Measurement of the Nonlinear Goos-Hänchen Effect for Gaussian Optical Beams

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The nonlinear Goos-Hänchen effect is experimentally isolated for the total reflection of a Gaussian beam at an interface between a linear medium and a negative Kerr-type nonlinear medium. The evolution of this nonlinear spatial shift versus light intensity is measured for both TE and TM polarizations. The results are in good agreement with a simple model based on nonlinear Artmann's formulas.

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Since the early development of nonlinear optics, the classical laws of reflection and refraction of light at an interface have been reconsidered for the case of an interface between a linear and a nonlinear medium [1]. On the one hand, in the case of an interface between a linear medium and a quadratic nonlinear medium, the laws of reflection, refraction [2], and conical refraction [3] have been reconsidered. On the other hand, in the case of the interface between a linear and a Kerr-type nonlinear medium, many theoretical [4] and experimental [5] studies have dealt with the existence and stability of different types of transmitted and surface waves. In the case of total internal reflection on an interface between *linear* media, a puzzling discrepancy with respect to the Snell-Descartes laws can be observed for finite width beams: the Goos-Hänchen effect [6]. It consists of a small longitudinal shift of the totally reflected beam with respect to specular reflection. The existence of the corresponding effect in the case of second-harmonic generation has been theoretically predicted [7]. In the case of positive ($\partial n/\partial I > 0$) Kerr-type nonlinearities, some theoretical studies have interpreted the deformations of the totally reflected beams in terms of a nonlinear Goos-Hänchen effect [4,8]. However, owing to the complicated beam deformations that occur in such cases [4], these effects seem difficult to observe. On the contrary, the aim of this Letter is to show experimentally that a simple nonlinear Goos-Hänchen effect can be isolated in the case of total reflection at the interface between a linear and a *negative* Kerr-type *nonlinear* medium, without any visible deformation of the incident Gaussian beam.

Let us consider the interface schematized in Fig. 1. A TE or TM polarized Gaussian beam impinges with an angle of incidence i from linear medium 1 (refractive index n_1) onto medium 2, with nonlinear refractive index $n_2 = n_0 + n_{\mathrm{NL}} I$, where I is the light intensity in medium 2. Let us first recall what happens in the case of linear optics ($n_{\mathrm{NL}} = 0$). For angles of incidence i larger than the critical angle $i_c = \arcsin(n_2/n_1)$, the incident beam is totally reflected and undergoes a spatial shift D_{TE} or D_{TM} with respect to what can be expected from specular reflection (represented by the dashed line in Fig. 1). This spatial shift is given by Artmann's formulas [6]:

$$D_{\mathrm{TE}} = \frac{\lambda_0}{\pi} \frac{\sin i}{(n_1^2 \sin^2 i - n_2^2)^{1/2}}, \tag{1a}$$

$$D_{\mathrm{TM}} = \frac{D_{\mathrm{TE}}}{(n_1^2/n_2^2 + 1)\sin^2 i - 1}, \tag{1b}$$

where λ_0 is the wavelength of light in vacuum. These equations are valid as long as all the plane wave components of the incident beam are totally reflected, i.e., as long as $i > i_c + \theta_1$, where θ_1 is the divergence angle of the Gaussian beam in medium 1. In the case where medium 2 is nonlinear, its refractive index is modified by the presence of the evanescent wave. This modifies the values of the Fresnel transmission coefficients $t_{\mathrm{TE}}(i, n_1, n_2)$ and $t_{\mathrm{TM}}(i, n_1, n_2)$, which, in the linear case, are given by Fresnel's laws [9]. If we make the rough approximations that all the considered waves can be regarded as plane waves and that the nonlinear refractive index modification is homogeneous in the evanescent wave and equal to the one at the interface, the value of the effective refractive index n_2 of the nonlinear medium at the interface can be obtained from the two following coupled equations:

$$E_j^{(2)} = t_j(i, n_1, n_2) E_j^{(1)}, \tag{2}$$

$$n_2 = n_0 + n_{\mathrm{NL}} \frac{n_0 c}{8\pi} |E_j^{(2)}|^2, \tag{3}$$

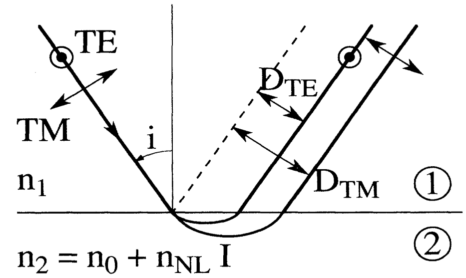

FIG. 1. Scheme of the considered interface between linear medium 1 and nonlinear negative Kerr-type medium 2.

