Dynamic Response of Isolated Aharonov-Bohm Rings Coupled to an Electromagnetic Resonator

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(Received 8 February 1995)

We have measured the flux dependence of both the real and the imaginary conductance of GaAs/GaAlAs isolated mesoscopic rings at 310 MHz. The rings are coupled to a highly sensitive electromagnetic superconducting microresonator and lead to a perturbation of the resonance frequency and quality factor. This experiment provides a new tool for the investigation of the conductance of mesoscopic systems without the need for invasive probes. The results obtained can be compared with recent theoretical predictions emphasizing the differences between isolated and connected geometries and the relation between ac conductance and persistent currents.

PACS numbers: 73.50.Jt, 05.30.Ch, 73.20.Fz

Mesoscopic metallic rings present a spectacular thermodynamic property: they carry a persistent nondissipative current when they are threaded by a magnetic flux [1–3]. The existence of such a persistent current is a consequence of the coherence of the electronic wave functions around the ring. However, unlike a superconductor, when connected to a voltage source, the same rings present a finite Ohmic conductance close to the classical value given by the Drude formula. In this formula the conductance depends only on the elastic scattering time (quantum interference effects give rise to contributions which are only a small fraction of this main classical contribution in the metallic diffusive regime). It has already been pointed out [4] that the existence of a finite Ohmic resistance for a phase coherent sample is not paradoxical when one properly takes into account the influence of the measuring leads. When connected to the sample, these macroscopic wires play the role of incoherent reservoirs where electron thermalization and dissipation take place. Such a strong coupling with a reservoir of electrons can be avoided by studying the current response of a mesoscopic ring to a time dependent flux, which induces an electric field around the ring. Since the early work of Büttiker and coworkers [5–7] (subsequently generalized [8–12]), it has been shown that the conductance measured by this last technique on an isolated ring is indeed fundamentally different from the conductance of the same sample connected to a voltage source. Essentially, it depends on the inelastic time τ_{in} (which describes the coupling of the electrons to the thermal bath). Furthermore, it is strongly related to the presence of persistent currents around the ring.

In its ac version the experiment consists of measuring the complex magnetic susceptibility of the rings $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ submitted to a small ac flux, which is in turn superimposed on a dc one Φ. In the linear response limit, this susceptibility is related to the complex ac conductance of the rings G by $\chi(\omega) \propto i\omega G(\omega)$. Let us summarize the main theoretical predictions [10–12]. An important time scale for the dynamics of the system is its thermalization time $\gamma^{-1} \approx \tau_{\mathrm{in}}$. In the adiabatic limit $\omega < \gamma$, $\mathrm{Im}(G)$ reduces to the derivative of the persistent current, while a relaxation term occurs at higher frequency. In the continuous spectrum limit $\hbar\gamma \gg \Delta$ (where Δ denotes the mean level spacing) and zero frequency, $\mathrm{Re}(G)$ is given by $\mathrm{Re}(G) = G_0 + \delta G(\Phi)$ where $\delta G \approx G_0\Delta/\hbar\gamma \ll G_0$ is the $\Phi_0/2$ periodic Altshuler-Aronov-Spivak (AAS) weak localization correction, positive in weak field [13,14]. Nevertheless, the same quantity in the discrete spectrum limit $\hbar\gamma \ll \Delta$ (and in the canonical statistical ensemble corresponding to isolated objects) may give oscillations of opposite sign (for $T < \Delta$, $\omega < \gamma$) and amplitude of the order of G_0 (the Boltzmann constant is taken equal to unity everywhere). These oscillations are predicted to reverse sign and become of the order of $(\Delta/\hbar\gamma)G_0$ when the temperature T increases.

Motivated by these findings, we have designed an experiment to measure the complex ac conductance of an array of GaAs/GaAlAs isolated rings. The discrete spectrum is much easier to reach in these samples (where Δ is of the order of a few tens of mK) than in metallic ones of comparable sizes (corresponding to Δ in the microkelvin range). The sample is an array of 10^5 isolated square rings, 2 μm in size, made using e-beam lithography. The electronic parameters of the rings are obtained from transport measurements on connected rings and wires fabricated together with the isolated rings. Moreover, because of depletion effects, the real width of the wires etched in the 2D electron gas is substantially smaller than the nominal one, and must be determined by weak localization measurements [15]. From these measurements we deduced the following parameters: $\Delta = 35$ mK, $E_c = hD/L^2 = 200$ mK, $M = 17$, $l_{\mathrm{tr}} = 3$ μm, $M_{\mathrm{eff}} = 4$, and $L_\phi(T = 50 \text{ mK}) = 7$ μm, where D denotes the diffusion coefficient, L the length of the rings, M their number of channels, M_{eff} their effective one, l_{tr} the transport length, and $L_\phi(T)$ the (temperature dependent) phase coherence length of electrons. The

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electronic motion is then diffusive around the rings and ballistic in the transverse direction. In terms of frequency, the energies are $\Delta/h = 630$ MHz and $E_c/h = 4.2$ GHz.

