One-Particle and Two-Particle Instability of Coupled Luttinger Liquids

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It is shown that the Luttinger liquid is unstable to arbitrarily small transverse hopping. The crossover temperatures below which either transverse coherent band motion or long-range order start to develop can be finite even when spin and charge velocities differ. Explicit scaling relations for the one-particle and two-particle crossover temperatures are derived in terms of transverse hopping, spin and charge velocities, and anomalous exponents. The special case of infinite-range transverse hopping is treated exactly and yields a Fermi liquid down to $T = 0$, unless the anomalous exponent $\theta > 1$.

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The one-dimensional electron gas with short-range interactions provides the best understood example of interacting electrons whose asymptotic low-energy behavior is *not* described by a Fermi liquid (FL) fixed point. Instead, the stable fixed point is the "Luttinger liquid" (LL). From the time of their discovery, high-temperature superconductors have lead to questioning the stability of the FL fixed point in *two* dimensions. In particular, Anderson [1] has put forward the idea that a Luttinger liquid could survive in two dimensions. It became clear that Luttinger liquids weakly coupled by interchain single-particle hopping $t_\perp$ would be a natural model to give a sound basis to this conjecture. In fact, the question of the stability of the Luttinger liquid in the presence of $t_\perp$ has been addressed long ago in the context of quasi-one-dimensional conductors [2–4]. It was then shown that $t_\perp$, as well as pair-hopping correlations, destabilize the Luttinger liquid fixed point. A series of recent works on two coupled chains [5] and on the many-chain problem [6] confirm this view but, nevertheless, the issue remains controversial [7]. In particular, going beyond the two chain case is a necessary requirement for phase transitions or long-range quasiparticle coherence to occur. The very few attempts to do so essentially deal with the situation where the spin and charge velocities are equal [3,4].

In this Letter we demonstrate the instability of the LL due to interchain coherence for an infinite number of coupled chains. We start from a new functional integral formulation where $t_\perp$ is the perturbation and the unperturbed system is the LL with both anomalous exponents *and* differing spin and charge velocities. We investigate single-particle spectral weight as well as induced two-particle correlations. For a given bare $t_\perp$ and temperatures much lower than the Fermi energy $T \ll E_F$, the typical phase diagram found here is sketched in Fig. 1 as a function of temperature and of the anomalous exponent θ, which is a measure of the interaction strength. As temperature is lowered, two types of crossovers can occur. For weak enough interaction, transverse one-particle coherent motion starts to develop, indicating that the crossover at the deconfinement temperature T_{x^1} is from LL to FL. On the other hand, for strong interaction, virtual pair hopping becomes the dominating process which eventually leads to long-range ordering below the two-particle dimensional crossover temperature T_{x^2}, even if there is confinement at the one-particle level ($T_{x^2} > T_{x^1}$). At temperatures such that $0 < T < T_{x^1}(T_{x^2})$, a true phase transition can occur in more than two dimensions.

Let us start with the full partition function for a set of $N_\perp$ fully interacting chains written in the interaction representation

$$Z = \mathrm{Tr}\left\{e^{-\beta\left(\sum_i \mathcal{H}_i^{1\mathrm{D}} + \sum_{ij} \mathcal{H}_{\perp ij}\right)}\right\} = Z_{1\mathrm{D}}\left\langle T_\tau e^{-\int_0^\beta d\tau \sum_{ij} \mathcal{H}_{\perp ij}(\tau)}\right\rangle_{1\mathrm{D}}, \quad (1)$$

where indices i, j run over all $N_\perp$ chains, $\mathcal{H}_i^{1\mathrm{D}}$ is the purely 1D Hamiltonian describing the interacting electrons along chain i, while the interchain hopping part $\mathcal{H}_{\perp ij}$ stands as the perturbation. The above thermodynamic average $\langle \cdots \rangle_{1\mathrm{D}}$ and partition function $Z_{1\mathrm{D}}$ only involve the pure 1D Hamiltonian. The hopping Hamiltonian is given by $\mathcal{H}_{\perp ij} = -\int dx \sum_{p,\sigma} t_{\perp ij} a_{p,i}^{\sigma\dagger}(x) a_{p,i}^{\sigma}(x)$,

