Measurement of the Solar Gravitational Deflection of Radio Waves Using Very-Long-Baseline Interferometry

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(Received 14 April 1995)

We made very-long-baseline-interferometry observations of the extragalactic radio sources 3C273B and 3C279 to measure the gravitational deflection of radio waves by the Sun. Cross-correlation of data recorded at antennas in California and Massachusetts at 2, 8, and 23 GHz during a ten-day period surrounding the October 1987 solar occultation of 3C279 yielded plasma-corrected group delays, from which we obtained $\gamma = 0.9996 \pm 0.0017$ (estimated standard error), corresponding to a gravitational deflection 0.9998 ± 0.0008 times that predicted by general relativity.

PACS numbers: 04.80.Cc, 95.85.Bh

There is wide recognition of the importance of testing theories of gravitation. This importance arises not only from the fundamental nature of the physics involved, but also from the astrophysical role that relativistic gravitation plays, for example, in cosmology, quasars, black holes, pulsars, and close binaries.

One of us first suggested nearly 30 years ago [1] that the gravitational deflection of light by the Sun—one of the three classic effects of general relativity analyzed by Einstein—could be measured more accurately at radio wavelengths with interferometry techniques than at visible wavelengths with available optical techniques. Since then, this deflection has been measured, with one exception [2], exclusively with radio interferometry (e.g., [3,4]). In this paper we present results of such a radio-wave deflection experiment using very-long-baseline interferometry (VLBI), at present the most accurate technique for measuring gravitational deflection.

VLBI involves simultaneous observations of (compact) radio sources at two or more antennas that are electrically independent of each other. The emissions received at each antenna are multiplied by ultrastable local-oscillator signals derived from an atomic clock and are then low-pass filtered, digitized (usually preserving only sign), and recorded on magnetic tape (see, e.g., Ref. [5]). These recorded data preserve the phase information of the incident radio signals. Data from the same frequency band obtained simultaneously at two antennas are cross-correlated to estimate the difference in the phases of the received signals as a function of time. This difference is the "fringe phase." The fringe phase divided by the observed angular frequency is the "phase delay," and the derivative of the fringe phase with respect to the observed angular frequency is the "group delay." Although phase-delay measurements are inherently more precise than group-delay measurements, the use of phase delays is problematic because such delays have narrowly spaced "ambiguities" stemming from the 2π ambiguities in the fringe phase; for that reason we used only group delays in our experiment. We determine the group delay (hereafter, "delay") from observations made at several (narrow) frequency intervals within a wider band [5]. For angular frequency ω and time t we model the delay as

$$\tau(\omega,t) = \tau_{\mathrm{geom}}(t) + \tau_{\mathrm{struc}}(\omega,t) + \tau_{\mathrm{plas}}(\omega,t) + \tau_{\mathrm{atm}}(t) + \tau_{\mathrm{inst}}(\omega,t) + \tau_{\mathrm{clk}}(t)\,, \qquad (1)$$

where $\tau_{\mathrm{geom}}(t)$ is the difference in propagation times along the vacuum signal paths from the radio source to the two antennas ("geometric delay") and $\tau_{\mathrm{struc}}(\omega,t)$, $\tau_{\mathrm{plas}}(\omega,t)$, $\tau_{\mathrm{atm}}(t)$, $\tau_{\mathrm{inst}}(\omega,t)$, and $\tau_{\mathrm{clk}}(t)$ are the contributions from, respectively, the brightness distribution ("structure") of the observed source, the dispersive elements ("plasma") in the signal propagation paths, the Earth's atmosphere, the receiver instrumentation, and the difference in the atomic-clock readings at the two sites.

The geometric delay includes the delay contribution from the gravitational deflections by the Sun and the Earth. Within the framework of the parametrized post-Newtonian (PPN) formalism (see, e.g., Ref. [6]), this contribution may be expressed as [7]

$$\tau_{\mathrm{grav}} \approx \frac{(\gamma+1)}{c}\left\{\frac{GM_S}{c^2}\ln\left[\frac{|\mathbf{d}_{ES}| + \mathbf{d}_{ES}\cdot\hat{\mathbf{s}} + \mathbf{r}_1\cdot(\hat{\mathbf{d}}_{ES}+\hat{\mathbf{s}})}{|\mathbf{d}_{ES}| + \mathbf{d}_{ES}\cdot\hat{\mathbf{s}} + \mathbf{r}_2\cdot(\hat{\mathbf{d}}_{ES}+\hat{\mathbf{s}})}\right] + \frac{GM_E}{c^2}\ln\left(\frac{1+\hat{\mathbf{r}}_1\cdot\hat{\mathbf{s}}}{1+\hat{\mathbf{r}}_2\cdot\hat{\mathbf{s}}}\right)\right\}, \qquad (2)$$

