy system" with an easily tunable transition parameter that controls the degree of chaos. In recent years, experiments with microwaves in metal cavities have been used to study chaos in billiard systems [2–5]. The electromagnetic vector Helmholtz equation is, in the case of sufficiently flat cavities, mathematically identical to the two-dimensional Schrödinger equation. Hence these experiments simulate two-dimensional quantum billiards. Since the level statistics of quantum spectra is well described by random matrix theory [6], the same is true for the statistics of these microwave resonances. It is interesting to investigate whether the spectral statistics of waves propagated by equations other than the Schrödinger equation also follow random matrix theory.

One example that allows spectra to be measured with high resolution is elastic waves in solids. The elastomechanical wave equation in three dimensions is tensorial and has complications not found for the Schrödinger equation. It is therefore important to show that the assumptions of random matrix theory carry over to a system like this. This question was addressed by Weaver [7] in a paper in 1989. He excited aluminum blocks acoustically by dropping steel balls on them. The spectral fluctuations he measured are very close to the predictions of the Gaussian orthogonal ensemble (GOE), which models a time reversal invariant system exhibiting level repulsion. Bohigas *et al.* [8] have shown that these results are consistent with the general assumptions of random matrix theory, provided that finite wavelength effects are taken into account. Recently, a new analysis of the data was performed from the viewpoint of periodic orbit theory [9].

In this Letter, we use Weaver's technique to study further the spectral statistics of acoustic resonances in aluminum blocks, and present three main results. First, we demonstrate that it is possible to obtain the Poisson type of behavior. It is not obvious, and was not shown in Weaver's work, that the regular system in acoustics yields Poisson statistics. Second, we show the transition from Poisson to GOE fluctuation properties by measuring blocks in the form of three-dimensional Sinai billiards. The transition parameter is easily controlled by the geometric deformation. Third, we present the first experimental example for this transition in a three-dimensional system.

In the case of isotropic materials, the elastomechanical equation of motion can be split into two wave equations for the longitudinal and the transverse parts of the displacement vector. These two modes are coupled through the reflections at the boundary between the medium and the air. In general, a purely longitudinal or transverse wave becomes, after the reflection, a mixture of both modes. This phenomenon is called mode conversion. Define c_X and $c_Y^{\prime}$ as the velocities before and after the reflection, respectively, and ϑ_X and $\vartheta_Y^{\prime}$ as the corresponding angles relative to the normal. Here X and Y stand for either longitudinal (L) or transversal (T). Snell's law [10] then reads $\sin\vartheta_X/\sin\vartheta_Y^{\prime} = c_X/c_Y^{\prime}$, which implies that the angle of reflection is generally not equal to the angle of incidence. Hence, acoustic resonances differ from the eigenstates of the scalar Schrödinger equation, and our experiments have no direct relationship to quantum mechanics, unlike the microwave experiments in two dimensions. Experiments under way with three-dimensional cavities [11] should conceptually have more in common with our work. Random matrix theory, however, rests on purely statistical assumptions formulated in energy space, or, in our context, frequency space. Besides linearity, there are no further restrictions on the form of the underlying wave equation.

Another significant difference from quantum mechanics and the microwave experiments becomes apparent when one tries to apply a ray picture analogous to semiclassics. In the acoustic experiments discussed here, there is, corresponding to the highest frequency measured, always a minimal wavelength that is not negligible compared to the size of the blocks. Thus the zero wavelength limit, essential for a discussion in the spirit of semiclassics, is not fulfilled; this has far reaching consequences for the interpretation of the spectral fluctuation properties [8].

The aluminum blocks in our experiments were manufactured from the alloy AlMgSi from NKT Metalgården;