where $E_j^{(1)}$ ($E_j^{(2)}$) is the field amplitude in the linear (nonlinear) medium (j = TE, TM), and c is the velocity of light in vacuum. Equations (2) and (3) can be solved numerically for all values of $E_j^{(1)}$ with $n_{NL} < 0$. Notice that we deliberately restrict ourselves to the case where the variation of the refractive index of medium 2 is small compared to its linear value ($n_0 - n_2 \ll n_0$) and also small compared to the difference between the refractive indices of the two media ($n_0 - n_2 \ll n_1 - n_0$), i.e., to the case in which former theoretical studies have predicted no particular effect [10]. However, if now we inject the value of n_2 obtained from the resolution of Eqs. (2) and (3) into Eqs. (1), a modification of the value of the Goos-Hänchen shift will occur. Indeed, because of the negative nonlinearity of medium 2, the evanescent wave reduces the value of n_2, thus reducing the value of the effective critical angle $i_c = \arcsin(n_2/n_1)$. This must lead to a reduction of the Goos-Hänchen shift D_{TE} or D_{TM}. This reduction of the longitudinal shift must be particularly significant in regions where the Goos-Hänchen effect is large in the linear case, i.e., for angles of incidence in the vicinity of the critical angle.

In order to check these predictions experimentally, we have chosen to measure the nonlinear Goos-Hänchen effect inside an argon ion laser cavity. The use of laser eigenstates has indeed already proven to be a very sensitive way to measure the linear Goos-Hänchen effect for a Gaussian beam [11]. A fused silica prism (refractive index $n_1 = 1.47$ at $\lambda_0 = 488$ nm) is introduced inside the cavity of a commercial argon ion laser. This cavity is 2 m long and is close by a spherical mirror M_1 (radius of curvature 10 m; transmission coefficient 3.5% at 488 nm) and a plane mirror M_2. The nonlinear medium introduced at the rear of the prism is a solution of Disperse Red 2 (Merck's D2 dye) in ethanol. Its linear refractive index at 488 nm is measured to be $n_0 = 1.37$, leading to a linear critical angle $i_c = 68.74°$. We independently measured the nonlinearity of this solution using single pass defocalization rings [12], leading to $n_{NL} = -2.1 \times 10^{-6}$ cm^2/W. In order to be able to observe the nonlinear Goos-Hänchen effect for both TE and TM polarizations, we introduce a half-wave plate (HWP) (see Fig. 2) between the discharge tube and the prism. Then, if the axes of this plate are aligned with (at ±45° to) the plane of incidence, the oscillating laser eigenstate is TM (TE) polarized between the half-wave plate and the plane mirror. In order to observe the variations of the Goos-Hänchen shift with the intracavity intensity, we first tune the discharge current so that the laser is slightly above threshold and choose an angle of incidence i at the nonlinear interface just above the critical angle. We then introduce a circular aperture (diameter 1.4 mm) between the prism and the plane mirror. To finely control the position of this aperture with respect to the center of the Gaussian beam, it is mounted on a piezoelectric transducer. It oscillates sinusoidally perpendicularly to the beam axis and in the plane of incidence of the nonlinear interface (see the arrow in Fig. 2). The frequency of oscillation of the aperture is $f_0 = 169$ Hz and its peak-peak amplitude is 4 μm. Thus the diffraction losses and the output power of the laser are modulated at f_0 and its harmonics. The laser output power is demodulated at f_0 using a lock-in amplifier. We then introduce an offset in the piezoelectric transducer driving voltage to make the demodulated first-harmonic signal vanish. We can therefore be sure that the axis of circular aperture coincides with the axis of the laser beam, because the Gaussian profile of the beam is an even function. If now we increase the discharge current inside the active medium, leading to an increase of the laser power, the effective refractive index of the nonlinear medium diminishes, leading to a decrease of the Goos-Hänchen shift. This leads to the observation of a nonzero demodulated signal at f_0 in the laser output power. The value of the variation of the Goos-Hänchen shift can then

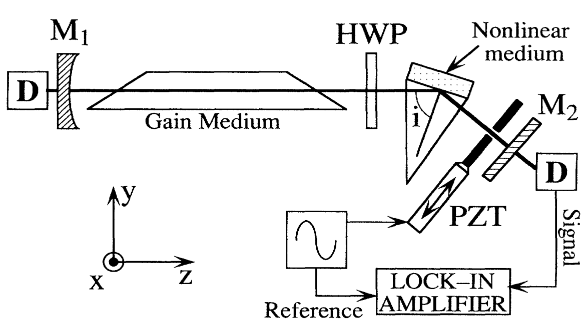

FIG. 2. Experimental arrangement. M's: laser cavity mirrors; HWP: half-wave plate; D: detectors; PZT: oscillating piezoelectric transducer.