where γ is the PPN parameter that characterizes the contribution of space curvature to gravitational deflection ($\gamma \equiv 1$ in general relativity), c is the vacuum speed of light, G is the gravitational constant, M_S and M_E are the masses of the Sun and Earth, respectively, $\mathbf{d}_{ES}$ extends from the Sun's center to the Earth's center, $\hat{\mathbf{s}}$ points towards the (distant) observed source (a caret designates a unit vector), and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the geocentric position vectors of the two antennas. We evaluate all coordinates in a solar-system barycentric reference frame. Equation (2) has picosecond accuracy, sufficient for our needs. Our goal is the estimation of γ.

We made VLBI observations of the extragalactic radio sources 3C273B and 3C279 on eight days surrounding the 8 October occultation of 3C279 by the Sun in 1987, a year of relatively low coronal activity. Observations of 3C279 provided delays with a large gravitational deflection signature; observations of 3C273B, $\sim 10°$ away from 3C279 on the sky, provided "reference" delays that reduced the sensitivity of our estimate of γ to several sources of possible systematic error. On 2–5 and 11–12 October we observed simultaneously in three widely separated frequency bands, centered at 2.3, 8.4, and 22.7 GHz, to correct for the dispersive effects of the propagation medium, to afford some redundancy in this correction, and to guard against frequency-specific nulls in the fringe visibility of either source. On 6 and 10 October we observed in the 8 and 23 GHz bands only, because we expected that large, rapid plasma-density fluctuations in the solar corona would make the coherence time of the signals from 3C279 at 2 GHz too short to allow detection. The antennas used were the 40 m (diameter) and one of the 27 m antennas at the Owens Valley Radio Observatory, Big Pine, California, and the 37 and 18 m antennas at the Haystack Observatory, Westford, Massachusetts. We used two antennas at each site because no single antenna was equipped for simultaneous observations in three frequency bands. We observed at 23 GHz with each larger antenna and at 2 and 8 GHz with each smaller one, located at each site ~ 1 km from its larger neighbor. On every observing day except 2 October we observed for the full ~ 6 h that the sources were visible to all antennas simultaneously; on 2 October we observed for only ~ 3 h because of a late start. We always alternated 4 min observations of 3C279 with 2 min observations of 3C273B in a repeating 8 min cycle, using 2 min in each cycle for slewing. All antennas were synchronized in both pointing and slewing.

We used the Mark III VLBI system [5,8] for data acquisition and correlation. At each site, signals from fourteen 8 MHz wide subbands, distributed among the three frequency bands, were down converted to baseband, one-bit sampled at the Nyquist rate, and recorded on tape. We processed [5] these data to obtain a delay measurement from each 4 s of 3C279 observations and each 10 s of 3C273B observations, the longer time for the latter being acceptable since these observations were less affected by the solar corona. We then fit a parametrized theoretical model of the delays to our measurements using a Kalman-filter estimator [9], and from this fit obtained an estimate of γ and its statistical standard error (SSE).

Each model delay consisted of an *a priori* value for each term on the right side of Eq. (1). For the geometric delay we used the model in the CALC (version 7.0) software package [10], with two improvements: we added terms to the model to account for diurnal and semidiurnal variations [11] in pole position (which describes the orientation of the Earth's crust relative to its spin axis) and in UT1−AT (the difference between the time inferred from the rotational phase of the Earth and atomic time); and we replaced the nutation series [12] used in CALC with a more accurate one [13] that we modified for consistency with our reference frame [10].

We calculated values of τ_{struc} at 23 GHz from brightness maps of each source [14,15], and we inferred values of τ_{struc} at 2 and 8 GHz for each source from the closure delays [16] and visibility amplitudes obtained from VLBI observations that we made at Big Pine, Westford, and Fort Davis, Texas on 14 October 1987. In estimating the error in our estimate of γ due to the errors in our model structure delays, we assumed that the correction to γ derived from each of the six sets of structure delays (one for each source at each frequency band) had a standard error equal to the magnitude of the correction, and that the correction from each set was independent of the corrections from the other five; thus the total estimated standard error due to source structure equals the root sum squared of the magnitudes of the corrections from each set.

