Formation of Dynamic Domains in Strongly Driven Ferromagnets

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Based on the dissipative Landau-Lifshitz equation, the spatiotemporal structure formation problem is investigated in the region far above the transverse ferromagnetic resonance instability. Apart from the external fields, the model contains an isotropic exchange field, a shape demagnetization field, and an anisotropy field. Numerical simulations exhibit in the rotating frame a stationary domain structure with a precessing motion in the wall regimes. Employing analytical methods, characteristic elements of this structure are explained. This driven dissipative system shows similarities to equilibrium systems of coexisting phases and organizes itself in such a way that the local dynamics tends to become Hamiltonian.

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Ferromagnetic systems strongly driven by external transverse magnetic fields have been under investigation for more than four decades (for the early work see [1]). Based on a nonlinear spin-wave expansion around the homogeneous ground state, Suhl [2] showed that certain modes become unstable for increasing amplitude of the pump field. Detailed measurements exhibit beyond threshold many of the phenomena predicted by the general theory of nonlinear dynamics. Based on [2], truncated spin-wave mode models have been published, which successfully describe the experimental findings (for a recent, comprehensive presentation see [3]).

Apart from single exceptions like the simulations of Elmer [4], who found dynamic domains for a special model, the existing investigations are limited to the weak nonlinear regime. The regime far above the instabilities with pump amplitudes of the order of the demagnetization field remains an open question (compare [5], p. 1075).

This question is of high interest in the general theory of spatiotemporal pattern formation in dissipative systems [5]. At the beginning of this modern and interdisciplinary field of research Anderson [6] proposed that a driven ferromagnet is the characteristic example for pattern formation and for the general problem of how the concepts of equilibrium phase transitions can be extended to driven dissipative systems. Recently, Cross and Hohenberg [5] raised doubts about this proposal.

It is mainly the dipole interaction that makes pattern formation in ferromagnetic materials a difficult problem. The difficulties already arise in the undriven case [7]. In a first step, the theory of static ferromagnetic structures [8] usually employs an approximation. Above all, stray fields are omitted and only the part of the dipole interaction which describes the shape demagnetization fields is considered. Even in this approximation many interesting results have been obtained [8] both for the formation of static structures and for the wall dynamics.

Using this approximation for the dipole field, this work addresses the pattern formation problem in driven magnetic systems. Focusing on the region far above threshold, it is the aim of the present investigations to work out general features and mechanisms by employing both analytical and numerical methods. It is beyond the scope of this paper to improve the existing near threshold treatments, as this would certainly require the complete dipole field used in the spin-wave approaches.

At a mesoscopic scale the dynamics is governed by the Landau-Lifshitz equation as was recently demonstrated by microscopic investigations [9,10]. In the frame rotating with the driving frequency ω around the $\mathbf{e}_z$ direction, this equation of motion takes the form

$$\dot{\mathbf{m}} = -\mathbf{m} \times (\mathbf{H}^{\mathrm{eff}} - \omega \mathbf{e}_z) - \Gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}^{\mathrm{eff}}). \quad (1)$$

$\mathbf{m}(\mathbf{r}, t)$ and $\mathbf{H}^{\mathrm{eff}}$ are the local magnetization and the effective field in the rotating frame, being related to the quantities $\mathbf{m}_{\mathrm{lab}}$ and $\mathbf{H}^{\mathrm{eff}}_{\mathrm{lab}}$ in the laboratory frame by $\mathbf{m}_{\mathrm{lab}} = \exp(\omega t \mathbf{e}_z \times)\mathbf{m}$ and by $\mathbf{H}^{\mathrm{eff}}_{\mathrm{lab}} = \exp(\omega t \mathbf{e}_z \times)\mathbf{H}^{\mathrm{eff}}$, respectively. Γ represents the Landau-Lifshitz damping rate. In reduced units the gyromagnetic ratio and the magnitude of the magnetization $m = |\mathbf{m}|$ are equal to 1.

Specifying the model under investigation, it is assumed for the effective field that

$$\mathbf{H}^{\mathrm{eff}} = h_{\parallel} \mathbf{e}_z + h_{\perp} \mathbf{e}_x - \overline{\mathbf{m}} + J \Delta \mathbf{m} + A m_z \mathbf{e}_z, \quad (2)$$

where $h_{\parallel}$ and $h_{\perp}$ are the amplitudes of the external static and the external circular driving rf field, respectively. The term $\overline{\mathbf{m}} = V^{-1} \int \mathbf{m}\, dV$ represents, again in reduced units, the demagnetization field of a sphere of volume V. The contribution $J \Delta \mathbf{m}$ results from the isotropic ferromagnetic exchange interaction. A uniaxial anisotropy is described by $A m_z \mathbf{e}_z$.

