Hadronic Light-by-Light Contribution to the Muon $g - 2$

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We present a calculation of the hadronic light-by-light contributions to the muon $g - 2$ in the $1/N_c$ expansion. We have used an extended Nambu-Jona-Lansinio model and introduced an explicit cutoff for the high energy region. We then have critically studied the relative size of the high energy contributions. Although we find them large, we can give an estimate of the light-by-light contribution to a_μ which is around 1×10^{-10}. This is smaller than previous estimates and the expected experimental uncertainty at the forthcoming Brookhaven National Laboratory experiment.

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The high precision measurement of the anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, combined with the CERN LEP results, is expected to provide valuable information on the electroweak sector of the standard model and maybe unravel new physics. See [1] for reviews of both the theoretical and experimental status. For carrying out this program, a new measurement of the muon anomalous magnetic moment at Brookhaven National Laboratory (BNL) [2] aims to reduce the experimental uncertainty to $\sim 4 \times 10^{-10}$. On the theoretical side, the error is dominated by the hadronic contributions and, in particular, by the hadronic vacuum polarization contribution. For a recent determination of this contribution and earlier references see [3]. The progress expected in measuring the total cross section $\sigma_{\text{total}}(e^+e^- \to \text{hadrons})$ will lower the theoretical uncertainty from this contribution down to the order of the experimental uncertainty quoted above [3].

There is another source of hadronic uncertainty in the theoretical calculation of a_μ which has recently raised some discussion about its reliable calculation [4–6]. It is the hadronic light-by-light scattering contribution where a full four-point function made out of four-vector quark currents is attached to a muon line with three of its legs coupling to photons in all possible ways and the fourth vector leg coupled to an on-shell external photon (see Fig. 1). The difficulty here is that this contribution

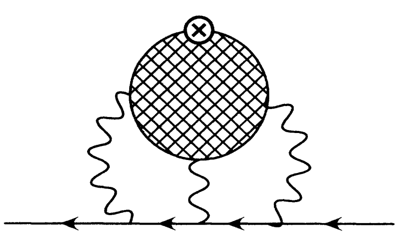

FIG. 1. Hadronic light-by-light contribution to a_μ. The bottom line is the muon line. The wavy lines are photons and the cross-hatched circle depicts the hadronic part. The circled crossed vertex is an external vector source.

cannot be expressed in terms of experimental observables, and thus one has to rely on our present knowledge in treating the strong interactions. There have been attempts to calculate this contribution in the past [7,8] and more recently in [9]. In this Letter we mainly address the calculation of the hadronic light-by-light scattering contributions to a_μ in the large N_c limit (N_c is the number of colors). These are $\mathcal{O}(N_c)$ in the $1/N_c$ expansion. We will use an extended Namub-Jona-Lasinio (ENJL) model and introduce an explicit cutoff. We defer a complete discussion and a more detailed presentation of our results to a forthcoming publication [10].

The momenta flowing through the three vector legs of the four-point function attached to the muon line run from zero up to infinity then covering both the nonperturbative and perturbative regimes of QCD. These two different regimes are naturally separated by the scale of the spontaneous symmetry breaking ($\Lambda_\chi \simeq 1$ GeV). Above this scale the strong interaction contributions have to match the perturbative QCD predictions in terms of quarks and gluons. At this point we would like also to check the current common wisdom statement that the bulk of the hadronic light-by-light contributions to a_μ is determined by the physics around the muon mass. In fact, this was assumed in all previous calculations. If this was correct, we could attempt to make a pure low-energy calculation that would saturate this contribution. Were the contributions not negligible at some high scale, we would need a more sophisticated model to calculate the vector four-point function. Indeed this appears to be the case from our results. However, at the level of accuracy needed in the near future, a rough estimate will turn out to be sufficient.

At very low energies (typically below the kaon mass), the framework to study the strong interactions is chiral perturbation theory (CHPT) and the relevant degrees of freedom are the lightest pseudoscalar mesons (π, K, and η). However, as emphasized in [6], there appear counterterms in the calculation of a_μ which are not determined by symmetry arguments alone. Instead we

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will use a low-energy model. We will use an ENJL model because it possesses the following features: It encodes all the chiral constraints and therefore satisfies all the QCD Ward identities (both anomalous [11] and nonanomalous); it has spontaneous symmetry breaking; both an $1/N_c$ expansion and a chiral expansion are possible; it reproduces the low-energy phenomenology and the success of the vector meson dominance (VMD) models. To a large extent, as shown by the Weinberg sum rules, it also has the correct matching with the high-energy QCD behavior. Models to introduce vector fields like the hidden gauge symmetry (HGS) model used in [9] do not always have such preferable behavior. These characteristics were emphasized for a_μ in [6].

