Broken Symmetry of Two-Component $\nu = 1/2$ Quantum Hall States

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(Received 1 March 1995)

We show that the $\nu = 1/2$ quantum Hall states in bilayer systems are triplet p-wave pairing states of composite fermions exactly the same as ^{3}He superfluids. The observed persistence (though weakening) of the $1/2$ state in the two- to one-component crossover region is identical to the well known A to A_1 transition in ^{3}He. This illustrates the remarkable phenomenon of "incompressible deformation" of quantum Hall states. The broken symmetry of the $\nu = 1/2$ state is a "pairing" vector $\mathbf{d}$, which implies a "pseudo" magnetization $\propto i\mathbf{d} \times \mathbf{d}^*$. In the presence of layer tunneling, the (331) state ($\mathbf{d}$ real) is unstable against other states with "magnetization" ($\mathbf{d}$ complex).

PACS numbers: 75.10.–b, 64.60.Cn, 73.20.Dx

The robust $\nu = 1/2$ quantum Hall state recently discovered in bilayer quantum well systems [1] and the $\nu = 5/2$ state discovered [2] some years ago in single-layer systems have illustrated the dramatic effect of internal degrees of freedom on the quantum Hall phenomenon, namely, the formation of multicomponent quantum Hall states. The $5/2$ state was discovered in single-layer systems with weak Zeeman energy, where both up and down spins play equal role in the makeup of the $5/2$ state. Bilayer systems are in effect pseudospin $1/2$ systems, even though the electron spins are frozen by the magnetic field. The two pseudospin states are the symmetric and antisymmetric states of the quantum well. Like the single-layer $5/2$ state, the bilayer $1/2$ state was discovered in the "two-component" regime [1] where the pseudospins have a weak "Zeeman" energy. Although our discussions below apply to both types of systems, for length reasons, we shall focus on bilayer systems from now on, and refer to pseudospins simply as "spins."

In the absence of disorder, bilayer systems have three energy scales: (a) the energy difference Δ between the symmetric and antisymmetric states (which is the "tunneling energy" between the layers); (b) intralayer Coulomb interaction e^2/ℓ, where ℓ is the magnetic length; and (c) interlayer interaction e^2/D, where D is the separation between the layers. These scales in turn lead to three distinct physical regimes: $\{1 + 1\}$, two separate one-component regime, $e^2/\ell \gg e^2/D, \Delta$; $\{2\}$, two-component regime, $e^2/\ell \sim e^2/D \gg \Delta$; $\{1\}$, one single-component regime, $e^2/\ell, e^2/D < \Delta$. Regime $\{1 + 1\}$ is realized for large D, where the system separates into two weakly coupled single-layer systems. Regime $\{2\}$ is the case where intralayer and interlayer interactions are comparable and dominate over the tunneling energy so that the two spin populations are nearly degenerate. Regime $\{3\}$ is the strong tunneling limit where all electrons lie in the symmetric state.

As mentioned before, the $1/2$ bilayer state [1] was first discovered in regime $\{2\}$. However, recent experiments [3] show that even though the $\nu = 1/2$ state is weakened and eventually disappears as the system is tuned from $\{2\}$ to $\{1\}$, it remains a good quantum Hall state (i.e., incompressible) for a sizable range of parameters $\Delta/(e^2/\ell)$. Numerical studies [4] indicate that the observed $\nu = 1/2$ state is likely to be the so-called (331) state [5]. There are also suggestions [3,6] that the observed persistence of the $\nu = 1/2$ state in the $\{2\}$ to $\{1\}$ crossover region correspond to the evolution of the (331) state into the so-called Pfaffian state [7]. No further analysis, however, has been made with this suggestion. Moreover, this picture seems to run counter of the conventional view that the (331) state cannot evolve continuously into the Pfaffian state as they have different "topological order" [8].

The purpose of this paper is to point out a fundamental broken symmetry of the bilayer $\nu = 1/2$ states and the important properties associated with it. This symmetry is not obvious in the conventional (331) and Pfaffian representation because they make reference to a specific spin quantization axis. To reveal this symmetry, it is necessary to use a general representation independent of spin quantization axes. The symmetry revealed leads to the following results.