In the first part of this Letter the results of numerical simulations in one spatial dimension with an arbitrary ξ direction $\mathbf{m}(\mathbf{r}, t) \rightarrow \mathbf{m}(\xi, t)$ are reported using periodic boundary conditions $\mathbf{m}(\xi, t) = \mathbf{m}(\xi + L, t)$. These investigations were performed on a vectorized supercomputer. The program uses a Euler integration scheme in time and is based on semispectral methods.

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These simulations exhibit a temporal evolution of $\mathbf{m}(\xi, t)$ which is characteristic for structure formation in dissipative systems. After a period of transient behavior, which strongly depends on the special initial state and which may be very complex, a tendency toward the formation of domains is found. In the early stage these domains interact with each other, merging into domains of larger size. With increasing time and increasing domain sizes, this process slows down and finally the structure becomes stationary on large time scales.

Figure 1 shows such a final domain state obtained by a long-time study starting from a randomly disturbed, homogeneous initial state. The magnetization is nearly everywhere constant, taking the values $\mathbf{m}_+$ or $\mathbf{m}_-$, respectively. In contrast to this behavior within the domains, a strong time dependency is found in the narrow wall regimes which separate the domains. As Figs. 2 and 3 demonstrate, the magnetization $\mathbf{m}$ at a fixed position is anharmonically oscillating with a characteristic internal period of rotation T_{per}, which is found to be independent of the specific position. This spatiotemporal wall structure is dynamically very stable. In the simulations no changes of any significance could be found for times corresponding to 10^4 periods of T_{per}.

It should be pointed out that this wall structure as well as the macroscopic quantities $\mathbf{m}_+$, $\mathbf{m}_-$, $\overline{\boldsymbol{m}}$, and T_{per} are independent of the specific initial state and thus are characteristic elements for the structure formation.

To analyze these characteristic features found in the simulations a perturbative approach has been worked out using the multiple-time scaling method and treating the contributions $J\Delta\mathbf{m}$ and $Am_z\mathbf{e}_z$ as perturbations [11]. The theory has been worked out up to the first order. The first order treatment is lengthy and rather technical [12]; thus I restrict myself here to the zeroth order.

In this order Eqs. (1) and (2) reduce to

$$\dot{\mathbf{m}}(\mathbf{r}, \mathbf{t}) = -\mathbf{m} \times \mathbf{H}_1 - \Gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_2) \tag{3}$$

with

$$\mathbf{H}_2 = \mathbf{H}_1 + \omega \mathbf{e}_z = h_\| \mathbf{e}_z + h_\perp \mathbf{e}_x - \overline{\boldsymbol{m}}(t). \tag{4}$$

Because of the $\overline{\boldsymbol{m}}$ term, the problem described by Eqs. (3) and (4) is of the mean field type and the usual technique can be applied. For the present case this would imply solving Eq. (3) for given $\overline{\boldsymbol{m}}(t)$ as a first step. Integration of the obtained solution $\mathbf{m}(\mathbf{r}, t)$ over the sample volume V would then lead to a self-consistency condition for $\overline{\boldsymbol{m}}(t)$, from which $\overline{\boldsymbol{m}}(t)$ and the complete solution can in principle be obtained. In general this procedure cannot be performed, as already the analytic solution of the first step is not known [13]. The fixed points of Eq. (3), however, and their stability can explicitly be determined by applying the mean-field procedure.

The results of this, basically straightforward, fixed point analysis show that marginally stable solutions exhibiting a domain state structure are possible. The local magnetization $\mathbf{m}(\mathbf{r})$ takes only two values $\mathbf{m}_+$ and $\mathbf{m}_-$ realized in the generally disconnected partial volumes $V_+ = n_+ V$ and $V_- = n_- V$, respectively.