The plasma delays obey approximately the relation

$$\tau_{\text{plas}}(\omega, t) = \frac{I(t)}{\omega^2}, \tag{3}$$

where $I(t)$ is proportional to the difference in total electron content along the signal propagation paths to the two sites. Using Eq. (3), we determined τ_{plas} from delays obtained at two frequencies, ω_1 and ω_2:

$$\tau_{\text{plas}}(\omega_1, t) = \frac{\omega_2^2}{\omega_1^2 - \omega_2^2}\left[\tau_{\text{res}}(\omega_2, t) - \tau_{\text{res}}(\omega_1, t)\right], \tag{4}$$

where τ_{res}, the residual delay, is the difference between measured and model delays prior to plasma correction. Implicit in Eq. (3) are assumptions of low electron densities and weak magnetic fields along the signal propagation paths; in our experiment the errors corresponding to these approximations amount to at most a few picoseconds [17], and are negligible, as are the contributions to our estimate of γ from the errors in the square-bracketed term of Eq. (4) (see below).

We characterized τ_{atm} as the sum of a component that assumes the atmosphere to be in hydrostatic equilibrium ("hydrostatic delay"), and a component that accounts for additional contributions from water vapor ("wet delay")

[18]. At each site we obtained for the zenith direction *a priori* estimates of the hydrostatic delays from barometric measurements and *a priori* estimates of the wet delays from measurements with a water-vapor radiometer (WVR) [19] or, as backup, with a spectral hygrometer [20]. We projected the hydrostatic and wet delays to the elevations of the sources using, respectively, the CfA-2.2 [18] and Chao [21] mapping functions.

Of the last two terms on the right side of Eq. (1), only τ_{inst} had nonzero *a priori* values; these were provided by the Mark III calibration system [5]. We determined τ_{clk} solely from the analysis of our VLBI data.

Our Kalman-filter estimator allows stochastic (or quasistochastic) parameters to be modeled as any combination of white-noise, random-walk, and integrated-random-walk Gauss-Markov processes. We characterized τ_{clk} and deviations from the *a priori* estimates of τ_{atm} at zenith as a combination of these processes using variances commensurate with instrument performances [9,15].

For the six days of observations on which 2 GHz data were available, we computed one set of plasma-corrected delays from 2 and 8 GHz data and one set from 2 and 23 GHz data. Although these two sets have the 2 GHz delays in common, the plasma-corrected delays at matching epochs from each set are virtually independent [15] because the error in each plasma-corrected delay is dominated by the error in the delay from the higher frequency band. For the two days of observations closest to occultation, when 2 GHz data were not collected, we used a single set of plasma-corrected delays based on the 8 and 23 GHz data. Unfortunately, the data sets utilizing 8 GHz delays included only ~2 h per day of useful 3C279 observations because of severe destructive interference at other times caused by milliarcsecond-spaced components of that evolving source.

In total we had 6488 plasma-corrected delays from the 2 and 8 GHz data, 13 823 from the 2 and 23 GHz data, and 1843 from the 8 and 23 GHz data. Using these delays with our Kalman-filter estimator, we obtained (i) an estimate of γ; (ii) corrections to the *a priori* values of source positions (except for the right ascension of 3C273B, which defines our right-ascension origin) and relative site positions; (iii) daily adjustments to the *a priori* values of pole position, UT1−AT, and nutation angles; and (iv) estimates, using stochastic modeling, of the delay contributions from the Earth's atmosphere and from the time-varying offset between the atomic clocks at the two sites. The *a priori* covariance matrix of the estimator constrained corrections (ii)–(iv) to be consistent with our estimates of the true standard errors of the *a priori* values [15]; we left γ unconstrained. Our result for γ is

$$\gamma = 0.9996 \pm 0.0017, \tag{5}$$

where the quoted uncertainty is the estimated standard error, obtained by adding in quadrature the SSE from the Kalman-filter estimator (0.0016) and the estimated standard error (0.0005) of our source-structure corrections, which contributed 0.0003 to our result. The mean SSEs of the plasma-corrected delays (all frequency-band pairs combined) are ~50 and ~80 ps for the 3C273B and 3C279 delays, respectively, the latter mean being 0.006 of the maximum predicted value of τ_{grav}.

We did numerous sensitivity studies to identify the significant error sources and to test the robustness of our γ estimate. We report here the most significant results of these studies. More thorough descriptions are given in Ref. [15].

