

Microwave-Induced “Somersault Effect” in Flow of Josephson Current through a Quantum Constriction

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We consider the supercurrent flow through gated mesoscopic semiconductor heterostructures in which a two-dimensional normal constriction is confined between superconducting electrodes. Photon-assisted Landau-Zener transitions between Andreev bound states in the constriction are shown to give rise to (i) a series of reversals in the direction of supercurrent flow (somersault effect) if the phase difference between the superconducting elements is fixed or (ii) a series of voltage spikes if the junction is current biased. We discuss necessary conditions for the described phenomena to be experimentally observable.

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The Josephson coupling between two superconductors is associated with charge transfer through the nonsuperconducting region that separates them. The coupling is therefore determined in a crucial way by the nature of the electron states in this region. Quantum mechanical tunneling through an insulating barrier superconductor-insulator-superconductor (SIS junction) [1], itinerant propagation of free electrons through a normal region [superconductor-normal-superconductor (SNS)] [2], and propagation of plasma waves (boson modes) through a 1D channel of strongly correlated electrons [3] are examples of charge transfer mechanisms in different types of weak links.

The electron states carrying the supercurrent in the non-superconducting region can be influenced by an external time-dependent field. This possibility raises interesting questions on the nature of Josephson coupling due to nonequilibrium electron states. This is the problem addressed here.

Nonequilibrium effects can be expected to be particularly important in situations where the Josephson coupling is mediated by a normally conducting microconstriction, where only a few electron states—Andreev bound states—carry the current. Weak links through ballistic microconstrictions in the two-dimensional electron gas of gated semiconductor heterostructures have recently been observed experimentally [4]. An important feature of this structure is that the electron energy levels in the constriction can easily be influenced by applying a potential to the gate electrodes. In this way the electron concentration (and hence the Fermi wave vector, k_F) in the channel can be controlled. The potential due to induced charges on the gate electrodes, furthermore, provides a mechanism for coupling the 2D electrons to an external electromagnetic field.

The geometry of the system to be considered in the following is shown in Fig. 1. The Josephson coupling between two superconductors can be expressed in terms of the phase difference ϕ between their respective order parameters. We will first consider this phase difference

to be a fixed quantity, which in a SQUID geometry can be controlled entirely by the magnetic flux (see inset of Fig. 1). Andreev reflection at the boundaries between superconducting and normal regions in a SNS configuration is known [5] to lead to a discrete set of energy levels within the energy gap of the superconductors. Because of spatial quantization in the transverse y direction, the energy spectrum of the Andreev bound states in a narrow and short constriction consists of a discrete set of pairs of levels labeled by the quantum number n [6],

$$E_{n,\pm} = \pm \Delta \sqrt{1 - T_n(k_F) \sin^2(\phi/2)}. \quad (1)$$

Energy is measured from the Fermi level, and the transmission eigenvalue $T_n(k_F)$ is related to the propagation of normal Fermi level electrons through the junction (along the x direction). For a short junction all of the Josephson current is carried by these Andreev states. It is important to note that states below ($E_{n,-}$) and above ($E_{n,+}$) the Fermi level carry current in different directions. In Fig. 2(a), the position of a pair of levels $E_{n,\pm}$ is indicated, and the directions of the corresponding partial currents are marked by arrows. The dashed arrow for the state $E_{n,+}$ above the Fermi energy illustrates that this state is unoccupied at zero temperature and hence does not contribute to the total current.

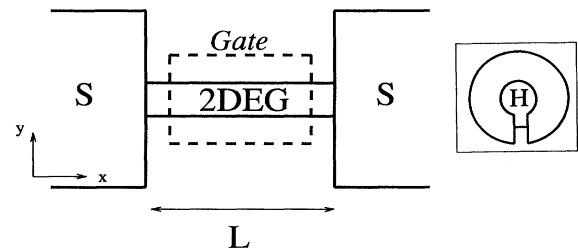


FIG. 1. Sketch of a gated semiconductor heterostructure in which a two-dimensional normal constriction is confined between superconducting electrodes. Inset: The phase difference between the two superconductors can be kept constant in the SQUID configuration shown.