(I) Both (331) and Pfaffian states are triplet p-wave BCS states of composite fermions [9,10] with wave functions identical to that of the A and the A_1 phase of superfluid ^{3}He [11]. The (331) to Pfaffian transformation corresponds to the well known A to A_1 transition in ^{3}He. The broken symmetry of the $\nu = 1/2$ states is a complex vector $\mathbf{d}$ (referred to as "pairing vector") denoting the direction of zero spin projection of the triplet pair, i.e., $\mathbf{d} \cdot \langle \mathbf{S} \rangle = 0$. The $\mathbf{d}$ vector of the (331) and the Pfaffian state is $\hat{\mathbf{x}}_3$ and $(\hat{\mathbf{x}}_3 - i\hat{\mathbf{x}}_2)/\sqrt{2}$, respectively.

(II) Contrary to the current topological order arguments [8], which implies that (331) cannot be deformed continuously into the Pfaffian because they have different degeneracies on a torus, we show (from the explicit solution of a model Hamiltonian) that *continuous* transition between them can indeed occur. In other words, there are neither topological nor intrinsic energetic obstructions to prevent continuous transformation between any two triplet pairing states [12].

(III) In the presence of tunneling (however weak), the (331) state ($\mathbf{d}$ real) is unstable with respect to the

 0031-9007/95/75(6)/1186(4)$06.00

appearance of an imaginary component in $\mathbf{d}$. This is because (as we shall see) layer tunneling acts as a "magnetic" field in the pseudospin space, and that a complex $\mathbf{d}$ will generate a magnetization $\mathbf{m} \propto i\mathbf{d} \times \mathbf{d}^*$. A complex $\mathbf{d}$ is therefore favored. As the (331) evolves towards the Pfaffian state, $|\mathbf{m}|$ grows from zero to a saturating value equal to half the full density ($\overline{\rho}/2 = \nu/4\pi = 1/8\pi$).

To begin, let us recall the Hamiltonian [13] of a bilayer system of $2N$ electrons, $\hat{H} = \hat{V} + \hat{H}_T$,

$$\hat{V} = \sum_{i>j=1}^{2N} \left[V_0(\mathbf{R}_i - \mathbf{R}_j) + \sum_{\alpha=1}^{3} V_\alpha(\mathbf{R}_i - \mathbf{R}_j) S_\alpha(i) S_\alpha(j) \right],$$
$$\hat{H}_T = -\Delta \sum_{i=1}^{2N} S_1(i), \tag{1}$$

where $\mathbf{R}_i$ is the guiding center coordinate of the ith electron. We have chosen a coordinate system $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3$ in spin space such that the symmetric and antisymmetric states are represented by the spinors $\binom{1}{1}$ and $\binom{1}{-1}$ respectively and that the layer tunneling Δ acts like a magnetic field along $\hat{\mathbf{x}}_1$. The states localized in the upper and lower layers will then correspond to $\binom{1}{0}$ and $\binom{0}{1}$, and $S_3^{\text{total}} \equiv \sum_i S_3(i) = (N_\uparrow - N_\downarrow)/2$ is the difference in electron number between the two layers. For typical parameters of the quantum wells (layer separation D, well depth, etc.), $V_3 \gg |V_1|, |V_2|$. The Coulomb interaction V_3 favors equal electron densities on both layers (i.e., $\langle S_3^{\text{total}} \rangle = 0$), while Δ favors a maximum $\langle S_1^{\text{total}} \rangle$.

In regime {2}, numerical studies [4] indicate that the $1/2$ state is the (331) state,

$$|\Psi_{331}\rangle = \int \prod_{i>j=1}^{N} (a_i - a_j)^3 (b_i - b_j)^3 \times \prod_{i,j=1}^{N} (a_i - b_j)^1 \prod_{i=1}^{N} \psi_\uparrow^+(a_i) \psi_\downarrow^+(b_i) |0\rangle, \tag{2}$$

where $\int \equiv \int \prod_{i=1}^{N} (d^2a_i d^2b_i) \exp[-\frac{1}{4} \sum_{i=1}^{N} (|a_i|^2 + |b_i|^2)]$. In regime {1}, the $1/2$ state is expected to be close to the Pfaffian form, which is a spin polarized state

$$|\Psi_{\text{Pf}}\rangle = \int \Phi^2 \, \text{Pf}\left(\frac{1}{z_i - z_j} \right) \prod_{i=1}^{2N} \psi^+(z_i) |0\rangle, \tag{3}$$

where $\psi^+(z) = [\psi_\uparrow^+(z) + \psi_\downarrow^+(z)]/\sqrt{2}$ represents the symmetric state of the quantum well, and $\Phi \equiv \prod_{i>j=1}^{2N} (z_i - z_j)$ is the Laughlin factor for all electrons. The Pfaffian of a matrix $M_{i,j}$ is defined as $\text{Pf}(M_{i,j}) = \sum_P \prod_{k=1}^{N} (-1)^P M_{P(2k-1),P(2k)}$, where P is a permutation of $2N$ objects.