Our largest sources of error are the uncertainties in our knowledge of pole position, τ_{atm}, and τ_{clk}, which contribute 0.0011, 0.0011, and 0.0010, respectively, to the SSE of our estimate of γ. Because of the negative correlations among these contributions, the combined contribution to the SSE is less than the root sum square of these individual contributions. In general, our estimate of γ is sensitive to those factors in the experiment that, like γ, affect the delays differently on different days (e.g., τ_{atm}), and is virtually insensitive to those factors that affect the delays in the same way each day (e.g., site positions).

The errors in the atmospheric mapping functions are largest at the lowest elevation angles. To test the mapping functions at low elevation angles, we examined the effects of successively deleting data from elevation angles below 5° (our lowest observation was at 4°), 10°, 15°, and 20°. The resulting changes in our estimate of γ were insignificant (≤ 0.0004).

The differences between the values of τ_{plas} derived from 2 and 23 GHz delays and the values derived at matching epochs from 8 and 23 GHz delays sometimes drifted systematically. After averaging these differences over the 2 or 4 min duration of each observation of each source, we found that these systematic drifts were typically ~10 ps in amplitude, lasted as long as a few hours, followed apparently random patterns that did not repeat from day to day, and were common to both sources. There was no evidence of these drifts in the postfit residuals, however, indicating that the drifts were being "absorbed" by the other estimated parameters (in particular, those parameters characterizing the Gauss-Markov processes). A sensitivity study [15] showed the maximum effect of these drifts on our estimate of γ to be insignificant (< 0.0001), so we made no changes to our estimation process to account for the drifts.

We also evaluated the robustness of our result by comparing estimates of γ from different data subsets. We made three (virtually) independent estimates of γ using, in turn, each of the different frequency-band combinations of plasma-corrected delays, and obtained $\gamma = 1.0044 \pm 0.0039$, 0.9982 ± 0.0021, and 1.0016 ± 0.0026 (SSEs) from the plasma-corrected delays based on the 2 and 8, 2 and 23, and 8 and 23 GHz data, respectively. We also estimated γ from each of the 45 pairs of single-day data subsets that included one day's data from before the solar occultation of 3C279 and one from after. These pairwise estimates ranged from $\gamma = 0.9440 \pm 0.0350$ to $\gamma = 1.0214 \pm 0.0338$ (SSEs); six of these estimates deviated from $\gamma = 1$ by more than 1.5 SSEs, and only one ($\gamma = 0.9903 \pm 0.0035$), which is based on 2 and 23 GHz data from 4 and 11 October, deviated from $\gamma = 1$ by more than 2.5 SSEs. Our estimates of γ from different data subsets thus show no statistically unusual deviations from each other and, as with our result in Eq. (5), are consistent with general relativity.

How does our value for γ compare with the most accurately measured values reported previously? Robertson *et al.* [4] found $\gamma = 1.000 \pm 0.002$ (estimated standard error). Their result was based on over 300 000 VLBI observations, accumulated over 10 years, of sources distributed over the sky; its sensitivity to γ stemmed from a very large number of relatively small solar deflection signals. Our result, by contrast, is based on a far smaller data set with larger deflection signals. The only other reported value of γ with standard error as low as 0.002 comes from a time-delay measurement [22]; that value, too, is consistent with general relativity.

Two of us (J. L. D. and I. I. S.) are currently working on an estimate of gravitational deflection using VLBI data collected for geodetic purposes. We plan to combine the data described in this paper with the geodetic VLBI data to lower substantially our uncertainty in γ. We are also contemplating, with colleagues at the Jet Propulsion Laboratory, an improved version of this experiment that would utilize several compact reference sources surrounding (on the sky) an occulted source and employ more, and more sensitive, antennas. Such an experiment, combined with an independent data base from ~20 years of VLBI observations of a large number of sources, should yield a result for gravitational deflection severalfold more accurate than the one presented here.

We thank J. Ball, R. Cappallo, W. Carter, J. Carter, T. Clark, D. Daugherty, N. Donald, M. England, D. Gordon, H. Hardebeck, M. Hayes, H. Hinteregger, M. Hodges, G. Hurford, P. M. Mathews, R. Phillips, M. Poirier, R. Potash, J. Salah, D. Smythe, M. Titus, A. Whitney, and J. D. Williams for their valuable assistance. We also thank S. Unwin for providing source brightness maps in advance of publication. This work was supported by the National Science Foundation under Grant No. PHY-84-09671.

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