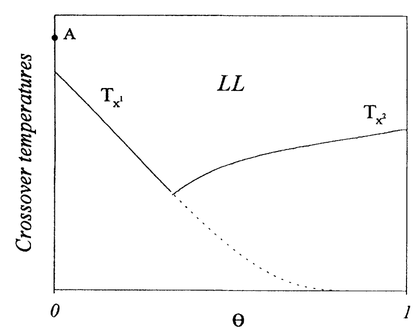

FIG. 1. One-particle (T_{x^1}) and two-particle (T_{x^2}) crossover temperatures shown qualitatively as a function of θ in the gapless case. The solid line becoming dotted indicates the one-particle deconfinement LL → FL for the exact infinite-range transverse hopping model. Point A gives the noninteracting value of T_{x^1} ($\sim t_\perp/\pi$), when $v_\rho = v_\sigma = v_F$.

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where x is the continuous coordinate along the chains while $p = \pm$ denote right- and left-going electrons, respectively. By analogy with the problem of propagation of correlations in critical phenomena, the propagation of one-particle transverse coherence is studied through an effective field theory which is generated by a Hubbard-Stratonovich transformation for Grassmann variables. This allows the partition function Z to be expressed as a functional integral over a Grassmann $\psi^{(*)}$ field:

$$Z = Z_{1\mathrm{D}} \int\int \mathcal{D}\psi^* \mathcal{D}\psi\, e^{-\int d(1) \sum_{ij} \psi_i^*(1) t_{\perp ij}^{-1} \psi_j(1)} \times \left\langle e^{\int d(1) \sum_i [a_i^\dagger(1)\psi_i(1) + \psi_i^*(1) a_i(1)]} \right\rangle_{1\mathrm{D}}, \quad (2)$$

with the notations $\int d(n)\psi_i(n) \equiv \int dz_n \sum_{p_n,\sigma_n} \psi_{p_n,i}^{\sigma_n}(z_n)$ and $\int dz_n \equiv \int dx_n \int_0^\beta d\tau_n$. The logarithm of the thermodynamic average in (2) is readily recognized as the generating function for the exact 1D connected Green's functions $G_c^{(n)}$. Changing variables from ψ_i to $\sum_j t_{\perp ij}^{1/2} \psi_j$ in the functional integral (2) allows us to ultimately write for the effective field theory

$$Z = Z_{1\mathrm{D}} \int\int \mathcal{D}\psi^* \mathcal{D}\psi\, e^{\mathcal{F}[\psi^*,\psi]}, \quad (3)$$

where the Grassmannian Landau-Ginzburg-Wilson functional, $\mathcal{F} = \mathcal{F}_0 + \sum_{n\geq 2} \mathcal{F}^{(n)}$, involves a quadratic part and a sum of effective interactions to all orders in the ψ field. To write a specific form for $\mathcal{F}$, let us consider the case where chains are lined up in a plane and let us Fourier transform in the direction transverse to the chains. Then, the Gaussian part describing the free propagation of the ψ field takes the form

$$\mathcal{F}_0 = -\int d(1) \int d(2) \sum_{k_\perp} \psi_{k_\perp}^*(1) \times [\mathbb{1} - t_\perp(k_\perp) G^{(1)}(1-2)] \psi_{k_\perp}(2), \quad (4)$$

where $G^{(1)}$ is the exact one-dimensional propagator. The interacting part is found to be

$$\mathcal{F}^{(n)} = \frac{1}{n!} \frac{1}{N_\perp^{n-1}} \sum_{k_{\perp 1},\ldots,k_{\perp 2n}} \int d(1) \cdots \int d(2n) \psi_{k_{\perp 1}}^*(1) \cdots \psi_{k_{\perp 2n}}(2n) \gamma_\perp(k_{\perp 1},\ldots,k_{\perp 2n}) G_c^{(n)}(1,\ldots,2n), \quad (5)$$