We now show that both $|\Psi_{331}\rangle$ and $|\Psi_{\text{Pf}}\rangle$ belong to the same family of triplet pairing states, $\{|N, \mathbf{d}\rangle \equiv |N, \chi\rangle\}$,

$$|N, \chi\rangle = \int \Phi^2 \prod_{i=1}^{N} \left[\frac{\chi_{\mu\nu} \psi_\mu^+(z_{2i-1}) \psi_\nu^+(z_{2i})}{z_{2i-1} - z_{2i}} \right] |0\rangle, \tag{4}$$

where χ is a symmetric 2×2 matrix (which can be normalized as $\text{Tr}\chi^+\chi = 2$ without loss of generality). Since the general representation of a 2×2 symmetric matrix is $\chi_{\mu\nu} = i\mathbf{d} \cdot [\vec{\sigma}\sigma_2]_{\mu\nu}$, where $\mathbf{d}$ is a complex vector, the triplet space is simply the space of the pairing vector $\{\mathbf{d}\}$ ($|\mathbf{d}|^2 = 1$). From the property $\langle \chi | \mathbf{S} | \chi \rangle / \langle \chi | \chi \rangle = \frac{1}{2} \text{Tr}[\chi^+ (\mathbf{S}(1) + \mathbf{S}(2)) \chi] = 2i\mathbf{d} \times \mathbf{d}^*$, one can see that $\mathbf{d}$ represents the direction of zero spin projection.

To see the relation between $|\Psi_{\text{Pf}}\rangle$ and Eq. (4), we note that each term in the Pfaffian contributes identically in $|\Psi_{\text{Pf}}\rangle$. Hence,

$$|\Psi_{\text{Pf}}\rangle = (2N!) \int \Phi^2 M_{12} M_{34} \cdots M_{2N-1,2N} [\psi^+(z_1)\psi^+(z_2)] \times [\psi^+(z_3)\psi^+(z_4)] \cdots [\psi^+(z_{2N-1})\psi^+(z_{2N})] |0\rangle,$$

where $M_{ij} = (z_i - z_j)^{-1}$. Apart from a normalization constant, this is precisely Eq. (4) with $\chi = 2^{-1/2} \binom{1\,1}{1\,1}$, or $\mathbf{d} = (\hat{\mathbf{x}}_3 - i\hat{\mathbf{x}}_2)/\sqrt{2}$. To see the relation between $|\Psi_{331}\rangle$ and (4), we make use of the Cauchy identity [14]

$$\prod_{i>j=1}^{N} [(a_i - a_j)(b_i - b_j)] = \prod_{i,j=1}^{N} (a_i - b_j) \times \text{Det}|(a_i - b_j)^{-1}|.$$

The integrand of $|\Psi_{331}\rangle$ can then be written as $\Phi^2 \, \text{Det}|M_{ij}|$. Proceeding similar to the Pfaffian case, we can write

$$|\Psi_{331}\rangle = (N!) \int \Phi^2 M_{12} M_{32} \cdots M_{2N-1,2N} [\psi_\uparrow^+(1)\psi_\downarrow^+(2)] \times [\psi_\uparrow^+(3)\psi_\downarrow^+(4)] \cdots [\psi_\uparrow^+(2N-1)\psi_\downarrow^+(2N)] |0\rangle,$$

where $\psi_\mu^+(i) = \psi_\mu^+(z_i)$. Because M_{ij} is antisymmetric, the product $\psi_\uparrow^+(1)\psi_\downarrow^+(2)$ in the integral can be replaced by the triplet state $\frac{1}{2}[\psi_\uparrow^+(1)\psi_\downarrow^+(2) + \psi_\downarrow^+(1)\psi_\uparrow^+(2)]$, which takes $|\Psi_{331}\rangle$ into the pairing form Eq. (4), with $\chi = \binom{0\,1}{1\,0}$, i.e., $\mathbf{d} = \hat{\mathbf{x}}_3$. Thus, we have $|\Psi_{331}\rangle = |N, \mathbf{d} = \hat{\mathbf{x}}_3\rangle$, $|\Psi_{\text{Pf}}\rangle = |N, \mathbf{d} = (\hat{\mathbf{x}}_3 - i\hat{\mathbf{x}}_2)/\sqrt{2}\rangle$.