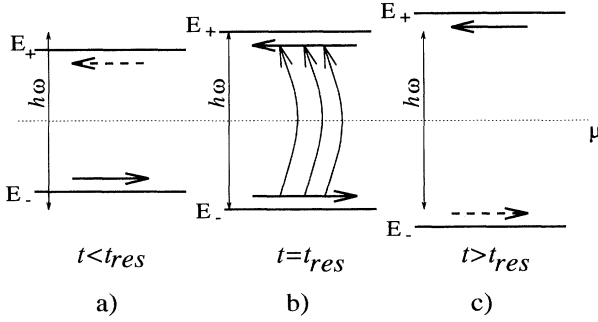


FIG. 2. Microwave induced transitions and currents carried by the Andreev levels in the normal constriction of Fig. 1 during a slow drift of the levels through resonance: (a) before reaching the resonance; (b) during the resonance; (c) after passing the resonance. Arrows show the direction of current, dashed arrows correspond to unpopulated level.

We shall consider a situation where a gate-induced time-dependent potential is a sum of one part $V_0(t)$, which varies slowly on a scale related to the interlevel spacing, and another rapidly oscillating part $V_\omega \cos(\omega t)$, where ω is of the order of the interlevel spacing. The high-frequency component of the induced potential produces interlevel transitions, which result in a strong coupling of pairs of Andreev states at resonance [7]. Because of the change in the interlevel distances resulting from a slow time variation of k_F , the condition for resonance, $\omega = \omega_{\alpha\beta} \equiv (E_\alpha - E_\beta)/\hbar$ ($\alpha = n, \pm$), can be met for any pair of levels at some time $t = t_{\text{res}}$ even if they are out of resonance with the electromagnetic field at $t = 0$, when the potential is switched on. Such a drift into and out of resonance is illustrated in Fig. 2, where panel (a) shows a nonresonant configuration. When the two levels drift into resonance, as in Fig. 2(b), resonant interlevel mixing results in a finite population of the upper level and a corresponding contribution to the partial Josephson current in the reverse direction.

If the Andreev levels pass slowly enough (see below) through the resonance, the result of the dynamic evolution is a complete depopulation of the lower level. Hence, as the levels drift out of resonance the Josephson current has been turned around and the current is flowing in the opposite direction [Fig. 2(c)]. After some time out of resonance, the system relaxes back to its ground state and the Josephson current returns to the forward direction. The microwave-induced “somersault” has been completed.

To discuss the above effects quantitatively we begin with the time-dependent Bogoliubov–de Gennes (BdG) equation;

$$i\hbar\dot{\Psi}(\mathbf{r}, t) = [\hat{H} + \hat{V}(t)]\Psi(\mathbf{r}, t). \quad (2)$$

Here \hat{H} is the standard Hamiltonian both for the normal electrons in the microconstriction part of the device $|x| < L/2$ (see Fig. 1) and for the electrons in the super-

conducting regions, $|x| > L/2$. The length of the junction L is assumed to be small in comparison with the coherence length $\xi_{\text{Sm}} = v_F/\Delta$ (v_F is the Fermi velocity in the constriction). The normal electron Hamiltonian includes an electrostatic potential that confines the electrons in the transverse direction and also describes the normal scattering from impurities and from the barriers at the Sm-S boundaries. The spatial variation of the superconducting order parameter due to the proximity effect is small and can be neglected, since the width of the constriction is much smaller than the superconductor coherence length ξ_S [8]. Hence, we use a step-function model for the pair potential, $\Delta(\mathbf{r}) = \Delta\Theta(|x| - L/2)\exp[i\text{sgn}(x)\phi/2]$. The time-dependent potential, $\hat{V}(t) = [V_0(t) + V_\omega \cos \omega t]\sigma_z\Theta(L/2 - |x|)$, where σ_z is a Pauli spin matrix, is confined to the normal region and does not mix the electronlike and holelike components of the wave function Ψ . If its high-frequency part V_ω is absent, the only effect of the external time-dependent field $V(t)$ is an adiabatic energy shift of the Andreev levels caused by the parametric dependence of E_α on $k_F = ((2m/\hbar^2)[\mu - V_0(t)])^{1/2}$ [see Eq. (1)]. If the field has a finite high frequency component, different Andreev states are mixed due to interlevel transitions described by the matrix element $V_{\alpha\beta} = V_\omega\langle\Psi_\alpha(\mathbf{r}), \sigma_z\Theta(L/2 - |x|)\Psi_\beta(\mathbf{r})\rangle$. In what follows we will consider the weak coupling limit $V_{\alpha\beta} \ll \hbar\omega_{\alpha\beta}$, in which almost all Andreev states are only slightly renormalized by the electromagnetic field. The exception is when the condition for resonance between two states, $\omega = \omega_{\alpha\beta}(t)$, is fulfilled at some time $t = t_{\text{res}}$. At resonance the interaction with the field cannot be treated perturbatively. Rather one must make the resonant approximation by finding a solution to the BdG equation (2) that is a mixture of the two adiabatic states, $\Psi_\alpha(\mathbf{r})$ and $\Psi_\beta(\mathbf{r})$, in resonance:

$$\begin{aligned} \Psi_{\alpha\beta}(\mathbf{r}, t) = & \exp\left(-i/2\hbar \int^t [E_\alpha(t) + E_\beta(t)]dt\right) \\ & \times [b_s^1\Psi_\alpha(\mathbf{r})e^{i(\omega/2)t} + b_s^2\Psi_\beta(\mathbf{r})e^{-i(\omega/2)t}]. \end{aligned} \quad (3)$$

Here the index $s = 1, 2$ labels independent solutions corresponding to different initial conditions, $b_s^r(t = 0) = \delta_{s,r}$. After substituting the wave function (3) into the BdG equation (2) and averaging over the fast oscillations, we readily find equations which describe the time evolution of the coefficients $b_s^{1,2}$. Introducing a vector coefficient $\mathbf{b}_s(t)$ we get

$$i\mathbf{b}_s(t) = \left[\frac{\delta\omega_{\alpha\beta}(t)}{2}\sigma_z + \left(\frac{V_{\alpha\beta}}{\hbar}\sigma_+ + \text{H.c.} \right) \right] \mathbf{b}_s(t), \quad (4)$$

where $\delta\omega_{\alpha\beta}(t) = \omega_{\alpha\beta}(t) - \omega \ll \omega$ and $\sigma_+ = (\sigma_x + i\sigma_y)/2$, $\sigma_{x,y}$ being Pauli spin matrices. A problem similar to Eq. (4) was first discussed by Landau and Zener in connection with molecular predissociation [9,10] (see also

[11]). It follows from their theory that the asymptotic solution of (4) valid at $t \gg t_{\text{res}}$ is $|b_s^r| = W_{\alpha\beta} + \delta_{rs}(1 - 2W_{\alpha\beta})$. The probability $W_{\alpha\beta}$ for the system to be in a different state after passing through the resonance,

$$W_{\alpha\beta} = 1 - e^{-2\pi\gamma_{\alpha\beta}^2},$$

$$\gamma_{\alpha\beta} = |V_{\alpha\beta}/\hbar| |\dot{\omega}_{\alpha\beta}(t_{\text{res}})|^{-1/2}, \quad (5)$$

is determined by the product of the interlevel transition frequency $|V_{\alpha\beta}|/\hbar$ and the characteristic time $\delta t_{\text{res}} = |\dot{\omega}_{\alpha\beta}(t_{\text{res}})|^{-1/2}$ the system spends on resonance. If the characteristic time is long, $\gamma \gg 1$ ($\delta t_{\text{res}} \gg \hbar/V_{\alpha\beta}$), we have $|b_s^r|^2 \simeq 1 - \delta_{rs}$ and the transition between modes α and β is complete.