where

$$\gamma_\perp(k_{\perp 1},\ldots,k_{\perp 2n}) = \left\{ \prod_\alpha^{2n} t_\perp(k_{\perp\alpha}) \right\}^{1/2} \delta_{\sum_\alpha k_{\perp\alpha},0}, \quad (6)$$

in which $t_\perp(k_\perp)$ is the Fourier transform of $t_{\perp ij}$. The Gaussian part gives the exact result: (a) In the noninteracting limit ($G_c^{(n\geq 2)} \to 0$) for arbitrary $t_\perp$. (b) For arbitrary interaction when $t_\perp = 0$. (c) In the limit of infinite-range transverse hopping, whatever the value of $t_\perp$ and of the interaction. Therefore $1/N_\perp$ may be used as a formal expansion parameter [8].

Quasiparticle pole at $T = 0$.—An instability of the LL at zero temperature and thus the possibility of a FL fixed point is already present in the free theory of the ψ field described by $\mathcal{F}_0$. At this level of approximation, the 2D one-particle Green's function in Fourier-Matsubara space, say for right-going electrons, is given by

$$\mathcal{G}^{2\mathrm{D}}(\mathbf{k}, \omega_n) = \frac{G^{(1)}(k,\omega_n)}{1 - t_\perp(k_\perp) G^{(1)}(k,\omega_n)}, \quad (7)$$

where $\omega_n = (2n+1)\pi T$, while $\mathbf{k} = (k, k_\perp)$, with k measured with respect to the right 1D Fermi point k_F^0. In order to show the existence of a quasiparticle pole at $T = 0$, we use the known general form for the asymptotic 1D propagator $G^{(1)}$ describing the Luttinger liquid in space and imaginary time [9],

$$G^{(1)}(x,\tau) = \frac{e^{ik_F^0 x}}{2\pi i \Lambda^\theta} \prod_{\nu=\rho,\sigma} S(z_\nu)^{-1/2-\theta_\nu/2} S(z_\nu^*)^{-\theta_\nu/2}, \quad (8)$$

where $S(z_\nu) = \xi_\nu \sinh(z_u/\xi_\nu)$ and $z_\nu = x + i v_\nu \tau$. Here, v_ρ and v_σ are the velocities of spin and charge excitations, respectively, while $\xi_{\rho,\sigma} = v_{\rho,\sigma}/\pi T$ are the corresponding thermal coherence lengths. The exponents $\theta_\rho > 0$ and $\theta_\sigma > 0$ are the spin and charge contributions to the anomalous dimension $\theta = \theta_\rho + \theta_\sigma$ of the 1D Green's function and Λ is an ultraviolet cutoff. Now, one of the central issues is to shed light on the influence of differing spin and charge velocities [1] on the stability of the LL. For this sake, let us consider (8) in a special case where things get simpler, that is, when $\theta_\rho = \theta_\sigma = 0$ but still $v_\rho \neq v_\sigma$. In this case, the $T = 0$ retarded 1D propagator, $G^{(1)}(k,\omega)$, has two square root singularities. At the Gaussian level, the corresponding spectral weight $A^{2\mathrm{D}} = -\pi^{-1} \mathrm{Im}\, \mathcal{G}^{2\mathrm{D}}$ obtained from the imaginary part of (7) is given by

$$A^{2\mathrm{D}}(\mathbf{k},\omega) = \frac{A^{1\mathrm{D}}(k,\omega)}{1 + [\pi t_\perp(k_\perp) A^{1\mathrm{D}}(k,\omega)]^2} + Z(t_\perp)\delta((\omega - \epsilon(\mathbf{k})), \quad (9)$$

