Charged Vortices in High Temperature Superconductors

D. I. Khomskii
Groningen University, 9747 AG Groningen, The Netherlands
and
P. N. Lebedev Physical Institute, Moscow, Russia

A. Freimuth
II. Physikalisches Institut, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany
(Received 2 September 1994)

It is argued that in the mixed state of a type II superconductor, because of the difference of the chemical potential in a superconducting versus normal state, the vortex cores may become charged. The extra electron density is estimated. The extra charge contributes to the dynamics of the vortices; in particular, it can explain in certain cases the change of the sign of the Hall coefficient below T_c frequently observed in the high temperature superconductors.

PACS numbers: 74.60.Ge, 74.72.–h

The transition of a system to a superconducting state leads to a change of the chemical potential of the electrons [1–3]. In a material which has two electronic subsystems, one of which becomes superconducting, the other remaining normal, this change of the chemical potential causes a charge redistribution between these subsystems below T_c [2]. This is qualitatively easy to understand: Below T_c there is an energy gain for the condensed charge carriers, and it is therefore energetically favorable to transfer some charge carriers from the normal to the condensate region. The effect is of general character and should occur in any superconductor. However, the charge transfer is determined by the magnitude of $(\Delta/\epsilon_F)^2$, where Δ is the energy gap and ϵ_F the Fermi energy. Therefore the charge redistribution is, in particular, important in the high temperature superconductors (HTSCs), because of their relatively large value of Δ/ϵ_F. It was shown in Ref. [2] that it can explain several anomalies observed in the HTSCs at and below T_c.

We want to point out in this Letter that the change of the chemical potential below T_c may also lead to charging of a vortex core in the mixed state of a type II superconductor. Assuming that the vortex core is a region of normal metal surrounded by superconducting material the corresponding difference in the chemical potential leads to a redistribution of the electrons. The extra charge of the cores gives rise to an additional force on vortices; in particular, for an appropriate sign of the charge this force can lead to a sign change of the Hall coefficient, which is frequently observed experimentally in the HTSCs [4–7].

The theoretical treatment carried out in [1,2] gives the following expression for the change of the chemical potential μ of the electrons below T_c for a model with a constant density of states:

$$\mu(T) = \mu_0 - \frac{\Delta^2(T)}{4\mu_0} \tag{1}$$

for a less than half-filled band. For $\mu_0 > D/2$, where D is the bandwidth, the term $-\Delta^2/4\mu_0$ in (1) is substituted by $+\Delta^2/4(D - \mu_0)$. For the general case of an arbitrary density of states $N(\epsilon)$ the corresponding formula has the form [2]

$$\mu(T) = \mu_0 - c\frac{1}{N(\epsilon_F)}\frac{\partial N}{\partial \epsilon}\bigg|_{\epsilon_F} \Delta^2(T), \tag{2}$$

where the coefficient $c \sim 0.3$.

When part of the system is normal, the condition of equality of the chemical potentials in different parts leads to a redistribution of electrons between the subsystems. Such a situation is realized in the mixed state of type II superconductors, where vortices are present. The superconducting order parameter is zero in the center of a vortex, and it increases to its bulk value on a length scale given by the coherence length ξ. In the region of a suppressed order parameter, a number of low lying normal excitations exist with a density of states comparable to that of a normal cylinder of radius ξ [8]. Accordingly, a commonly used model for the vortex core is that of a normal cylinder of radius ξ in a superconducting surrounding [9]. Because of the electron redistribution, the normal core would acquire an extra electron density δn (per unit length), which for the *neutral* system would be

$$\delta n = \pi\xi^2 2N(\epsilon_F)\delta\mu = -2\pi\xi^2 c\frac{dN}{d\epsilon}\bigg|_{\epsilon_F} \Delta^2(T). \tag{3}$$

There is yet another contribution to the chemical potential—the kinetic energy of the superfluid motion around the vortex core [10,11]

$$\delta\mu = \frac{n_s}{n}\frac{mv_s^2(r)}{2}. \tag{4}$$

Here n_s and n are the superfluid and total electron densities. Note that the terms (1) and (4) perfectly match at $r = \xi$: The condition that the kinetic energy of the superfluid motion equals the condensation energy, or that the corresponding velocity is equal to the depairing velocity, determines the size of the normal core.