The triplet pairing family [Eq. (4)] can be written in a more concise form in terms of the composite fermion operator $\phi_\mu^+(z) = \psi_\mu^+(z) U(z)^2$, where $U(z)$ is the quasihole operator [15] defined as (in first quantized form)

$$U(a)\Psi(\mathbf{r}_1\mu_1, \ldots, \mathbf{r}_{2N}\mu_{2N}) = \prod_{i=1}^{2N} (a - z_i) \times \Psi(\mathbf{r}_1\mu_1, \ldots, \mathbf{r}_{2N}\mu_{2N}),$$

is the wave function of the system, and $\{\mathbf{r}_i\mu_i\}$ denote the position and the spin of the ith electron. In terms of ϕ_μ^+, Eq. (4) reduces to

$$|N, \chi\rangle = (Q^+)^N |0\rangle,$$
$$Q^+ = \int \chi_{\mu\nu}(z, z') \phi_\mu^+(z) \phi_\nu^+(z'), \tag{5}$$

where $\chi_{\mu\nu}(z, z') = (z - z')^{-1} \chi_{\mu\nu}$, and $\int \equiv \int d^2z \, d^2z' \, e^{-(|z|^2 + |z'|^2)/4}$. In this form, it is clear that $|N, \chi\rangle$ is a triplet p-wave BCS state of composite fermions. If the composite fermions in Q^+ were ordinary fermions, $|\Psi_{331}\rangle$ and $|\Psi_{\text{Pf}}\rangle$ reduce to the ground states of

^{3}He-A and ^{3}He-A_1 in a thin slab. The slab geometry fixes the orbital angular momentum axis of the Cooper pairs of ^{3}He-A and ^{3}He-A_1 so that their orbital wave functions are given by $x - iy$. [The slab geometry has no effect on the pairing vector, which is $\hat{\mathbf{x}}_3$ for ^{3}He-A and $(\hat{\mathbf{x}}_3 - i\hat{\mathbf{x}}_2)/\sqrt{2}$ for ^{3}He-A_1 [12].]

Note that the entire triplet pairing family Eq. (4) has filling factor $\nu = 1/2$, because of the factor Φ^2. Since the triplet pairing space $\{\mathbf{d}\}$ is simply connected, any two states in this space can be deformed *continuously* into one another in an *incompressible* manner unless prevented by energetic reasons. In other words, in the absence of energy obstructions, quantum phase transitions between two triplet pairing states are either nonexistent or at most of second (but not first) order. From the expressions of $|\Psi_{331}\rangle$ and $|\Psi_{\rm Pf}\rangle$, one can see that the relative angular momentum L between two electrons of like spins is at least three in the (331) state, whereas it starts with $L = 1$ between in the Pfaffian state. The incompressible deformation between these two states illustrates clearly the fact that *electrons can release and capture relative angular momentum in a continuous and incompressible manner through spin rotations.*

An intrinsic "energy obstruction," however, is nonexistent. To show this, we construct below a *continuous* family of model Hamiltonians $\{\tilde{H}(\mathbf{d})\}$ with $\{|N, \mathbf{d}\rangle\}$ as their ground states. (One has in mind that the effect of changing the external parameters is to change the vector $\mathbf{d}$ in the Hamiltonian.) The continuity of $\tilde{H}(\mathbf{d})$ then implies that the system can evolve from one $\mathbf{d}$ state to another continuously without changing its filling factor while remaining in the ground state during the entire process.