where $A^{1\mathrm{D}} = -\pi^{-1} \mathrm{Im} G^{(1)}$ is the exact one-dimensional spectral weight, $Z(t_\perp)$ is the quasiparticle residue, and $\epsilon(\mathbf{k})$ the pole of (7). For the case at hand, the undamped quasiparticle spectrum given by the pole of (7) is $\epsilon(\mathbf{k}) = k\overline{v} - \mathrm{sgn}\{t_\perp(k_\perp)\}\sqrt{(k\Delta v)^2 + t_\perp^2(k_\perp)}$, where $\overline{v} = (v_\rho + v_\sigma)/2$ and $\Delta v = (v_\rho - v_\sigma)/2$, while the quasiparticle residue takes the form

$$Z(t_\perp) = \frac{|t_\perp(k_\perp)|}{\sqrt{(k\Delta v)^2 + t_\perp^2(k_\perp)}}. \quad (10)$$

The residue is readily seen to satisfy $Z \to 0$ when $t_\perp \to 0$ while $Z \to 1$ when $\Delta v \to 0$ as one expects for free electrons. Note that the infinite lifetime of the quasiparticles in (9) should become finite when one goes beyond the Gaussian approximation thus allowing the ψ's to interact [8], except at the Fermi surface where the lifetime should remain infinite because of the usual phase space arguments. One can also check that for wave vectors close to the new Fermi surface, given by $(v_\rho v_\sigma)^{1/2} k = t_\perp(k_\perp)$, the single-particle spectral weight has the following frequency dependence. For nearest-neighbor hopping and $k_\perp < \pi/2$, a quasiparticle peak is first encountered as the frequency is increased, followed by an incoherent background which is a smoothed version of the original LL. In other words, remnants of spin-charge separation are left at high energies.

One-particle dimensional crossover.—In the more general case $\theta \neq 0$, $Z(t_\perp)$ cannot be found analytically although we can prove from known 1D spectral weights [10] that a pole appears in regions where the 1D spectral weight is zero, leading to results qualitatively similar to those just discussed. Nevertheless, we can determine the temperature scale T_{x^1} at which the pole in $\mathcal{G}^{2\mathrm{D}}$ becomes perceptible and transverse single-fermion coherence starts to develop. Using the natural change of variables $\tau' = \pi T \tau$ and $x = (\xi_\rho \xi_\sigma)^{1/2} x'$ to evaluate the Fourier transform of the 1D Green's function (8) at the Fermi level $G^{(1)}(0, \pi T)$, and substituting in the 2D Gaussian propagator (7) one readily finds

$$T_{x^1} \sim E_F \left(\frac{t_\perp}{E_F}\right)^{1/(1-\theta)} \left(\frac{v_F}{v_\rho}\right)^{\theta_\rho/(1-\theta)} \times \left(\frac{v_F}{v_\sigma}\right)^{\theta_\sigma/(1-\theta)} F_1\left(\{v_\sigma/v_\rho\}^{1/2}\right), \qquad (11)$$

where $F_1(x)$ is a temperature independent function that satisfies $F_1(x) = F_1(1/x)$ and which also depends on $\theta_{\rho,\sigma}$. As long as $\theta < 1$, or equivalently if $G^{(1)}(x)$ decays more slowly than x^{-2}, the coupling $t_\perp$ is relevant and T_{x^1} is finite, although smaller than the noninteracting value $T_{x^1} \sim t_\perp/\pi$ [2,4,6]. The condition $\theta < 1$ is satisfied for the Hubbard model with a non-half-filled band where one has the exact result $\theta \leq 1/8$ [11]. For more specialized 1D models (forward scattering only, half filling, etc.) one can have $\theta = 1$ and $\theta > 1$, where transverse hopping becomes marginal and irrelevant, respectively. In these cases, transverse band motion does not develop and the electrons remain spatially confined along the chains at all temperatures. As seen from (11), the effect of $\Delta v \neq 0$ is to decrease the deconfinement temperature but not to make it vanish. The vanishing of T_{x^1} is expected for sufficiently strong coupling, since spin and charge degrees of freedom must recombine for an electron to tunnel on a neighboring chain. Indeed, it can be shown that $F_1(\{v_\sigma/v_\rho\}^{1/2}) \overset{v_\sigma/v_\rho \to 0}{\sim} \{v_\sigma/v_\rho\}^{1/2}$ and correspondingly, $T_{x^1} \to 0$ when $\theta_\sigma < \frac{1}{3}(1 - \theta_\rho)$.