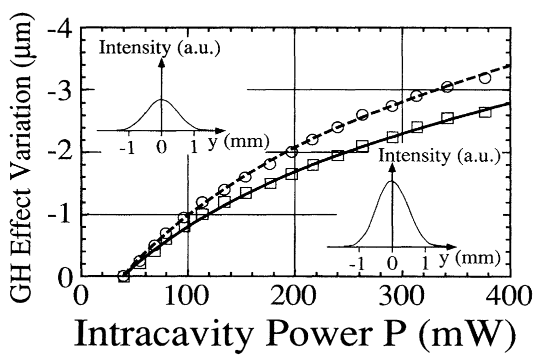

FIG. 3. Variation of the Goos-Hänchen shift versus intracavity power P. The squares (circles) are measurements for the TE (TM) polarization. The full line (dashed line) is the corresponding TE (TM) theoretical curve obtained with $i = i_c + 0.0105°$. The insets are experimental recordings of the output power profiles obtained by translating the detector located at the output of the spherical mirror along the y axis for two values of the intracavity power (P = 126 and 400 mW).

be obtained by measuring the value of the extra offset voltage needed to compensate for this demodulated signal. Typical results of such measurements are reproduced in Fig. 3 for both the TE (squares) and TM (circles) polarizations versus intracavity power. The error in the measurement is estimated to be $\pm 0.5\ \mu$m. The intracavity power P in the abscissa is the total average power of the standing wave, i.e., twice the value of the measured laser output power divided by the output mirror transmission coefficient. The reference intracavity power for which the aperture is initially centered on the beam is equal to 40 mW. We have checked experimentally that when the nonlinear medium is replaced by pure ethanol, no variation of the Goos-Hänchen shift can be observed to $\pm 0.5\ \mu$m. We have also checked by performing several measurements for different positions of the aperture along the beam axis that the measured effect is really a longitudinal shift and cannot be attributed to an angular effect. The theoretical curves (full and dashed lines of Fig. 3) are obtained using the simple model of Eqs. (1)–(3) with an intracavity intensity $I = P/\pi w^2$ with $w = 790\ \mu$m. The only parameter that has been adjusted is the value of the angle of incidence i, which is difficult to measure accurately. The value of i obtained for both curves in the case of Fig. 3 ($i = i_c + 0.0105°$) is in the domain of validity of Artmann's formulas (1). Indeed, the divergence angle at $1/e$ of the Gaussian beam in the linear medium is $\theta_1 = 7.7 \times 10^{-3}$ deg. Anyway, the agreement with the experimental results is good. Notice that the validity of our simple model based on Artmann's formulas is not so surprising, since the nonlinear variations of the refractive index in our experiments are relatively small $[(n_0 - n_2)/n_0 \lesssim 10^{-4}]$, allowing us to isolate the nonlinear Goos-Hänchen effect from other complex phenomena, like the ones predicted in Ref. [10]. This point is confirmed by the fact that the totally reflected beam always remains fairly Gaussian, as can be seen from the experimental spatial profiles shown in the insets of Fig. 3, obtained by translating the detector placed in the plane of incidence. No important deformation can be seen on these profiles, which were obtained for two different values of the intracavity power.

In conclusion, we have isolated and measured the nonlinear Goos-Hänchen effect at total reflection on the interface between a linear medium and a negative Kerr-type nonlinear medium. These measurements, obtained for a Gaussian beam for both TE and TM polarizations, are in good agreement with a straightforward model based on a nonlinear extension of Artmann's formulas. Such a nonlinear Goos-Hänchen effect could have implications in the physics of lasers with nonlinear interfaces. It could also be extended to the other areas of wave phenomena such as acoustics, quantum mechanics, and plasma physics [6], where the linear Goos-Hänchen effect has already been observed.

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