 0031-9007/95/75(7)/1384(3)$06.00

For the electrons in metals it is the electrochemical potential $\mu_{el} = \mu(r) + e\varphi(r)$ which has to be spatially constant. Therefore the position-dependent terms (1) and (4) in the chemical potential lead to an electric potential (cf. [10])

$$e\varphi(r) = \frac{\Delta^2(r)}{4\mu_0} - \frac{n_s}{n}\frac{mv_s^2(r)}{2} \tag{5}$$

and accordingly to a charge redistribution. Taking into account an ordinary metal screening (which is essentially the same in superconductors and in normal metals), we find for the case of small Debye screening length $r_D \ll \xi$ the charge density q_v at the vortex from the usual condition $q_v/\epsilon_0 = \nabla^2\varphi(r)$ (we use SI units below). Using (5) we estimate the total charge Q_v per unit length inside the vortex core:

$$Q_v \simeq \pi\xi^2 q_v = \epsilon_0 \pi\xi^2\nabla^2\varphi \sim \epsilon_0 \frac{\Delta^2}{e\epsilon_F}. \tag{6}$$

Taking for the typical HTSCs $\Delta/\epsilon_F \sim 10^{-1}$, we get for a pancake vortex of length $\simeq$5Å a charge of order $10^{-3}e$. This charge may be treated as bound to the core (the corresponding wave functions are predominantly those localized in a core [8]). According to Eqs. (5) and (6) this charge is screened on a length scale λ. Note that in typical situations the sign of the core charge is opposite to the sign of the dominant charge carriers, so that for HTSCs with hole conductivity the core should be charged negatively. The corresponding radial electric field is such as to provide the force necessary for the circular motion of electrons in a vortex: It has to overcompensate the Lorentz force on the circulating electrons, which is directed *away* from the core [11].

There should exist several consequences of the effect of charge redistribution around the vortex. For example, the potential $\varphi(r)$, in Eq. (5), on a vortex leads to a certain shift of ions, so that the total density of the material at the vortex will be different from the bulk. This is the microscopic origin of the effect previously discussed phenomenologically [12]. This density change contributes, e.g., to the pinning and can lead to a long-range elastic interaction between vortices (see, e.g., [13]).

More interesting may be the consequences of the charging for the dynamics of vortices. In particular, an extra force $\mathbf{f}_q$ on a *moving* vortex should be present. One can obtain $\mathbf{f}_q$ using the following arguments. The motion of a vortex with the velocity $\mathbf{v}_L$ yields a current density $\mathbf{j}_v = q_v\mathbf{v}_L$, when there is an extra charge density $q_v \simeq Q_v/\xi^2$ in the vortex core. Charge neutrality then requires a backflowing supercurrent $\mathbf{j}_b = -\mathbf{j}_v$, which in turn exerts a Lorentz or Magnus force on the vortex, given by

$$\mathbf{f}_q = -q_v\Phi_0\mathbf{v}_L \times \mathbf{n}, \tag{7}$$

where $\mathbf{n}$ is a unit vector in the direction of the vortex axis. This force drives vortices parallel ($q_v > 0$) or antiparallel ($q_v < 0$) to the electric current $\mathbf{j}||\mathbf{E}|| - \mathbf{v_L} \times \mathbf{n}$. It gives rise to a Hall voltage, and it can lead to a sign change of the Hall coefficient. A sign change of the Hall coefficient below T_c in weak magnetic fields is frequently observed in the HTSCs. It is one of the most puzzling experimental findings regarding the dynamics of vortices in these systems [4–7].

Note that the derivation of the force shows that $\mathbf{f}_q$ is not of electromagnetic origin, but results from the hydrodynamic interaction of the backflowing particle current with the circulating supercurrents around the vortex. A similar force should be present in superfluid He, where the particle density in the vortex cores should differ from that of the surrounding material.