To motivate this model Hamiltonian, it is useful to look at the general structure of the pair potential $\hat{V}(ij)$. Decomposing it into relative angular momentum channel (L) and diagonalizing it in spin space within each L channel, $\hat{V}(ij)$ can be expressed in the diagonal form

$$\hat{V}(ij) = \sum_{L=0,2,4,\ldots} P_L(ij) V_L^s |S=0\rangle\langle S=0| + \sum_{L=1,3,5,\ldots} P_L(ij) \sum_{\alpha=1}^{3} V_{L,\alpha}^t |\mathbf{d}_{L,\alpha}\rangle\langle \mathbf{d}_{L,\alpha}|, \quad (6)$$

where $P_L(ij)$ is the projection operator for relative angular momentum L, and $\{|\mathbf{d}_{L,\alpha}\rangle, \alpha = 1,2,3\}$ are the three triplet eigenstates in channel L. (The spin wave function of the singlet is $(\sigma_2)_{\mu\nu} = \langle \mu\nu \,|\, S=0\rangle$. The wave functions of the triplet are $(\chi_L^{(\alpha)})_{\mu\nu} = \langle \mu\nu \,|\, \mathbf{d}_{L,\alpha}\rangle = i\mathbf{d}_{L,\alpha} \cdot [\vec{\sigma}\sigma_2]_{\mu\nu}$. Orthogonality between different triplet states implies $\mathbf{d}_{L,\alpha} \cdot \mathbf{d}_{L,\beta}^* = \delta_{\alpha\beta}$.) $V_L^s, V_{L,\alpha}^t$ are the pseudopotentials for the singlet and the triplet in channel L. Like the triplet states $\{|\mathbf{d}_{L,\alpha}\rangle\}$, they depend on the external parameters.

Consider now a model potential $\hat{W}(ij)$ which consists only of s and p channels. All pseudopotentials are positive definite except for V_1^t, which is zero. Our model Hamiltonian is then

$$\tilde{H}(\mathbf{d}_1) = \sum_{i>j} \hat{W}(ij),$$

$$\hat{W}(ij) = V^s P_0(ij) |S=0\rangle\langle S=0| + \sum_{\alpha=2,3} V_\alpha^t P_1(ij) |\mathbf{d}_\alpha\rangle\langle \mathbf{d}_\alpha|,$$

where $V^s, V_2^t, V_3^t > 0$. We now show that $|N, \mathbf{d}_1\rangle$ is the ground state of $H(\mathbf{d}_1)$. Since $\tilde{H}(\mathbf{d}_1)$ is positive, it is sufficient to show that $|N, \mathbf{d}_1\rangle$ is annihilated by the interaction of any particular pair, say, $\hat{W}(12)$. Defining $\chi^{(\alpha)} = \langle \mu\nu \,|\, \mathbf{d}_\alpha\rangle$, the wave function of $|N, \mathbf{d}_1\rangle$ is

$$\Psi\binom{z_1,\ldots,z_{2N}}{\mu_1,\ldots,\mu_{2N}} = \Phi^2[\chi^{(1)}(12)\chi^{(1)}(34)\cdots\chi^{(1)}((2N-1)(2N)) - \chi^{(1)}(13)\chi^{(1)}(24)\cdots \times \chi^{(1)}((2N-1)(2N)) + \text{etc.}], \quad (7)$$

where $\chi^{(1)}(ij) = (z_i - z_j)^{-1}\chi^{(1)}_{\mu_i\mu_j}$. There are two kinds of terms in Eq. (7); those that contain $\chi^{(1)}(12)$ and those that do not. The former will be annihilated by $\hat{W}(12)$ as it is orthogonal to $\hat{W}(12)$ in spin space. The latter will be annihilated by the angular momentum projections in $\hat{W}(12)$ as they contain factors like $(z_1 - z_2)^p(z_1 + z_2)^n$ with $p \geq 2$. We have thus shown that $|N, \mathbf{d}_1\rangle$ is the ground state of $\tilde{H}(\mathbf{d}_1)$, and have by now established the I and II mentioned in the opening.

For bilayer system, the actual path of evolution from (331) to Pfaffian depends on how the external parameters are varied. To determine the general feature of this path, as well as to understand other properties of the triplet pairing states, we note the important property that $\mathbf{d}$ gives rise to a magnetization

$$\mathbf{m} = \frac{1}{A}\frac{\langle N, \mathbf{d}|\mathbf{S}^{\rm total}|N, \mathbf{d}\rangle}{\langle N, \mathbf{d}\,|\,N, \mathbf{d}\rangle} = \lambda(i\mathbf{d} \times \mathbf{d}^*), \quad (8)$$