Two-particle dimensional crossover.—We now proceed beyond the Gaussian level by taking into account the $\mathcal{O}(t_\perp^2)$ quartic term in the functional (5) which describes correlated transverse pair tunneling [8]. We argue that the system will undergo a two-particle dimensional crossover towards charge density wave (CDW), or spin density wave (SDW) ordered states if the interaction is repulsive and singlet or triplet superconducting states if it is attractive. Focusing on the $2k_F^0$ particle-hole channel, we rewrite the partition function at the quartic level

$$Z = Z_{1\mathrm{D}} \left\langle e^{\sum_{\mu,q_\perp} \int \{dz\} O^*_{\mu,q_\perp}(z_3,z_1) \gamma_\perp(q_\perp) R_\mu(\{z\}) O_{\mu,q_\perp}(z_2,z_4)} \right\rangle_{\psi^*\psi},$$

with the obvious notation $\{z\} = \{z_1, z_2, z_3, z_4\}$ and where $\langle \cdots \rangle_{\psi^*\psi}$ is the average with respect to $\mathcal{F}_0$. The composite fields

$$O_{\mu,q_\perp}(z_3, z_1) = N_\perp^{-1/2} \sum_{k_\perp} \sum_{\alpha\beta} \psi^{\alpha *}_{-,k_\perp}(z_3) \sigma_\mu^{\alpha\beta} \psi^{\beta}_{+,k_\perp+q_\perp}(z_1)$$

describe CDW ($\mu = 0$) and SDW ($\mu = 1,2,3$) correlations. For nearest-neighbor hopping, we approximate the transverse pair tunneling amplitude as $\gamma_\perp(q_\perp) \approx (2t_\perp)^2 \cos(q_\perp)$, where $q_\perp$ is the transverse momentum of the particle-hole pair, by setting the incoming momenta to 0 or π, since this leads to the highest value for T_{x^2}. In the above, $\sigma_0^{\alpha\beta} = \delta^{\alpha\beta}$, $\sigma_{\mu=1,2,3}^{\alpha\beta}$ are the Pauli matrices and the $q_\perp$ independent function $R_\mu(\{z\})$ is the 1D connected correlator for charge or spin fluctuations.

To examine the possibility of phase transition, we perform a Hubbard-Stratonovich transformation on the O_μ fields. Let $\zeta^*_{\mu,q_\perp}(z_3, z_1)$ be the complex field conjugate to $O_{\mu,q_\perp}(z_3, z_1)$. The partition function Z then takes the form

$$Z = Z_{1\mathrm{D}} \iint \mathcal{D}\zeta^* \mathcal{D}\zeta \, e^{-\int \{dz\} \sum_{\mu,q_\perp} \zeta^*_{\mu,q_\perp}(z_3,z_1)[\mathbb{1} - \gamma_\perp(q_\perp) R_\mu(\{z\})] \zeta_{\mu,q_\perp}(z_2,z_4) + \mathcal{O}(\zeta^4)}. \qquad (12)$$