These values determine the interesting range of frequency: from a few hundred megahertz ($\hbar\omega < \Delta$) to a few gigahertz ($\hbar\omega \approx E_c$). The inelastic parameter γ, of course, cannot be *a priori* deduced from such transport experiments, since it represents a property of the isolated rings. Nevertheless, assuming that γ is of the order of $1/\tau_\phi$ (where τ_ϕ is the phase coherence time of electrons measured by weak localization in wires having the same width as the rings), we expect that it is smaller than $\Delta/\hbar$ below 50 mK (such an assumption is in agreement with the results obtained by Sivan *et al.* [16] on the tunnel spectroscopy measurements of quantum dots).

Our aim was to be able to detect the in-phase and out-of-phase response of this array of rings to a small magnetic excitation. We recall that such an experiment, which deals with electrically isolated objects, is very different from the ac measurement of the complex conductance of connected rings [17] and also from dc conductance measurements in the presence of a high frequency electromagnetic field [18]. Since the estimated amplitude of the signal was extremely small, we had to design a special experimental setup. We have used a resonant technique in which the rings are magnetically coupled to an electromagnetic multimode resonator, whose performance is affected by the perturbations caused by the rings. The resonance frequencies f_n and quality factors Q_n are modified by the presence of the rings according to

$$\frac{\delta f_n}{f_n} = \frac{2\pi N \mathcal{M}^2 f_n \mathrm{Im}[G(f_n)]}{\mathcal{L}},$$
$$\frac{\delta Q_n}{Q_n^2} = -\frac{2\pi N \mathcal{M}^2 f_n \mathrm{Re}[G(f_n)]}{\mathcal{L}}, \tag{1}$$

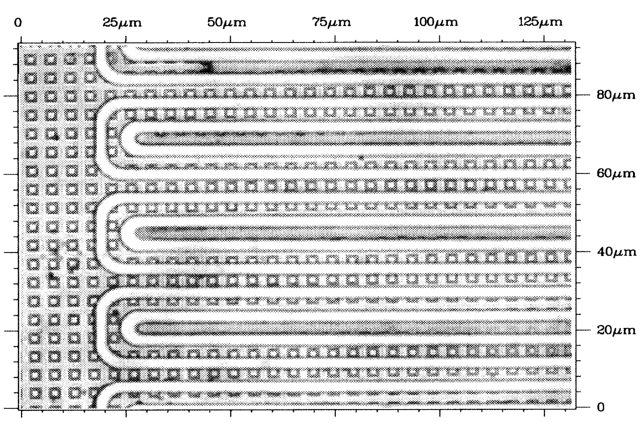


FIG. 1. Optical photography of a piece of the resonator coupled to the GaAs/GaAlAs mesoscopic rings. One notes the two folded Nb lines (1 μm thick, 2 μm wide, and 20 cm long) on the sapphire substrate.

where N is the number of rings and $\mathcal{M}$ is the mutual inductance between one ring and the resonator of self-inductance $\mathcal{L}$. Considerations of coupling optimization between the samples and the detector have led us to use the meander strip line resonator depicted in Fig. 1. The array of rings is placed on the top of this resonator. In this geometry each ring is close to the resonating line, which ensures a good mutual coupling between them. The line, open at both ends, has resonances each time its length is a multiple value of $\lambda_0/2$, where λ_0 is the electromagnetic wavelength. The resonator is coupled to an rf synthesizer via capacitances made by lithography on the same substrate as the meander.

Typical superconducting Nb resonators produced on sapphire substrates have a fundamental resonant frequency of 380 Mhz and a $Q = 80\,000$ at temperatures below 1 K. The injected power (1 nW) is small enough to avoid the heating of electrons, and corresponds to an ac magnetic flux less than $0.1\Phi_0$ through the rings. The sensitivity of our experiment is determined by the precision with which we can detect a small deviation of f_n and Q_n, actually [15] $\delta f_n/f_n \approx 10^{-9}$ and $\delta Q_n/Q_n^2 \approx 10^{-10}$.