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the sizes were about 10 cm (see below). The blocks were supported by a foam pad. For the measurements we used piezoelectric transducers made of Ferroperm Pz34 (modified lead titanate ceramic), with dimensions $8 \times 3 \times 1.195$ mm^3. The lowest resonances of the transducers were about 1.8 MHz. For the coupling to the blocks, microcrystalline wax was used. This and the foam support appeared to have only a minimal effect on the resolution of the resonances. The signal was amplified and filtered using a Stanford Research Low Noise preamplifier RS560. It had very little distortion, of the order of 0.01%, so as to prevent the generation of spurious harmonics. Upon dropping a steel ball on the block, the amplified, transient time signal was measured with a Hewlett Packard 3562A spectrum analyzer; the measurement was triggered by the signal and the length of the time series captured by the analyzer was approximately 100 ms. A fast Fourier transform then gave the frequency spectrum. Our measurements were restricted by the frequency range of the spectrum analyzer, 0 to 100 kHz. To obtain one spectrum for the study of the level statistics, we averaged over ten measurements with the same impact position of the steel balls, and then repeated this procedure for ten different impact positions. This was done to minimize the effect of accidentally hitting a node of a resonance. For the same reason, the positions of the transducers were also varied. The exponential decay of the time signal yields a Lorentzian line shape in frequency space, allowing an estimate of the resolution $Q = f/\Delta f$, where f is the position and Δf the width of a given resonance. By scanning small frequency intervals (using a "flat-top" windowing function), we determined the Q values of several peaks. These all lie between 5000 and 10 000.

The measurements were performed on a block machined into three successive shapes. In the first experiment, we used a block of rectangular shape, with dimensions $60.6 \times 98.0 \times 158.6$ mm^3. For the ratios of the side lengths we chose the golden mean, i.e., approximately 0.618, in order to prevent accidental degeneracies in the spectra. In the second and the third experiments, octants of spheres of radii 10 and 20 mm, respectively, were removed from one corner. The block thereby acquired the shape of a three-dimensional Sinai billiard. A section of a typical spectrum is shown as the inset in Fig. 1. The most difficult part, as in all experiments of this type, was the identification of the resonances. This was done by comparing the ten spectra for different impact positions of the steel balls. In a spectrum with many near lying levels, there is always a certain fraction of levels which are not seen due to the finite width of the resonances. The only remedy for this "missing level effect" is an improvement of the quality factor Q. The ratio of Δf to the mean level spacing at 100 kHz, i.e., in the worst case, is about 1:5. In the three experiments, 391, 436, and 439 levels were identified, respectively, yielding three sequences of frequencies to be used for the statistical analysis. In each case, the cumulative eigenfrequency density $N(f)$ was computed, which is the number of frequencies in the sequence less than or equal to f. A Weyl type of formula

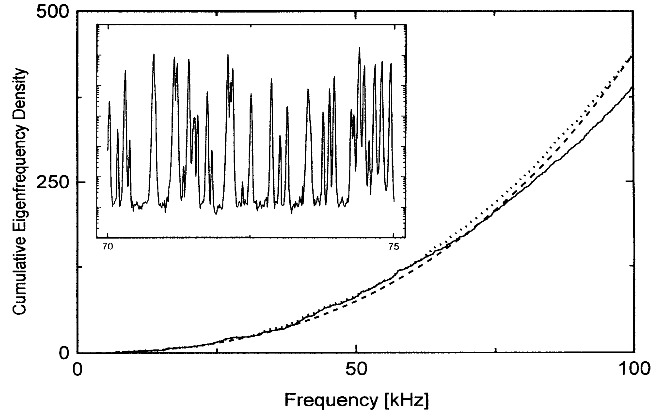

FIG. 1. Cumulative eigenfrequency densities: the solid line corresponds to the rectangular block; the dotted line corresponds to the Sinai billiard with 20 mm octant radius. The dashed line is the curve given by formula (1). The inset displays on a linear-logarithmic plot the section between 70 and 75 kHz from a spectrum of the first experiment, showing no level repulsion. Since a "Hanning" windowing was used, the widths of the peaks are artificially broadened.

$$N_{\mathrm{av}}(f) = \frac{4\pi V}{3}\left(\frac{2}{c_T^3} + \frac{1}{c_L^3}\right) f^3 + \frac{\pi S}{4}\, \frac{2 - 3(c_L/c_T)^2 + 3(c_L/c_T)^4}{c_L^2[(c_L/c_T)^2 - 1]}\, f^2 \tag{1}$$

was proposed [7,12], where V and S are the volume and the surface area of the block. Since this formula assumes periodic boundary conditions in two (out of the three) spatial directions, it cannot be directly applied to our blocks. Nevertheless, it does seem to describe the trend of our data—as can be seen in Fig. 1; our values for c_L and c_T were calculated from the values of the mechanical properties of aluminum that were used in Ref. [13].