See [12] for more details, definitions, and technical points about the specific model we are using. Its major drawback is the lack of confinement. This can be smeared out by calculating with constituent quarks far off-shell and color singlet observables. This model has three parameters plus the current light quark masses. These are fixed as to reproduce the experimental pion and kaon masses. The three parameters can be chosen to be the couplings of the spin zero G_S and spin one G_V four-quark terms in the ENJL model (see [12] for details) and, since it is a nonrenormalizable model, the cutoff Λ of the regularization which we chose to be proper time. Although this regulator breaks, in general, the Ward identities, we impose them by adding the necessary counterterms including those for the anomaly [11]. The values of the parameters we use are the ones obtained in the first reference in [12] from a fit to low energy data: $G_S = 1.216$, $G_V = 1.263$, and $\Lambda = 1.16$ GeV. Then the constituent quark masses solution of the gap equation are $M_u = M_d = 275$ MeV and $M_s = 427$ MeV. The hadronic vacuum polarization contribution was estimated within the ENJL model in [6] using a procedure similar to the one below. The result agreed within about 15% with the one in [3].

Let us proceed to the calculation itself. We calculate to all orders in the chiral expansion. Notice that this is needed, since we are integrating over three of the vector momenta. In previous calculations the lowest order CHPT result was convoluted with a naive VMD propagator. It is not clear how this procedure preserves the QCD Ward identities and the CHPT expansion itself.

At leading order in $1/N_c$ there are two classes of hadronic light-by-light diagrams contributing to a_μ. The first one is shown in Fig. 2(a). This is a pure full four-point function with a constituent quark loop and the three vector legs attaching to the muon line dressed by full two-point functions. These full two-point functions are the sum of strings with one, two, $\cdots$, ∞ constituent quark loops and can be found in [12]. There are six possible permutations for each quark flavor. The leading order in the CHPT expansion of this contribution is $\mathcal{O}(p^8)$ and thus potentially sensitive to the high-energy

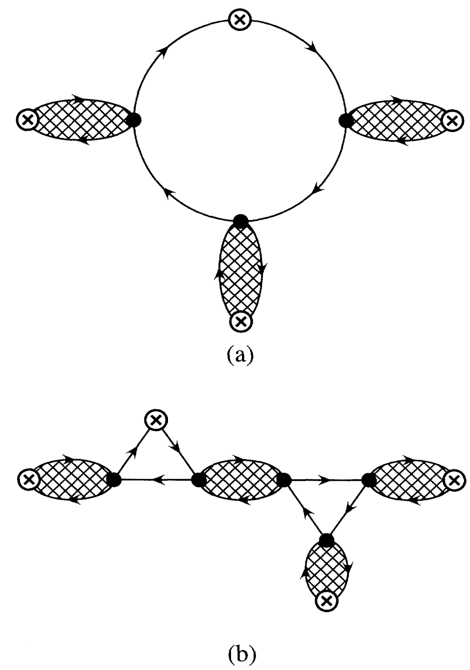

FIG. 2. The two classes of hadronic light-by-light contributions to a_μ at leading $\mathcal{O}(N_c)$. (a) The four-point functions class. (b) The product of two three-point functions class. The dots are ENJL vertices. The circled crossed vertices are where photons connect. The cross-hatched loops are full two-point functions and the lines are constituent quark propagators.

region. The other class is shown in Fig. 2(b). Here we have two one-loop three-point functions with two vector legs on each one and glued with a full two-point function that can be either pseudoscalar, scalar, mixed pseudoscalar axial vector, or axial vector. The vector one does not contribute. The three vector legs attaching to the muon line are dressed with full two-point functions. The leading order in the CHPT expansion is $\mathcal{O}(p^6)$ for the pseudoscalar exchange and $\mathcal{O}(p^8)$ for the others. There are 12 possible permutations for each quark flavor.

Although the sum of the contributing terms is UV finite, each of them can be logarithmically divergent and one has to rely on potentially dangerous numerical cancellations. Instead, we used the method proposed in [13] to construct individually UV safe quantities. This is achieved by making use of the gauge invariance in the on-shell photon leg. We then construct the quantity in (2.9) of [13]. Momentum integrals are performed numerically in Euclidean space. This allows us to impose physically relevant cutoffs on the photons' momenta.

The contribution of the first class of diagrams in Fig. 2(a) can be written as a seven-dimensional integral, which we have evaluated using the Monte Carlo routine VEGAS. As a check we have reproduced the constituent quark and muon loops results in [8] and the electron loop results in [14]. Since we are dealing with a low-energy model, we want to study the dependence on a high-energy cutoff μ on the vector legs' momenta. The result only stabilizes at a rather high value of μ. For a bare constituent quark with a mass of 300 MeV, the change between a cutoff of 2 GeV to a cutoff of 4 GeV is still around 20%. The change from 0.7 to 2 GeV is typically a factor of 1.8. This invalidates the use of any low-energy model to calculate the complete hadronic light-by-light contribution to a_μ. The bulk of these contributions does not come from the dynamics at scales around the muon mass as it is often stated. This also explains the rather high sensitivity to the damping provided by the vector two-point functions observed in [8,9]. Although the only rigorous result is for scales smaller than 0.6–1 GeV, one still obtains an estimate. Mimicking the high-energy behavior of QCD by a bare constituent quark loop with a mass of about 1.5 GeV gives only an addition of $\sim 0.2 \times 10^{-10}$, and if there is any VMD suppression it will be even smaller. This we will take then as the uncertainty due to the high-energy region contribution and the ENJL result where it stabilizes as our estimate.