The fact that the net charge of a vortex integrated over distances $>\lambda$ is zero due to screening does not modify this conclusion: The charge inside any given radius R ($\xi < R < \lambda$) is nonzero (and is always of the same sign), so that the interplay of a backflow with circulating supercurrents will be present at all distances. The treatment of screening by different methods [14] also leads to similar conclusions. Note also a certain analogy of this problem with the problem of mobile charged impurities—e.g., protons—in metals. Despite often controversial claims in the literature (see, e.g., [15,16]), the general conclusion is that screening does not preclude that an electric field acts on protons, leading, e.g., to the electromigration. Moreover, even a proton Hall effect was observed experimentally.

In order to estimate the Hall angle resulting from the force $\mathbf{f}_q$ we include $\mathbf{f}_q$ as an additional *driving* force in the equation of motion of a vortex [9,17], which then reads

$$\Phi_0\mathbf{j} \times \mathbf{n} - q_v\Phi_0\mathbf{v}_L \times \mathbf{n} - \eta\mathbf{v}_L = 0, \tag{8}$$

where $\eta\mathbf{v}_L$ is the friction force. Note that $\mathbf{f}_q$ is exactly of the form usually taken for the Hall term of the equation of motion of a vortex [9,17]. The solution of Eq. (8) yields $\tan\alpha_q = q_v\Phi_0/\eta$ and $\eta \simeq \Phi_0 B_{c2}/\rho_n$, where ρ_n is the normal resistivity. With this and $Q_v \simeq q_v\xi^2$ one finds

$$\tan\alpha_q = \frac{Q_v\rho_n}{\Phi_0} \tag{9}$$

In addition to the contribution of $\mathbf{f}_q$ to the Hall angle there is the usual Hall force acting on one vortex [9,17], which arises from a Hall effect in the normal core [9]. This Hall force is of magnitude $f_h = f_L\tan\alpha_n$, where f_L is the usual Lorentz or Magnus force on a vortex and α_n is the normal state Hall angle. This force yields a Hall angle of the same sign as in the normal state. Its direction for positive charge carriers is therefore parallel to the transport current, i.e., $\mathbf{f}_h||\mathbf{j}$, and it drives vortices in the direction of the electric current [6].

In order to have a sign change of the Hall angle in the mixed state $\mathbf{f}_q$ must be directed antiparallel to $\mathbf{f}_h$ and it must be larger. The first condition is obviously met for holes if the vortex is negatively charged—which according to the arguments presented above should be the typical situation. For $Q_v < 0$, $\mathbf{f}_q$ acts antiparallel to $\mathbf{E}$ and thus antiparallel to $\mathbf{j}$ and $\mathbf{f}_h$. The magnitude of $\tan\alpha_q$ according to Eq. (9) with $Q_v \approx 10^{-13}$ C/m, $\rho_n \approx 10^{-6}\ \Omega$ m, and $\Phi_0 = 2 \times 10^{-15}$ T m^2 is $\tan\alpha_q \approx 10^{-4}$. Typical values of $\tan\alpha_n$ are of the same order at low magnetic fields (less than $\sim$1 T). Thus a sign change of the Hall coefficient in low fields is likely to occur.

The temperature and magnetic field dependence of $\tan\alpha_q$ according to Eq. (9) is captured in that of the electric resistivity. Ignoring the weak field dependence of ρ_n, we conclude that $\tan\alpha_q$ is field independent. Since the normal contribution to the Hall coefficient arising from the usual Hall effect in the vortex cores increases linearly with magnetic field, a crossover to a positive Hall angle at large magnetic fields should occur. In other words, we expect that the Hall angle is given by

$$\tan\alpha = \tan\alpha_n(T,B) + \tan\alpha_q(T),$$

where α_q is independent of the magnetic field and α_n is the normal state Hall angle ($\tan\alpha_n \propto B$). This is in agreement with the experimental data [18]. Regarding the temperature dependence we note due to the decrease of ρ_n with decreasing temperature $\tan\alpha_q$ should decrease with decreasing temperature also, so that the negative contribution to the Hall angle should vanish, in agreement with the experimental data. On the other hand, the Hall angle may remain negative in the dirty limit when ρ_n is large and relatively temperature independent [14].