In such domain states the fields $\mathbf{H}_1$ and $\mathbf{H}_2$ become orthogonal,

$$\mathbf{H}_1 \cdot \mathbf{H}_2 = 0, \tag{5}$$

and it is advantageous to write the results in terms of the internal coordinates introduced by $\mathbf{e}_1 = \mathbf{H}_1/H_1$, $\mathbf{e}_2 = \mathbf{H}_2/H_2$, and $\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$. The transformation from the internal to the primary frame is found to be

$$\begin{pmatrix} \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z \end{pmatrix} = \frac{1}{uw} \begin{pmatrix} -dh_\perp & -du & \Gamma w \\ \Gamma h_\perp & \Gamma u & dw \\ -u^2 & uh_\perp & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix}, \tag{6}$$

where

$$d = h_\| - \omega, \qquad u = (d^2 + \Gamma^2)^{1/2}, \tag{7}$$

$$v = (u^2 - \Gamma^2 h_\perp^2)^{1/2}, \qquad w = (u^2 + h_\perp^2)^{1/2}, \tag{8}$$

were introduced. The magnitudes of the fields $\mathbf{H}_1$ and $\mathbf{H}_2$ are calculated to take the values

$$H_1 = \omega u w^{-1}, \qquad H_2 = \omega h_\perp w^{-1}, \tag{9}$$

and the domain magnetizations are found to be given by

$$\mathbf{m}_\pm = u^{-1}(\mp v \mathbf{e}_1 + \Gamma h_\perp \mathbf{e}_3). \tag{10}$$

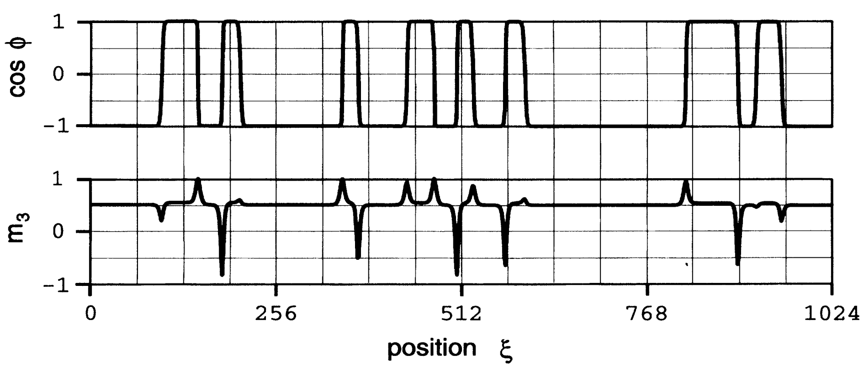

FIG. 1. Spatial dependence of a stationary domain state in the rotating frame. Plotted are $\cos\phi = m_1(m_1^2 + m_2^2)^{-1/2}$ and m_3, where m_i are the components of the local magnetization in the internal coordinates, defined by Eq. (6). The simulation uses 1024 mesh points and the parameter values $h_\| = \omega = 2$, $h_\perp = 0.5$, $J = 0.01$, $A = -0.005$, $\Gamma = 0.1$, and $L = 1024$.

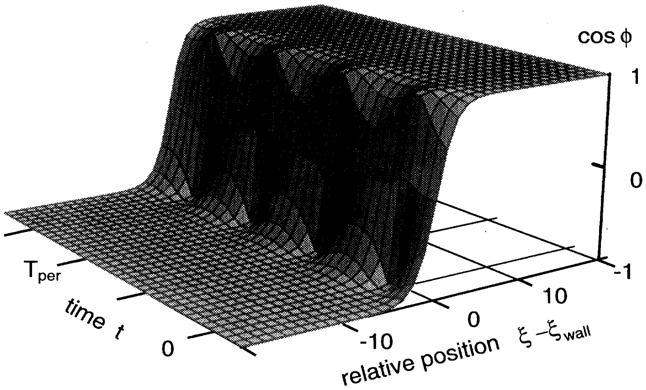

FIG. 2. Spatiotemporal structure of $\cos\phi$ in the vicinity of a wall located at $\xi_{\rm wall}$ with $T_{\rm per} = 18.6$. Definitions and parameter values as in Fig. 1.

For the bulk magnetization in a domain state, the self-consistency condition leads to

$$\overline{\mathbf{m}}_{\rm dom} = n_+ \mathbf{m}_+ + n_- \mathbf{m}_- , \tag{11}$$

and the $n_\pm$ are calculated to be

$$n_\pm = \frac{1}{2} \pm \frac{dw^2 + \omega u^2}{2wv} . \tag{12}$$

The domain states exist only in a subspace of the parameter space spanned by $h_\parallel$, $h_\perp$, ω, and Γ. This subspace is implicitly determined by the condition $0 < n_+ < 1$. For parameter values outside this subspace Eq. (3) has a different stable fixed point solution characterized by a homogeneous magnetization $\mathbf{m}(\mathbf{r}) = \mathbf{m}_{\rm hom}$. These homogeneous states, being of minor interest for the present work, are unstable in the regime where the domain states exist. Apart from the domain and the homogeneous states, no other stable fixed points of Eq. (3) exist.