Ideally all the rings should be exposed to the same ac magnetic field and therefore should have a very well-controlled position tightly coupled to the resonator, but for the moment this is very difficult to achieve, since for reasons of lithography the line and the rings are sitting on different substrates. However, as long as we are concerned only with the linear response, it is not necessary that all the rings experience the same ac field (if it is small enough). The problem of the homogeneity of the dc magnetic field is somewhat more serious. Since the characteristic signature of the effects we are looking for are periodic oscillations with the dc flux through the rings, it is crucial that they see essentially the same dc field. Because of the Meissner effect, the dc field just above the resonator is strongly spatially inhomogeneous. However, these field inhomogeneities decrease exponentially with the distance between the rings and line substrates and are reduced to 10% when a 1.5 μm thick, Mylar film is inserted between the detector and the ring substrate. We have estimated that in this geometry the typical mutual inductance $\mathcal{M}$ between one ring and the resonator is of the order of 1.5×10^{-13} H. We have checked this value by measuring the susceptibility of an array of superconducting aluminum rings. The most serious difficulty we had to overcome in order to realize this experiment was the existence of spurious losses coming from the partially etched GaAlAs top layer of the heterostructure. The first attempt was made with very slightly etched samples, for which we observed a drop of the quality factor of the resonator from 80 000 to 10. By etching the samples more deeply we could decrease these losses by a factor of 100 and we obtained the results depicted below where $Q = 1650$ for the fundamental frequency. As yet it has not been possible to work on the higher harmonics . We hope to reduce further these residual losses in future experiments.

Let us now describe the magnetic field and temperature dependence of the complex susceptibility of the array of GaAs/GaAlAs rings measured at 310 MHz. The measurements were done using two different resonators and were reproducible. The amplitudes of the measured signals δQ and δf are independent of the injected power below 1 nW but decrease at higher excitation. Further investigation of these nonlinearities is necessary to separate heating effects from possibly more interesting ones. The dc magnetic field was modulated at 3 Hz with an amplitude of 1 G. The resulting signals are proportional to the derivatives of f_1 and Q_1 with respect to the dc magnetic field. In Fig. 2 we show the field dependence of $-\partial f_1/\partial H$ averaged 40 times. In low field one clearly notices the oscillations associated with the rings superimposed on the linear dependence of the Nb diamagnetism. The 5 G periodicity corresponds to a flux of amplitude $\Phi_0/2 = h/2e$ in each square. The oscillations are not visible at fields larger than 10 G, which is due to the rather small aspect ratio of the rings ($1\Phi_0$ through the area of the wires corresponds to 15 G). One deduces from Fig. 2

$$-\partial f_1/\partial H = \alpha H + \beta_1 \sin 4\pi\Phi/\Phi_0 + \beta_2 \sin 8\pi\Phi/\Phi_0 , \tag{2}$$

with $\alpha = 13$ Hz/G^2, $\beta_1 = 27 \pm 2$ Hz/G, and $\beta_2 = 10 \pm 2$ Hz/G. According to Eq. (1) this amplitude of the oscillations corresponds to an imaginary conductance of the order of $2.5 \times 10^{-3}\ \Omega^{-1}$ per ring for which the estimated Drude conductance is $G_0 = 5 \times 10^{-4}\ \Omega^{-1}$. This is clearly shown in Fig. 3. The temperature dependence of the $h/2e$ periodic component of the signal (see Fig. 4) is compatible with an exponential decay with a characteristic energy of 200 mK over a range of temperature corresponding to $1.5\Delta - 2E_c$.

Since the frequency is smaller than the level spacing, we can assume that the variations of f_1 reflect only the dc orbital magnetism of the rings: $-\partial f_1/\partial H \propto \partial^2 I_{\mathrm{per}}/\partial \Phi^2$. Thus the amplitude of the oscillations corresponds to a value of 1.5 nA per ring, which is of the order of $2E_c/\Phi_0 = 1.4$ nA (the factor 2 standing for spin), as compared to $2\sqrt{E_c\Delta}/\Phi_0 = 0.5$ nA or $2\Delta/\Phi_0 = 0.2$ nA. However, under the same assumption, our result implies a diamagnetic zero field persistent current. One possible explanation of this diamagnetism could be the presence of interactions that modify the orbital magnetism of the rings [19]. Although the existence of an attractive interaction between electrons in the GaAs/GaAlAs heterojunction, necessary to explain our experimental results, is not very likely, it cannot be completely ruled out. Furthermore, we cannot rule out that we are in the regime $\gamma < \omega$, implying a possible contribution from relaxation processes to Im(G). However, according to Ref. 10, this last hypothesis does not explain the observed sign either.