In the absence of relaxation, the Josephson current is solely determined by the dynamic evolution of the electron-hole wave function $\Psi(\mathbf{r}, t)$ discussed above. Before the high-frequency field is switched on, the Andreev levels are in equilibrium and the Josephson current has its equilibrium value I_0 . The deviation of the time averaged current, $\delta\bar{I} = \bar{I} - I_0$, is found using the asymptotic result for the coefficients $b_s^r(t)$:

$$\delta\bar{I} = \sum_{\{\alpha, \beta\}_{\text{res}}} W_{\alpha\beta} (n_\alpha - n_\beta) (I_\beta - I_\alpha), \quad (6)$$

where $I_{\alpha, \beta} = (2e/\hbar)dE_{\alpha, \beta}/d\phi$, and $n_{\alpha, \beta}$ is the Fermi function, $n_{\alpha, \beta} = n_F(E_{\alpha, \beta})$. The summation in (6) is over pairs of levels, $\{\alpha, \beta\}_{\text{res}}$, which have passed through a resonance [12]. As we can see from Eq. (6) only transitions between states with energies of opposite signs (the Fermi level is at zero) contribute to $\delta\bar{I}$. If the Josephson current through the channel is carried by a single pair of Andreev levels, the direction of the Josephson current is reversed for $W \geq 1/2$. Of course, for $t \gg t_{\text{res}}$ there is no resonant coupling with the electromagnetic field to maintain the inverse population. Any inelastic relaxation mechanism will bring the system back to its equilibrium state and restore the original direction of Josephson current. This process of dynamically switching on the inverse population of Andreev states and the subsequent inelastic relaxation manifests itself as a somersault of the Josephson current.

Equation (6) is significantly simplified if the microconstriction joining the two superconductors is an adiabatically smooth ballistic channel with quantized energy levels (modes) corresponding to the transverse electron motion. In this situation each propagating mode produces one pair of Andreev levels of the type (1)—one level above and one below the Fermi energy—and the transmission eigenvalue T_n reduces to the normal electron transmission coefficient of the n th mode,

$$T_n(k_F) = \left[1 + \frac{4R}{(1-R)^2} \sin^2(k_n L) \right]^{-1}, \quad (7)$$

where $k_n = \sqrt{k_F^2 - \pi^2 n^2/d^2}$, and R is the probability for electron backscattering at the Sm-S boundary. The electromagnetic field couples levels only within these

pairs, i.e., if $\sigma = \pm$ one has $V_{\alpha\alpha'} = \delta_{n,n'}\delta_{\sigma,-\sigma'}V_n$, and consequently $\omega_{\alpha\alpha'} = \delta_{n,n'}\delta_{\sigma,-\sigma'}\omega_n$. The matrix element V_n has the form

$$|V_n|^2 = V_\omega^2 \frac{L^2}{\xi_{\text{Sm}}^2} T_n^3 \frac{R}{(1-R)^2} \frac{\cos^2(k_n L) \sin^2 \phi}{1 - T_n \sin^2(\phi/2)}. \quad (8)$$

In the ballistic case the total current at zero temperature, taking into account Eq. (6), can be written in the form

$$\bar{I} = \sum_n I_n (1 - 2W_n), \quad (9)$$

where the summation is formally done over all transverse modes and W_n is assumed to be zero for those Andreev levels which have not passed through a resonance.

As one can see from Eq. (8), the electromagnetic field can only mix Andreev states if the probability for electron backscattering at the Sm-S boundaries is finite. A somersault which changes the total momentum of the system can evidently only occur if backscattering is possible. It is interesting that if there is a geometric resonance, $k_n L = n\pi$, the somersault effect still takes place according to (6) even though the microconstriction is fully transparent for electrons in mode n , $T_n(k_F) = 1$. This fact provides an important possibility of distinguishing between resonant transmission through the double barrier structure and ballistic propagation in the absence of normal backscattering from Sm-S boundaries.

It is also interesting to consider the case when there is no drift with time of the Andreev levels, i.e., when $V_0(t) = 0$. A permanent resonant coupling of Andreev states is now possible and leads to an equalization of the average level population [13]. Solving equations (4) we find a result for the Josephson current that deviates from its stationary value:

$$\bar{I} = \sum_n I_n \frac{|\delta\omega_n|}{\sqrt{\delta\omega_n^2 + (2V_n/\hbar)^2}}. \quad (10)$$

In particular, we find that resonant coupling, i.e., when $\delta\omega_n = 0$, results in a complete blockade of the Josephson current. This phenomenon can be observed as a sequence of dips if the Josephson current is plotted as a function of the frequency of the electromagnetic field. The number of dips should equal the number of propagating modes in the ballistic constriction, and the dip amplitude should equal the single mode contribution I_n to the stationary Josephson current.