We conclude that the inhomogeneous charging of the vortex can, in principle, account for the experimentally observed change of sign of the Hall coefficient below T_c. The effect is universal and does not apply only to HTSCs. The numerical estimates presented above show that it is sufficiently large to explain the behavior of the Hall effect in the HTSCs.

After completion of this work we became aware of the work of Feigelman, Geshkenbein, Larkin, and Vinokur [14] in which the authors also notice the possibility to explain the negative Hall anomaly if the electron density at the vortex is different from the bulk. The authors, however, do not discuss the origin of this electron redistribution. The detailed discussion of the Hall effect in their paper is also different from ours: The additional force on a vortex arises from momentum transfer to the lattice and a corresponding backward reaction. As a result, the sign of the extra force is opposite to ours and the authors of Ref. [14] get a negative Hall effect if the density of the carriers at the core is *larger* than far away from it. As we argued above, the typical situation is rather opposite: Simple physical considerations show that the charge of the core is opposite to the charge of the charge carriers. This *decrease* of the carrier density in the core is just the condition for the negative Hall effect in our treatment.

Summarizing, we have presented arguments that in the mixed state of a type II superconductor the vortex cores may acquire an additional charge. This effect is due to the difference of the electronic chemical potential in a superconducting versus normal state, which causes a charge redistribution between the normal vortex cores and the condensate region. The electric potential arising leads to the modification of the density of the material at the core and thus contributes to the pinning. The extra charge leads to an additional force on vortices, which modifies the Hall effect and can explain the change of sign of the Hall coefficient observed in high temperature superconductors below T_c.

We thank B. Büchner, W. Brenig, V. B. Geshkenbein, A. P. Kampf, A. I. Larkin, and D. van der Marel for useful discussions. D. Kh. is especially grateful to M. Mostovoi, M. Simkin, and I. Vekhter for the discussion of the problem of screening. This work was supported by the Deutsche Forschungsgemeinschaft through SFB 341 and by the Foundation for the Fundamental Research of Matter (FOM), The Netherlands.

[1] D. van der Marel, Physica (Amsterdam) **165C**, 35 (1990).
[2] D. I. Khomskii and F. V. Kusmartsev, Phys. Rev. B **46**, 14 245 (1992).
[3] G. Rietveld, N. Y. Chan, and D. van der Marel, Phys. Rev. Lett. **69**, 2578 (1992).
[4] M. Galffy and E. Zirngiebl, Solid State Commun. **68**, 929 (1988).
[5] J. Luo, T. P. Orlando, J. M. Greybeal, X. D. Wu, and R. Muenchhausen, Phys. Rev. Lett. **68**, 690 (1992).
[6] S. J. Hagen, C. J. Lobb, R. L. Greene, M. G. Forrester, and J. H. Kang, Phys. Rev. B **41**, 11 630 (1990).
[7] A. Freimuth, in *Superconductivity,* Frontiers in Solid State Sciences, Vol. 1, edited by L. C. Gupta and M. S. Multani (World Scientific 1993), p. 393.
[8] C. Caroli, P. G. de Genees, and J. Matricon, Phys. Lett. **9**, 307 (1964).
[9] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).
[10] F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950).
[11] A. G. van Vijfeijken and F. A. Staas, Phys. Lett. **12**, 175 (1964).
[12] R. Labush, Phys. Rev. **170**, 470 (1968).
[13] G. Kogan, L. N. Bulaevskii, P. Miranovič, and L. Dobrosavljevič-Grujič (to be published).
[14] M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, in Proceedings of the Grenoble Conference on Materials and Mechanisms of Superconductivity, July 1994; (to be published).
[15] A. K. Das and R. Peierls, J. Phys. C **6**, 2811 (1973).
[16] L. Turban, P. Nozieres, and M. Gerl, J. Phys. **37**, 159 (1976).
[17] P. Nozieres and W. F. Vinen, Philos. Mag. **14**, 667 (1966).
[18] S. J. Hagen, C. J. Lobb, R. L. Greene, and M. Eddy, Phys. Rev. B **43**, 6246 (1991).