Comment on "Behavior of a Falling Paper"

In a recent paper [1], Tanabe and Kaneko propose a model to describe a number of interesting regimes of motion of a falling two-dimensional object in a fluid. As might be expected, the general topic of their Letter has received considerable prior study, which is not reflected in the literature they cite. The specific problem of the free fall of a strip of paper in air was the subject of an 1854 paper by Maxwell [2]. The study of this and related problems is broadly classified under the heading autorotation [3]. The general problem of the motion of a solid body in an inviscid fluid has been studied for at least a century following seminal work by Kelvin and Kirchhoff around 1870. Central to this theory is the replacement of the scalar mass by a renormalized added-mass tensor that quantifies the inertial interaction of the fluid with the moving solid, and whose components reflect the geometry of the body. The resulting equations of motion generalize Euler's equations for the motion of a rigid body in a vacuum and are presented in many places, e.g., the treatise by Lamb [4]. The effect of the inertia of the fluid set in motion by a body moving through it is neglected in [1].

Even in the absence of gravity, the motion of a two-dimensional solid in an infinite expanse of inviscid, incompressible fluid, at rest except for the motion induced by the body, exhibits sinuous modes [4] not unlike those shown in Fig. 3 of [1]. In this case, the resulting dynamical system is integrable. When gravity is included, Kozlov [5] has rigorously proved the existence of periodic solutions for small initial data ("periodic flutter" in the terminology of [1]). He also brings to attention the work of Joukowski and Chaplygin who studied aspects of this problem during the early part of this century. The authors of [1] allude to none of this work.

The study in [1] omits any treatment of the inertial interaction terms and introduces phenomenological formulas to model the drag and lift forces. As a consequence, in the absence of drag and lift the rotational motion will decouple from the translational motion in Eq. (4) of [1], while in the formulation of the problem within the realm of inviscid hydrodynamics, the coupling persists due to the presence of the added-mass effect. This coupling between rotation and translation for anisotropic bodies such as a flat plate is at the origin of most of the interesting behavior of the model examined in [1]. We believe that several (and possibly all) of the interesting regimes of motion found in Tanabe and Kaneko's model [1] are already present in simpler models that have the advantage of being cast in a physically consistent framework consisting of (a) the two-dimensional, incompressible Euler equations for the fluid motion, (b) the equations of motion for a two-dimensional solid coupled to the fluid motion by (c) kinematical conditions at the surface. While it is not presently known whether all the regimes reported in [1] appear within this established simpler description, the related problem of the impulsive motion of a three-dimensional solid body in an inviscid, incompressible fluid is known to be chaotic for certain ranges of the anisotropy of the added-mass tensor [6].

Additionally we remark that the gravitational body force that contributes to the motion of the solid when the solid and fluid densities differ is not introduced correctly. This term should be proportional to the density difference and vanishes when the solid and fluid densities are equal. From Eq. (4) of [1], even a body whose density is less than or equal to that of the surrounding fluid will sink when released from rest.

Although the results derived from the model described in [1] seem plausible for heavy bodies, given the assumptions of the study, the model is itself flawed and inconsistent with the physics of the phenomena it seeks to describe. Since simpler descriptions [5–7] do show some of the interesting behavior, it would seem prudent to explore these carefully first before introducing further *ad hoc* effects.

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