Next, the findings of the linear stability analysis about the domain state fixed points are presented. Employing the usual $\exp(\lambda t)$ ansatz for the deviations, one finds for all modes with the exception of four

$$\lambda = \pm i\Omega \quad \text{with} \quad \Omega = \omega v w^{-1}, \tag{13}$$

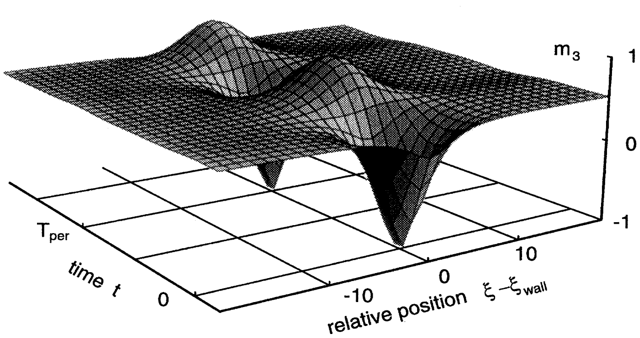

FIG. 3. Spatiotemporal structure of m_3 in the vicinity of a wall. Definitions and parameter values as in Figs. 1 and 2.

which implies that they are undamped and oscillating with Ω. The four remaining modes are of collective character as they describe disturbances of the form $V^{-1} \int_{V_\pm} [\mathbf{m}(\mathbf{r}, t) - \mathbf{m}_\pm] \, dV$. The characteristic equation for these collective modes is found to be given by

$$0 = |\Lambda_+ \Lambda_-|^2 - 2n_+ n_- v^4 u^{-4} [\mathrm{Re}(\Lambda_+ \Lambda_-^*) - n_+ n_-] - n_+^2 n_-^2 - 2n_+ n_- \Gamma^4 h_\perp^4 u^{-4} [\mathrm{Re}(\Lambda_+ \Lambda_-) - n_+ n_-], \tag{14}$$

where $\Lambda_\pm = (\lambda \mp i\Omega)/(\Gamma + i) + n_\pm$ was introduced. It has been shown analytically for the weak damping case ($\Gamma \ll 1$) and numerically for the general case that the collective modes are damped everywhere. Note that this implies relaxation of $\overline{\mathbf{m}}(t)$ to $\overline{\mathbf{m}}_{\rm dom}$.

Under the constraint $\overline{\mathbf{m}}(t) = \overline{\mathbf{m}}_{\rm dom}$, the analysis of Eqs. (3) and (4) can be extended to the nonlinear regime. Employing Eq. (9), this leads to

$$\dot{\mathbf{m}} = -\omega w^{-1} [u \, \mathbf{m} \times \mathbf{e}_1 + \Gamma h_\perp \mathbf{m} \times (\mathbf{m} \times \mathbf{e}_2)] . \tag{15}$$

For this equation of motion the quantity

$$m_1(\mathbf{r}, t) [\Gamma h_\perp m_3(\mathbf{r}, t) - u]^{-1} = VC(\mathbf{r}) \tag{16}$$

is a constant of motion and Eq. (15) is integrable, as in addition $m_1^2 + m_2^2 + m_3^2 = 1$ holds. Equation (16) represents a plane and thus the resulting motion of $\mathbf{m}(\mathbf{r}, t)$ is a precession on a cone at an arbitrary but fixed position $\mathbf{r}$. This precession is found to be anharmonic with the period of rotation of $T_{\rm per} = 2\pi\Omega^{-1}$ independent of $\mathbf{r}$.

The values of $C(\mathbf{r})$ are restricted by the condition $|C(\mathbf{r})| \le 1$, as otherwise the plane (16) does not intersect with the sphere. For the limiting values $C(\mathbf{r}) = \pm 1$ the plane becomes tangential and consequently $\mathbf{m}$ becomes time independent taking the fixed point values $\mathbf{m}_\pm$ of Eq. (10). Thus a wall is characterized by a change from $C = -1$ to $C = 1$. Considering now the $V \to \infty$ limit, the constraint $\overline{\mathbf{m}}(t) = \overline{\mathbf{m}}_{\rm dom}$ can be satisfied, as long as the entire wall volume goes to zero faster than V.