$\mathbf{S}^{\rm total} = \sum_{i=1}^{2N} \mathbf{S}(i)$, where A is the surface area, λ is a function of $|\mathbf{d}^2|^2$ and is of the order of $\overline{\rho}/2$, and $\overline{\rho} = \nu/2\pi = 1/4\pi$ is the total electron density of the two layers. (This result of $\mathbf{m}$ can be derived in a straightforward manner using Eq. (4), noting that the numerator of $\mathbf{m}$ is of the form $\mathrm{Tr}[\vec{\sigma}(\chi\chi^+)^n]\mathrm{Tr}[(\chi\chi^+)^m]$.) Analytic evaluation of $\lambda(\mathbf{d}^2)$ in the thermodynamic limit turns out to be difficult. However, since the Pfaffian state (which has $\mathbf{d}^2 = 0$) is completely spin polarized, λ must satisfy $\lambda(0) = \overline{\rho}/2$. Using symbolic computation, we have found that for a four electron system, $\lambda = (\overline{\rho}/2)(1 + \frac{49}{260}|\mathbf{d}^2|^2)^{-1}$.

In the same way of deriving $\mathbf{m}$, one can show that the energy (per unit area) of the system is $E(\mathbf{d}) = \langle N, \mathbf{d}|\hat{H}|N, \mathbf{d}\rangle\langle N, \mathbf{d}\,|\,N, \mathbf{d}\rangle$ is

$$E(\mathbf{d}) = f - g_0|\mathbf{d} \cdot \hat{\mathbf{x}}_3|^2 - g_1(\mathbf{d}^2(\mathbf{d}^* \cdot \hat{\mathbf{x}}_3)^2 + \text{c.c.}) + g_2|\mathbf{d} \times \mathbf{d}^* \cdot \hat{\mathbf{x}}_3|^2 - \Delta\lambda i\mathbf{d} \times \mathbf{d}^* \cdot \hat{\mathbf{x}}_1, \quad (9)$$

where f, g_0, g_1, g_2, and λ are functions of $|\mathbf{d}^2|^2$. The f and $\{g_i\}$ terms are contributions from V_0 and V_3 in $\hat{V}$, respectively. Note that g_2 must be positive, otherwise the two layers will have different electron densities in

equilibrium, i.e., $\mathbf{m} \cdot \hat{\mathbf{x}}_3 \neq 0$. The fact that the system is in a (331) state in the absence of layer tunneling [4] also means that the functional forms of f and g's are such that $E(\mathbf{d})$ has a minimum at $\mathbf{d} = \hat{\mathbf{x}}_3$.

From the expression of $E(\mathbf{d})$, it is easily seen that the (331) state is unstable against the appearance of a small imaginary component in $\mathbf{d}$, i.e., $\mathbf{d} \rightarrow \hat{\mathbf{x}}_3 - i\epsilon\hat{\mathbf{x}}_2$, $\epsilon \ll 1$. This is because the tunneling energy gain [the Δ term in $E(\mathbf{d})$] by the resulting magnetization is first order in ϵ, whereas the cost in Coulomb energy [the f and g terms in $E(\mathbf{d})$] is quadratic in ϵ. Thus, in the presence of tunneling, bilayer 1/2 states will not be exactly of the (331) form. Moreover, the evolution of the (331) to the Pfaffian state (as the system is tuned from regime {2} to regime {1}) must lie within the family $\mathbf{d}(s) = \hat{\mathbf{x}}_3 \cos(\pi s/4) - i\hat{\mathbf{n}}(s)\sin(\pi s/4)$, $0 \leq s \leq 1$, where $\hat{\mathbf{n}}(s)$ is a real unit vector in the x_2-x_3 plane such that $\hat{\mathbf{n}}(1) = \hat{\mathbf{x}}_2$. The magnetization of this family is along $\hat{\mathbf{x}}_1$, meaning that the electron densities in both layers remain identical during this process.

A natural question is what the typical magnitude of $\mathbf{m}$ in the one- to two-component crossover region. Although a precise answer cannot be given because the functions f, g_i, and λ in $E(\mathbf{d})$ are difficult to calculate, one can estimate the magnitude of $\mathbf{m}$ from the recent studies of Wigner crystal states in bilayer systems [16], which allows the spins to rotate freely in response to the tunneling field and the Coulomb interaction. These studies show that the Wigner crystal at $\nu = 1/2$ (as well as other fillings) acquire a sizable magnetization (a substantial fraction of the full magnetization $\overline{\rho}/2$) in a large region of parameter space which covers the two- to one-component crossover region.