The magnetic field oscillations of the dissipative conductance, obtained by integration of $\partial Q_1/\partial H$, are presented in Fig. 3. Their period is also $h/2e$, and their amplitude, which is an order of magnitude smaller than the oscillations of Im(G), corresponds to $0.2G_0$, i.e., 10 times greater than the AAS $h/2e$ periodic weak localization magnetoconductance, as measured in identi-

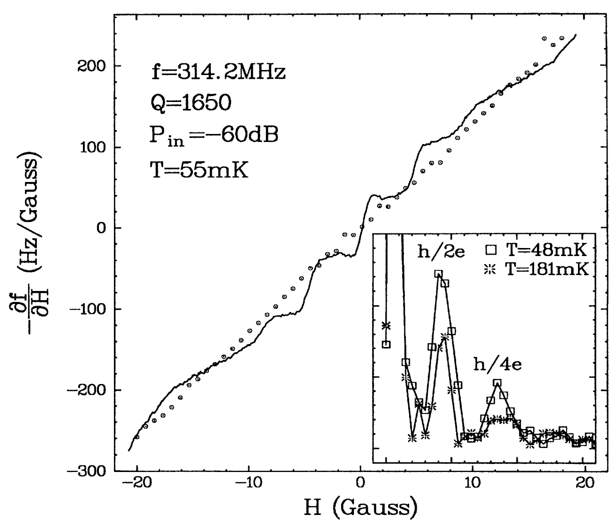


FIG. 2. Evolution of the derivative of the fundamental resonance frequency f_1 of the line as a function of the dc magnetic field. The linear background observed in the absence of the rings (open symbols) corresponds to the diamagnetism of the Nb, on which are superimposed the $h/2e$ oscillations due to the mesoscopic rings. This curve is averaged 40 times. Inset: Fourier transform of the signal in the presence of the rings for two different temperatures.

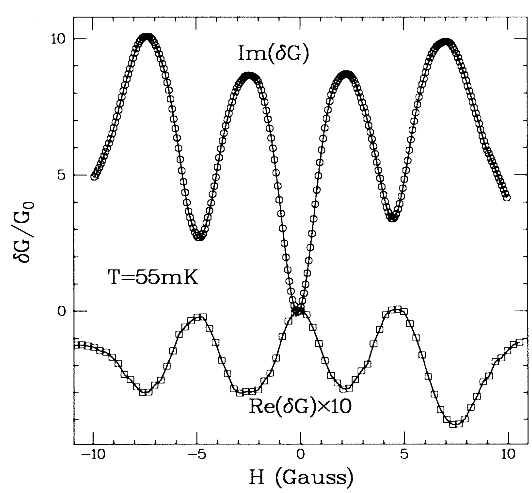


FIG. 3. Magnetic field dependence of the imaginary and real components of the conductance of the rings expressed in units of the Drude conductance G_0. These two curves have been obtained by integrating the measured quantities $\partial f_1/\partial H$ and $\partial Q_1/\partial H$ after subtraction of the background.

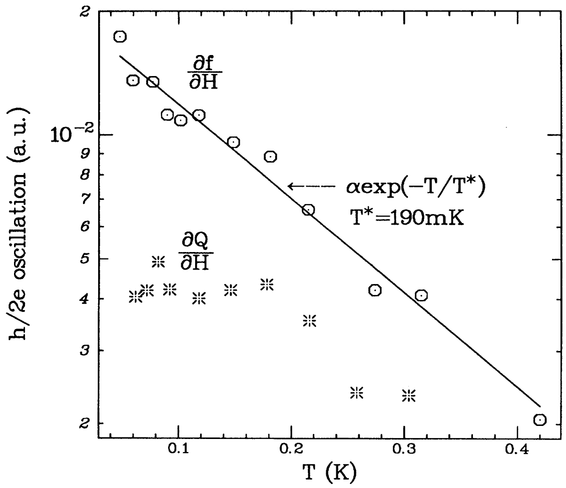


FIG. 4. Temperature dependence of the $h/2e$ periodic components of $\partial f_1/\partial H$ and $\partial Q_1/\partial H$.