In order to obtain the smooth part of $N(f)$, a third order polynomial with four free coefficients was fitted to each experimental cumulative eigenfrequency density. Each sequence was then mapped onto a new one by introducing a dimensionless frequency scale $x = N_{\mathrm{av}}(f)$ with unit level density everywhere. Finally, the nearest neighbor spacing distribution $P(s)$ and the spectral rigidity $\Delta_3(L)$ were extracted from those sequences using the procedure described in Ref. [14].

In Fig. 2(a) the nearest neighbor spacing distribution is plotted for the first experiment. The data are divided into a lower and a higher frequency sector, i.e., into the first 196 and the remaining 195 levels, respectively. The low frequency sector is clearly rather close to

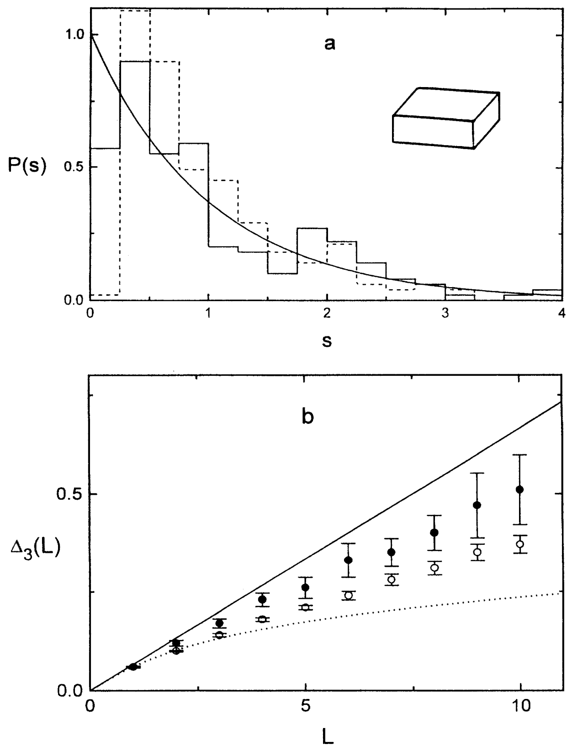

FIG. 2. (a) The nearest neighbor spacing distribution for the rectangular block. The solid histogram is for the lowest 196 resonances; the dashed histogram is for the highest 195. The smooth curve is the Poisson distribution. (b) The spectral rigidity for the rectangular block. The solid dots are for the lowest half resonances; the open dots are for the highest half. The solid curve is the Poisson case; the dashed curve is the GOE case.

a Poisson distribution, whereas the higher frequency deviates from the Poissonian, but only at small spacings. This is precisely the missing level effect. Since the level density increases quadratically with frequency, it becomes more and more difficult to resolve and identify the higher frequencies. The same effect can be seen in Fig. 2(b) where the spectral rigidity is plotted. Again, the high frequency sector is further away from the Poisson prediction. Assuming that the Poissonian is the true distribution, an estimate yields that we miss 40 or so levels. Such a considerable number of missing levels mimics level repulsion. However, the fact that we see Poisson statistics in the low sector is a strong indication that there are no further mechanisms besides the missing level effect causing deviations from the Poisson statistics. The quality of these results is comparable to room temperature microwave measurements performed by the Boston group [3,15], but inferior to the Darmstadt group [4] measurements on superconducting cavities.

Since our block still has mirror symmetries, the spectrum is a noninteracting superposition of eight irreducible spectra. Because of mode conversion, which is a strong effect [16], we cannot exclude the possibility that each irreducible spectrum exhibits non-Poisson statistics. However, the superposition of several independent spectra has near-Poisson statistics, even in the extreme case where each irreducible spectrum has GOE statistics [6]. In the present context we were interested in any system that efficiently produces spectra with Poisson statistics, as a starting point for measuring the transition to a system with GOE fluctuation properties.

The spacing distribution and the spectral rigidity for the Sinai billiard with a 10 mm radius octant are shown in Figs. 3(a) and 3(b), respectively. Again, the data are divided into a low and a high frequency sector. The results are somewhere between the Poisson and the GOE prediction. In the case of the spacing distribution, we used the Wigner surmise as theoretical prediction, which is known [6] to be very close to the GOE result. We may conclude that the removal of the octant from the block induced level repulsion, characteristic of chaotic spectra. In a zero wavelength limit, i.e., in a ray picture, the spectral fluctuation properties are a consequence of defocusing induced by the octants. However, as mentioned above, the wavelength in our spectra has a lower limit, which is about 4 cm in the low frequency sector and about 3 cm in the high frequency sector.