Summing up, the analysis of Eqs. (3) and (4) has shown that stable solutions of domain type do exist. In the domain regions the local magnetization is stationary but precesses with $T_{\rm per}$ in the wall regime. It is obvious that these results are qualitatively in accord with the numerical findings. Using the parameter value of the simulation, quantitative agreement is found for $\overline{\mathbf{m}}_{\rm dom}$, for $\mathbf{m}_\pm$, for $n_\pm$, and for $T_{\rm per}$. The deviations are less than 1%. Recall also that the numerical results of Figs. 1–3 were already presented in terms of the internal coordinates (6). Furthermore, the numerical results show that the motion in the wall regime is planar and well described by Eq. (16). Finally, the first order treatment [12] of Eqs. (1) and (2) is able to explain with high accuracy the complete spatiotemporal wall structure of Figs. 2 and 3.

Some features of the present investigation which may be of general interest should be pointed out.

The behavior of this dissipative system driven away from equilibrium in the stationary domain state is analogous to a thermodynamic system *at* a first order equilibrium transition exhibiting coexisting phases. The translational symmetry is broken for the domain structure. Macroscopic variables, such as in the present case $\overline{\boldsymbol{m}}_{\text{dom}}$, $\overline{\boldsymbol{m}}_{\pm}$, and $n_{\pm}$, are independent of the initial conditions and only depend on the external parameters like $h_{\|}$, $h_{\perp}$, and ω. In this context Eq. (5) has to be interpreted as the criterion for existence of the domain states. Note that this criterion reduces for the static case ($\omega = 0$) to $h_{\|}\mathbf{e}_z + h_{\perp}\mathbf{e}_x - \overline{\boldsymbol{m}}_{\text{dom}} = 0$, which means that the internal field vanishes. As this is the usual criterion for a ferromagnet to be at the equilibrium phase transition, some aspects of the present work can be interpreted as an extension of the concepts of phase transitions to driven systems.

Another interesting aspect of the present work is the local generally nonlinear dynamics within the domain regimes. As long as the sample is sufficiently large, this local dynamics is integrable or is Hamiltonian according to the investigations of Eq. (15). Although the system becomes weakly dissipative in the first order treatment [12], it is worthwhile to point out this tendency of the system to organize itself on the global macroscopic scale in such a way that the local mesoscopic dynamics becomes at least nearly Hamiltonian.

In this context it is interesting to go back to the microscopic scale [9], where the system is a quantum mechanical many body spin system in contact with a bath of sufficiently low temperature. It is basically the competition between the short ranged exchange interaction and the long ranged repulsive dipole interaction which leads to the formation of structure and to the nearly Hamiltonian dynamics on the mesoscopic scale.

Ferromagnets are representative of a whole class of many body systems characterized by short ranged attractive interactions in competition with long ranged repulsive interactions. Thus the question arises whether the tendency toward a local Hamiltonian dynamics is generic for all these rather common systems. Should this be the case, it would perhaps become clear why structure and Hamiltonian dynamics are so widespread in nature.

Coming back to the ferromagnet, it is pointed out that neither the location nor the geometric shape of the walls is determined by the present approach. By analogy with the static case [8], it is expected that the dipolar stray field and the usual boundary effects will reduce this freedom. The formation of regular patterns in driven ferromagnets seems to be possible.

For completeness, the results of this work are specialized to the weakly nonlinear regime $h_{\perp} \ll 1$. For simplicity, the discussion will be restricted to $\omega = h_{\|}$ and to $\Gamma \ll 1$. A critical value of the pump field $h^{\text{crit}} = \Gamma(h_{\|}^2 - 1)^{1/2}$ results. For $h_{\perp} < h^{\text{crit}}$ the homogeneous state is stable and for $h_{\perp} > h^{\text{crit}}$ the domain states arise. As in this work the dipolar stray field is neglected, h^{crit} differs from the threshold values of [2], which under usual conditions are smaller than h^{crit}. Lastly, the findings for the power absorption P will be given. For the homogeneous state $P = \Gamma^{-1}h_{\perp}^2$ and for the domain states $P = \Gamma h_{\|}^2 h_{\perp}^2(\Gamma^2 + h_{\perp}^2)^{-1}$ are obtained. This result implies an experimentally well established [1,3] saturation of P for $h_{\perp} \gg \Gamma$. Thus, as a side product of this work it must be concluded that this saturation is not solely explained through spin-wave approaches.

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