In the discussion so far we have assumed that the phase difference between the two superconductors is constant in time. For the somersault effect to develop as described above, it is necessary that ϕ be fixed to a high degree of accuracy, otherwise the system will escape from resonance. This fact implies that the change of the interlevel distance due to the phase shift $\delta\phi$ should be smaller than the nonlinear resonance width: $\hbar(\partial\omega_{\alpha\beta}/\partial\phi)\delta\phi \lesssim V_{\alpha\beta}$. In our estimation $\delta\phi \sim (\mathcal{L}/\xi_S)(1/N_\lambda)$, where \mathcal{L} is the ring length, ξ_S is the superconductor coherence length, and N_λ is the number

of transverse modes carrying the Meissner current in the superconducting ring. The criterion for the phase difference to be stable enough to observe the resonance is thus $(V_{\alpha\beta}/\Delta)L/\mathcal{L} \gtrsim 1/N_\lambda$.

When the Josephson junction is connected to a current source, the current rather than the phase difference is held constant. In this situation another interesting phenomenon occurs; the somersault effect causes phase slips at times when Andreev levels are resonantly coupled. The phase slips in turn manifest themselves as voltage spikes. The characteristic time scale of a phase slip is determined by the Josephson plasma frequency $\omega_J = [(\Delta/\hbar)(1/R_0C_J)]^{1/2}$, which for the geometry of Fig. 1 and $R_0 = \hbar/e^2$ can be written as $\omega_J = (\Delta/\hbar)(\xi_S L/d^2)^{1/2}$. This relationship allows us to estimate the amplitude and the duration of the voltage spike as $U = (\hbar/2e)\dot{\phi} \sim (\hbar/e)\omega_J$ and $\Delta t \sim 1/\omega_J$, respectively, in the case where the duration of the interlevel transition, δt_{res} , is shorter than Δt , i.e., if $1/\omega_J \gg \delta t_{\text{res}}$. In the opposite case, $1/\omega_J \ll \delta t_{\text{res}}$, a more complex analysis of the joint dynamics of the phase and interlevel transition gives the estimates $U \sim V_{\alpha\beta}^{4/3}/e\Delta^{1/3}$ and $\Delta t \sim \hbar/V_{\alpha\beta}^{2/3}\Delta^{1/3}$.

The somersault effect discussed above relies on the preservation of phase coherence during the dynamical evolution of the Andreev states and the redistribution of level populations at resonance. This implies that the phase breaking time τ associated with inelastic relaxation should be the largest time scale in the problem:

$$1/\omega_{\alpha\beta} \sim \hbar/\Delta \ll \hbar/V_{\alpha\beta} \ll \delta t_{\text{res}} \ll \tau, \\ 1/\omega_J \ll \tau. \quad (11)$$

Since the main part of the Andreev state wave function is localized in the superconducting electrodes, the most important relaxation mechanisms are recombination processes due to electron-phonon interaction in the superconductors. According to Ref. [14] a typical estimate of the recombination time in a superconductor is $\tau\Delta/\hbar > 10^2$. This value makes it possible to fulfill the inequalities (11) in a device with normal region length $L \approx \xi_{\text{sm}}$ if the amplitude V_ω is of the order of 10 mV. Meanwhile, an amplitude of the voltage spikes in a current biased junction is of order of 1–10 mV.

In conclusion, we have shown that a microwave electromagnetic field can drastically change the Josephson current carried by quantized modes in a mesoscopic constriction, giving rise to sequential reversals of the current in a phase biased junction and to voltage spikes in a current biased junction. While it may be difficult to measure the reversal of current directly, the voltage spikes are a different matter. Even though the considerable impedance mismatch between our device ($\approx 10 \text{ k}\Omega$) and a high-frequency

amplifier ($\approx 50 \Omega$) will reduce the signal by a factor of 100 to, say, $50 \mu\text{V}$, the noise level of low noise cooled microwave amplifiers available today is an order of magnitude lower (see, for example, [15]). Observation of the somersault effect, which hence seems to be possible in currently available gated semiconductor heterostructure devices, would be a direct manifestation of the discrete nature of the Josephson current in a quantum constriction.

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