To conclude our symmetry discussions, we show that the triplet pairing family possesses off diagonal long range order (ODLRO) similar (but not identical) to those of single-component quantum Hall fluids. Proceeding as in the single-component case [15], we first note that

$$\langle \rho(z')\rho(z)\rangle_{N+1} = 4(N+1)^2 \chi^*_{\alpha\beta}\chi_{\mu\nu} \times \langle \xi_\beta(z')\psi_\alpha(z')\psi^+_\mu(z)\xi^+_\nu(z)\rangle_N ,$$

where $\rho(z) = \psi^+_\mu(z)\psi_\mu(z)$ is the density operator, $\langle \rho(z')\rho(z)\rangle_{N+1} \equiv \langle N+1, \chi|\rho(z')\rho(z)|N+1, \chi\rangle$, and $\xi_\nu(z) \equiv U^2(z)\int (z-a)^{-1}\phi^+_\nu(a)d^2a$. Defining the operator $\overline{\xi}^+_\mu(z)$ which operators on any state $|\Psi\rangle$ as

$$\overline{\xi}^+_\mu(z)|\Psi\rangle \equiv \xi^+_\mu(z)|\Psi\rangle \times \langle\Psi|[\psi^+_\nu(z), Q]^+[\psi^+_\nu(z), Q]|\Psi\rangle^{-1/2}, \quad (10)$$

it is easy to show that the matrix

$$W(z'z)_{\alpha\beta,\mu\nu} \equiv \langle N, \chi|\overline{\xi}_\beta(z')\psi_\alpha(z')\psi^+_\mu(z)\overline{\xi}^+_\nu(z)|N, \chi\rangle$$

satisfies

$$\chi^*_{\alpha\beta}W(z', z)_{\alpha\beta,\mu\nu}\chi_{\mu\nu} = \frac{\langle \rho(z)\rho(z')\rangle_{N+1}}{\langle\rho(z)\rangle^{1/2}_{N+1}\langle\rho(z')\rangle^{1/2}_{N+1}} \rightarrow \overline{\rho} = 1/4\pi , \quad (11)$$

as $|z - z'| \rightarrow \infty$, where $\overline{\rho} \equiv \langle\rho(z)\rangle_N/\langle 1\rangle_N = (4\pi)^{-1}$. This is a statement of ODLRD. It says that the "density matrix" $W(z, z')_{\alpha\beta,\mu\nu}$ becomes a product of two functions $F(z')_{\alpha\beta}\overline{F}(z)_{\mu\nu}$ whose overlap with $\chi_{\mu\nu}$ is exactly the square root of the density.

I thank X. G. Wen for discussions of topological order. Part of this work was done during the 1994 HKUST Summer School on Strongly Interacting Electron Systems. I would like to thank HKUST for generous support.

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[10] That the Pfaffian state can be written in a pairing form was pointed out in Moore and Read, Ref. [7].

[11] Both ^{3}He-A and ^{3}He-A_1 consist of Cooper pairs with orbital wave functions $Y_{1,-1}(\mathbf{r}/r) = (x - iy)/r$, where $\mathbf{r}$ is the displacement vector between the two atoms in the Cooper pair. The spin state of ^{3}He-A $|++\rangle - |--\rangle = |\uparrow\downarrow + \downarrow\uparrow\rangle$, where $|\pm\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. The spin state of ^{3}He-A_1 is $|++\rangle$. In the absence of an external magnetic field, ^{3}He-A_1 does not appear in the phase diagram. However, a nonvanishing magnetic field $\mathbf{H} = H\hat{x}$ will stabilize A_1 over A near T_c [11]. See, for example, D. Vollhardt, in *The Superfluid Phases of Helium 3*, edited by P. Wolfle (Taylor & Francis, London, 1990).

[12] Our explicit proof shows that degeneracies in toroidal geometries do not truly reflect the topological properties of the quantum Hall states with internal degrees of freedom. The different degeneracies at different points of the triplet pairing space can reconcile with each other inside the triplet space. The situation is similar to the topological stability of the vortices in x-y spin systems. Vortices with different winding numbers are topological distinct if the spins are confined in the x-y plane. However, they can be deformed into each other if the spins are given a z component to become Heisenberg spins.

[13] We shall use the notation in T. L. Ho, Phys. Rev. Lett. **73**, 874 (1994).

[14] See T. Muir, *A Treatise on the Theory of Determinant* (Dover, New York, 1960), p. 348.

[15] N. Read, Phys. Rev. Lett. **62**, 86 (1989).

[16] S. Narasimhan and T. L. Ho (to be published).