Softening of the ζ field first occurs for a value of $q_\perp = \pi$ corresponding to the usual staggered order. The temperature T_{x^2} at which the ζ field softens must be greater than T_{x^1} to retain its meaning as a two-particle crossover temperature. For definiteness, let us now consider the gapless case. As shown below, T_{x^2} scales differently in the weak- and strong-coupling limits. In the strong-coupling limit [8], the correlator $R_\mu(z_1 - z_3, z_2 - z_4, z_1 - z_2)$ decays faster than the square of the electron-hole separations $|z_1 - z_3|$ and $|z_2 - z_4|$, so that the scaling is determined by $z = z_1 - z_2$ only, leading to the asymptotic form

$$R_\mu(z) \sim -\Lambda^{\gamma_\mu} \cos 2k_F^0 x \prod_{\nu=\rho,\sigma} [S(z_\nu)S(z_\nu^*)]^{-\gamma_{\mu,\nu}/2},$$

with $\gamma_\mu = 2 - \gamma_{\mu,\rho} - \gamma_{\mu,\sigma}$. The strong-coupling regime is defined by $2 - 2\theta - \gamma_\mu < 0$ [3,4], where γ_μ is the critical exponent which governs the temperature behavior of the 1D susceptibility $\chi_\mu^{1\mathrm{D}}(T) \sim T^{-\gamma_\mu}$. In this regime,

$$T_{x^2}^\mu \sim E_F \left(\frac{t_\perp^2}{E_F^2}\right)^{1/\gamma_\mu} \left(\frac{v_F}{v_\rho}\right)^{\gamma_{\mu,\rho}/\gamma_\mu} \left(\frac{v_F}{v_\sigma}\right)^{\gamma_{\mu,\sigma}/\gamma_\mu} F_2^\mu((v_\sigma/v_\rho)^{1/2}), \tag{13}$$

where the dimensionless function $F_2^\mu(x)$ vanishes at $x = 0$ and $E_F \sim \Lambda v_F$.

In the weak-coupling regime [8], $2 - 2\theta - \gamma_\mu > 0$, the correlator essentially scales as the square of the one-particle propagator (8) so that

$$T_{x^2}^\mu \sim T_{x^1} F_3^\mu((v_\sigma/v_\rho)^{1/2}, \{g\}). \tag{14}$$

where $\{g\}$ stands for the set of dimensionless electron-electron interaction vertices and $F_3^\mu(x,y)$ vanishes when either of its arguments vanishes. It is only for sufficiently weak coupling that $T_{x^2} < T_{x^1}$ and that deconfinement takes place. In both regimes, the effect of $v_\rho \neq v_\sigma$ is to decrease T_{x^2}. Finally, (13) and (14) reduce to the known results [4] when $v_\rho = v_\sigma$. This is also compatible with the discussion in [6].

Higher-order vertices, $\mathcal{O}(t_\perp^3), \ldots$, involve higher-order connected functions $G_c^{(3)}, \ldots$ of the Luttinger liquid. Since these functions do not contain any new relevant exponent, they do not change the above scaling results for either T_{x^1} or T_{x^2}. In the latter case, higher-order connected functions modify mode-mode coupling terms included in the functional (12).

Infinite range transverse hopping: An exact result.— The case where the transverse hopping amplitude has an infinite range can be solved exactly [12]. For the energy to be independent of the number of perpendicular chains $N_\perp$ in the thermodynamic limit $N_\perp \to \infty$, the transverse hopping matrix must be scaled as $t_{\perp ij} = t_\perp / N_\perp$. This implies that in Fourier space, $t_\perp(k_\perp) = t_\perp \delta_{k_\perp,0}$, so the n-body interaction is such that $\mathcal{F}^{(n\geq 2)} \propto t_\perp (t_\perp/N_\perp)^{n-1}$. Consequently, in the limit $N_\perp \to \infty$, all effective interactions vanish as $t_\perp^n / N_\perp^{n-1} \to 0$. Hence, the Gaussian propagator (7) becomes exact. There can be *no* interchain pair tunneling and consequent long-range order. Either $\theta \geq 1$ and the electrons remain spatially confined along the chains at all temperatures or $\theta < 1$ and the single-particle propagator acquires a pole for wave vectors along the chain $(k, 0)$ at all temperatures below T_{x^1}.

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