Let us now turn to the second type of contributions in Fig. 2(b). This contribution can be factorized into a five-dimensional integral, which we have evaluated using the Monte Carlo routine VEGAS times two two-dimensional integrals and one one-dimensional integral that we have evaluated using Gaussian integration. Here, again we have followed the prescription in [13] to calculate the contribution to a_μ. We have used two different approaches to calculate the quantity in (2.9) of [13]. The first one is using the Ward identities for four-point functions and the second one is using the Ward identities for three-point functions. Both agree exactly. We have done the same study of the cutoff dependence as for the four-point function contribution finding essentially the same conclusions. In Table I we have listed the $\mathcal{O}(N_c)$ hadronic light-by-light two leading contributions to a_μ and their sum for the up and down quarks as a function of the cutoff together with the errors quoted by VEGAS. Since the integrand is rather irregular, this error estimate is somewhat on the small side (see also [15]) and will be largely superseded by the error in our final result. Notice that the result for a_μ in Table I is more stable after adding the two contributions than separately. The contribution from the strange quark is in the range of the quoted errors in Table I. Here we used nonet symmetry. The effects of the $U(1)_A$ breaking tend to lower the η and η' exchange contributions [10]. The charm quark contribution we calculate with a bare quark loop damped with $c\bar{c}$ meson dominance propagators in the photon legs. This contribution is very small. Both scalar and axial-vector exchange contributions are again in the range of the quoted errors in Table I. We therefore take as an estimate of the leading $\mathcal{O}(N_c)$ hadronic light-by-light contributions to a_μ the result in Table I plus the strange and charm quarks contributions. We also add the scalar and axial-vector exchange contributions:

TABLE I. Results for the $\mathcal{O}(N_c)$ in the $1/N_c$ expansion two dominant hadronic light-by-light contributions to a_μ in the ENJL model for up and down quarks.

Cutoff (GeV)	$a_\mu \times 10^{10}$ from Constituent quark loop in Fig. 2(a)	$a_\mu \times 10^{10}$ from Pseudoscalar exchange in Fig. 2(b)	$a_\mu \times 10^{10}$ Sum
0.7	1.14 ± 0.02	-0.36 ± 0.01	0.78
1.0	1.44 ± 0.03	-0.46 ± 0.01	0.98
2.0	1.78 ± 0.04	-0.63 ± 0.01	1.15
4.0	1.98 ± 0.05	-0.75 ± 0.03	1.23
8.0	2.00 ± 0.08	-0.88 ± 0.05	1.12

$$(a_\mu^{\text{light-by-light}})_{\mathcal{O}(N_c)} = (1.2 \pm 0.5) \times 10^{-10}. \qquad (1)$$

The error includes the one from the integration routine VEGAS as shown in Table I multiplied by 5 plus the estimate of the high energy contributions uncertainty.

In addition to the leading $\mathcal{O}(N_c)$ result above, there are the contributions from pion and kaon loops. These are $\mathcal{O}(1)$ in the $1/N_c$ expansion and have to be added to the $\mathcal{O}(N_c)$ result in (1). We have seen that the lowest order CHPT result is damped by roughly the same factor in both the constituent quark loop and the pseudoscalar meson exchange contributions and that the high-energy region contributes significantly. This can be used to estimate that the result in [9] for the pion and kaon loops is in the right ballpark when vector mesons are included. As a first estimate we take the number and error from [9]

$$(a_\mu^{\text{light-by-light}})_{\mathcal{O}(1)} = (-0.45 \pm 0.80) \times 10^{-10}. \qquad (2)$$

We will return to this contribution in [10]. Adding the above $\mathcal{O}(N_c)$ and $\mathcal{O}(1)$ results, we get our final estimate

$$a_\mu^{\text{light-by-light}} = (0.8 \pm 0.9) \times 10^{-10}. \qquad (3)$$

A more general comment is that although a HGS model can be derived from the ENJL model, this is only true after a series of approximations. In HGS models the consistency between the parameters in the anomalous and nonanomalous sectors is not obvious. In the ENJL model we are using the same parameters appearing in both sectors. This is particularly important for the flavor anomaly contribution to the light-by-light scattering. For instance, the calculation in [9] assumes complete VMD for the anomalous sector. It was shown in [11] that complete VMD breaks the anomalous Ward identities. A prescription to include vector and axial-vector couplings was given there and was used in the present work. We find the pseudoscalar-exchange contribution to be negative but roughly between 5 times and 1 order of magnitude lower than the values quoted in [8,9]. It, however, agrees with the order of magnitude estimate in [5] and is then of the same order as the pseudoscalar meson loop contribution.

Our calculation establishes that, contrary to previous calculations [7–9], the contribution to a_μ from light-by-light scattering is positive and smaller than or around 1×10^{-10}. This is safely in the range of both the aimed experimental uncertainty at BNL and the theoretical error from the vacuum polarization contribution. It thus removes an important theoretical uncertainty in the interpretation of the muon $g - 2$ results from the planned BNL experiment.

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