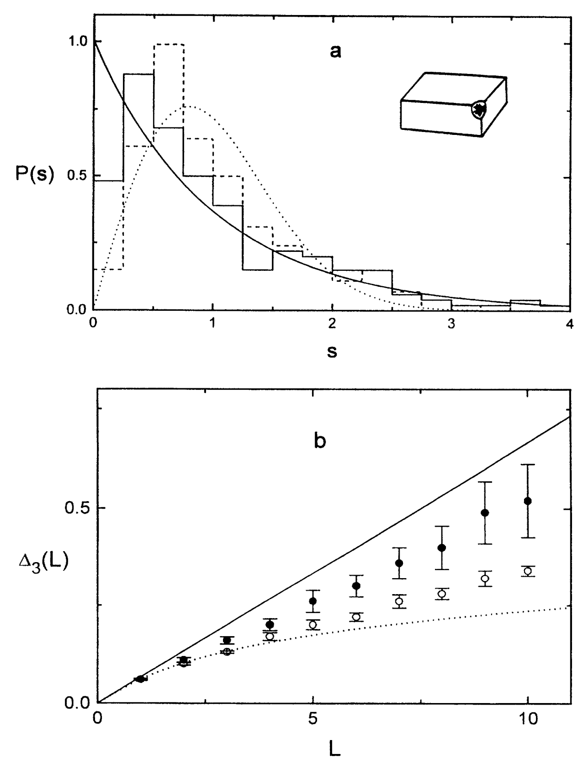

FIG. 3. As for Fig. 2 but for the small octant removed from the block. The dotted curve in (a) is the Wigner surmise.

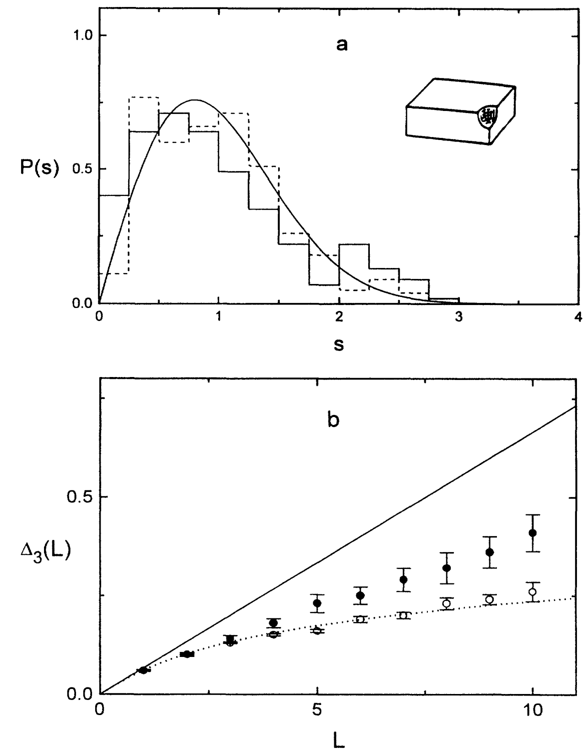

FIG. 4. As for Fig. 2 but for the large octant removed from the block. The solid curve in (a) is the Wigner surmise.

Structures much smaller than these lengths cannot be resolved by the acoustic waves. This explains why the level statistics of the high frequency sector are closer to the GOE prediction. As expected, the GOE features become considerably stronger in the results for the third experiment, plotted in Figs. 4(a) and 4(b), where the radius of the octant was 20 mm. Upon the assumption that the Wigner surmise is the true distribution, the number of missing levels in this experiment can be estimated as 2 or 3, which is 10 times lower than in the first experiment.

In conclusion, we have shown that it is possible to obtain Poisson statistics in acoustic experiments, and have studied the transition to GOE statistics. This is, to our knowledge, the first time that this transition has been measured in a three-dimensional geometry.

We thank J. Bondorf for suggesting this line of work to us. We are grateful to R.L. Weaver for helpful comments regarding his experiment. We acknowledge fruitful discussions with S.C. Creagh, P. Cvitanović, P. Dimon, A. Kudrolli, G.E. Mitchell, P. Persson, N. Whelan, and S. Åberg. T.G. acknowledges financial support from the Danish Research Council.

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