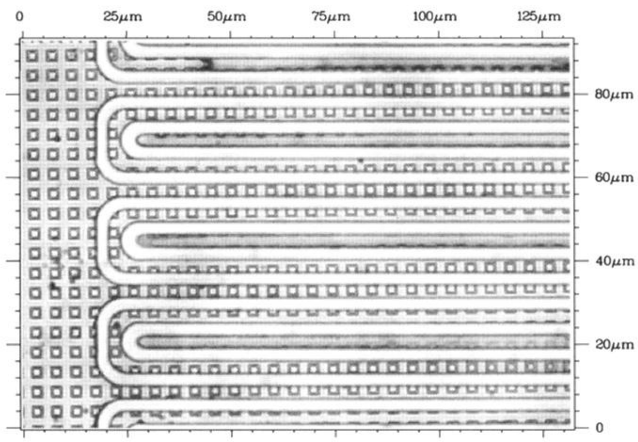
cal connected rings. This is the first indication that the conductance measured in isolated rings is different from that measured in connected ones. The temperature dependence of the $h/2e$ periodic component of $\partial Q_1/\partial H$ depicted in Fig. 4 is notably different from the temperature dependence of $\partial f_1/\partial H$; it is nearly independent of temperature until $T = 200$ mK and strongly decreases at higher temperature. The sign of $\partial Q/\partial H$ is positive in small field which according to Eq. (1) corresponds to a negative magnetoconductance in low field. It is thus opposite to the AAS result. The observation of a negative sign for the low field magnetoconductance at $T > \Delta$ could be another indication that $\gamma < \omega$ [10]. In this limit, interlevel absorption processes are the dominant contribution to $\mathrm{Re}(G)$. The magnetoconductance is then related to the flux dependent spectral properties of the rings and, according to random matrix theory, has negative sign in low field. These results invite us to pursue the experiment in a wider range of frequency and temperature.

Up to now we have only discussed the magnetic coupling of the rings to the detector. However, we also expect the resonator to be affected by the polarizability of the rings, which experience a strong electric field in their plane. This field is screened by the electrons on a scale of the order of the Thomas-Fermi length. However, since it is not infinitely small (of the order of 200 Å) compared to the dimensions of the ring, one cannot exclude the existence of quantum corrections to the polarizability of the rings contributing to the flux dependence of f_1 with Aharonov-Bohm-like oscillations. A quantitative estimation of this effect is clearly needed and is underway.

In conclusion, we have designed an experiment sensitive enough to measure the complex conductance of an array of rings in the relevant frequency range. We have given evidence for $h/2e$ flux oscillations for both the real and imaginary parts of the conductance. The sign of the oscillations of the imaginary conductance corresponds to diamagnetism in low field, which is for the moment difficult to interpret. The periodic component of the real part of the conductance has an opposite sign to the weak localization oscillations measured in connected rings, and is at least 10 times larger.

This work has strongly benefited from the help and suggestions of L. P. Lévy, M. Nardonne, P. Pari, and C. Urbina. This work was partly supported by DRET Grant No. 92/181.

[1] L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[2] V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).
[3] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).
[4] R. Landauer, Phys. Scr. **42**, 110 (1992).
[5] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[6] R. Landauer and M. Büttiker, Phys Rev. Lett. **54**, 2049 (1985).
[7] M. Büttiker, Ann. N.Y. Acad. Sci. **480**, 194 (1986).
[8] Y. Imry and N. S. Shiren, Phys. Rev. B **33**, 7992 (1986).
[9] N. Trivedi and D. A. Browne, Phys. Rev. B **38**, 9581 (1988).
[10] B. Reulet and H. Bouchiat, Phys. Rev. B **50**, 2259 (1994).
[11] A. Kamenev, B. Reulet, H. Bouchiat, and Y. Gefen, Europhys. Lett. **28**, 391 (1994).
[12] A. Kamenev and Y. Gefen (unpublished).
[13] B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].
[14] D. Y. Sharvin and Y. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 285 (1981) [JETP Lett. **34**, 272 (1981)].
[15] B. Reulet, H. Bouchiat, and D. Mailly (unpublished).
[16] U. Sivan, F. P. Milliken, K. Milkove, S. Rishton, Y. Lee, J. M. Hong, V. Boegli, D. Kern, and M. deFranza, Europhys. Lett. **65**, 829 (1994).
[17] J. B. Pieper and J. C. Price, Phys. Rev. Lett. **72**, 3586 (1994).
[18] S. A. Vitkalov, G. M. Gusev, Z. D. Kvon, G. I. Leviev, and V. I. Fal'ko, Pis'ma Zh. Eksp. Teor. Fiz. **94**, 376 (1988) [JETP **67**, 1080 (1988)]; J. Liu and N. Giordano, Phys. Rev. B **41**, 9728 (1990).
[19] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).



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