**Tanabe and Kaneko Reply:** In our Letter [1], we have proposed a simple phenomenological model for the behavior of a falling paper. As Mahadevan, Aref, and Jones [2] commented, there is a beautiful formulation by Kirchhoff [3] for the motion of a rigid body in an inviscid fluid. Adopting this formulation, Aref and Jones found chaotic motion of a rigid body for a given class of initial conditions [4]. Kozlov's recent study is important as a choice of direction in the falling process during the acceleration by gravity [5]. We would like to thank the authors for pointing out these previous works.

It should be noted, however, that these studies are all based on the assumption of a perfect fluid, and belong to the category of a Hamiltonian dynamical system. On the other hand, ours is the first model to address the question of dissipation, albeit in a phenomenological manner. It seems natural to assume that an irregular fall is explained in terms of a chaotic attractor, not as a Hamiltonian system. The aim of our study is to construct a simple model to check if the irregular fall of a paper can be explained in terms of dissipative chaos.

As we stated in the Letter, our model is not derived from the Navier-Stokes equation. Rather we constructed our model as simply as possible by composing forces (friction, lift, and gravity) that we thought essential and should not be omitted. We believe that the viscosity effect is essential to the falling of a paper, which leads to many complex phenomena. The relevant part of the effect would mainly be replaced by friction. In our Letter we divided the friction terms into two parts considering the direction of the wake. [Indeed this division was found to be important in producing chaotic falling patterns (cf. Fig. 2 of [1]).]

As pointed out in the Comment, our choice of a friction term that is proportional to the velocity leads to an apparent contradiction, since the proportionality was assumed on the basis of a low Reynolds number, while we borrowed the lift term from the perfect fluid formulation. However, we do not think it is appropriate to use a friction term proportional to the square of the velocity [6], because the fluttering motion in real life is usually very slow (as well as in our simulation). Instead, we think the approximation for a lift term may be rather rough.

We believe that our model, albeit a crude approximation for the natural processes, has succeeded in reproducing the falling patterns observed in nature, and in associating the irregular falls with chaotic attractors. We also believe that our result is rather robust and independent of the details of the models. Thus we feel that our conclusion is (qualitatively) invariant even if a refined model is constructed.

We agree that Kirchhoff's formulation should be developed further for the study of a motion of a rigid body in inviscid fluid. In the previous model the overdamped limit of the fluid motion is taken by assuming that the fluid becomes stationary at each instance [7]. On the other hand, when the inertia of the fluid is included too much, the correspondence with the fall in nature may be lost. We know from our experience that two papers fall almost independently, even if they are close to each other. This implies that the interaction between the papers is negligible, and thus the fluid inertia might not be so important. A better description of the fluid viscosity may be required to have a better approximation. A study of the combination of the inertial effects included in the previous work with dissipation is currently underway.

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[6] Both friction formulas (with linear or square dependence on velocity) assume that the flow of fluid is static. If the velocity is time dependent, no general formula for the friction exists as far as we know. Indeed the choice of a friction term is not so critical in our problem, since the paper tends to orient itself so as to minimize the friction effect in our simulation (and in nature).
[7] The criticism on the lack of the buoyancy in our model is totally irrelevant to our behavior. The buoyancy leads to another parameter multiplying the gravitational term, which is easily scaled out by a simple reparametrization. (In any case, the buoyancy is not important, since we are concerned with the fall of a one-dimensional sheet.)

  