Thermal Conductivity of ^{4}He near the Superfluid Transition in a Restricted Geometry

A. M. Kahn and Guenter Ahlers

Department of Physics and Center for Nonlinear Science, University of California, Santa Barbara, California 93106-9530

(Received 4 August 1994)

We report measurements of the thermal conductivity of ^{4}He at vapor pressure confined in a glass capillary array of thickness 3 mm with holes 2μm in diameter. The finite geometry rounds the superfluid transition, resulting in a finite thermal conductivity at the bulk transition temperature T_λ. Below T_λ the thermal conductivity becomes effectively infinite when the bulk correlation length becomes small compared to the size of the confining holes.

PACS numbers: 67.40.Pm, 64.60.Ht, 66.60.+a

Experiments on bulk helium have provided a number of tests of the modern theory of critical phase transitions in three dimensions [1]. However, critical behavior in restricted geometries is less well understood and continues to be an active area of interest [2,3]. The superfluid transition in liquid helium is particularly useful for studying the effects of confining geometries because the bulk transition has been carefully studied and, except for gravity rounding [4], has been shown to be sharp [5] within experimental resolution. While there have been several measurements of the finite-size effects on the *static* properties of ^{4}He such as the specific heat [6,7] and the superfluid density [6,8], to our knowledge no work has been done on transport in well-defined geometries. In this Letter we report measurements of the thermal conductivity λ of ^{4}He confined in a glass capillary array (GCA) of thickness 3 mm with holes 2 μm in diameter. In bulk helium, λ diverges as the bulk transition temperature T_λ is approached from above [5]. Although *detailed* theoretical calculations of finite-size effects upon λ have not yet been carried out, these seem possible in principle [9]. On *general* theoretical grounds, one would expect long cylindrical samples to show a transition from three-dimensional to one-dimensional behavior when the correlation length becomes comparable to the cylinder radius. Consistent with the expectation that the one-dimensional system has no phase transition, we find that the conductivity is finite at T_λ. The experiment suggests that λ diverges exponentially as T decreases below T_λ, without any singularity near T_λ.

For bulk helium the correlation length is given by $\xi = \xi_0 t^{-\nu}$, where ξ_0 is a length of atomic dimensions, $t = (T - T_\lambda)/T_\lambda$ is the reduced temperature, and $\nu = 0.6705$ is the critical exponent [10]. A temperature of merit is that for which the bulk correlation length becomes equal to the scale of the confining geometry. Using $\xi_0 = 3.4$ Å [11], ξ equals the radius of the pores in the GCA used in this work for $t = 7 \times 10^{-6}$.

Measurements of finite-size effects on transport require an especially uniform confining geometry, since there is a preferred orientation in transport measurements. Further, the presence of bulk helium can have a larger effect than in experiments on static properties because it can provide a thermal short below the bulk transition temperature. The GCA used in this work was manufactured by Galileo Electro-Optics Company and has been characterized [12] by density, gas-flow impedance, and optical microscopy measurements. The average hole diameter determined from these different techniques was 2.1, 2.1, and 1.8 μm, respectively [12]. The good agreement among these different measurements indicates that the holes are quite uniform. The GCA is approximately 60% open by volume, 13 mm in diameter, and 3 mm thick, giving a length/radius aspect ratio for each hole of approximately 3000. The holes are arranged in a hexagonal array and do not cross.

The sample cell is cylindrical, consisting of OFHC copper anvils at the top and bottom and a 0.1 mm thick stainless steel sidewall. There is a 1 mm epoxy layer between the GCA and the sidewall to ensure that there are no parallel conduction paths of helium between the GCA and the sidewall. If a parallel bulk helium path had existed, then the conductivity of the cell would have been effectively infinite below T_λ, severely limiting the range of this experiment. A separate bulk conductivity cell is attached to the cell bottom and has a fill line connected to the GCA cell, allowing us to measure the bulk λ point at the sample pressure with a precision of 30 nK. The bulk superfluid transition temperature depends on the vertical position due to the hydrostatic effect of gravity [4]. We chose to reference all of our data to the bulk transition temperature at the center of the sample cell, which resulted in a gravity correction of 1.54 ± 0.02 μK. All of our measurements were done at vapor pressure. The liquid-vapor interface was set in a separate overflow volume connected to and just above the cell, using a capacitive height gauge.

The cryostat is a modified version of one described previously [13]. The cooling power for the cryostat comes from a ^{4}He refrigerator at 1.4 K, with two subsequent stages of thermal isolation above the sample cell. The first of these stages is temperature controlled using germanium thermometry and the second stage, from which the sample cell is cooled, is temperature controlled at

 0031-9007/95/74(6)/944(4)$06.00

approximately 2.0 K and is held stable to within 10 nK rms using a ^{4}He melting curve thermometer [14]. A copper enclosure maintained at 2.2 K shields the cell from external radiation. The thermal conductivity is measured by temperature controlling the cell top and then measuring the change in the bottom temperature when a heat current is applied to the cell bottom. For each temperature the applied heat current is chosen to be small enough that nonlinear effects, due to a finite heat current and due to the variation in the conductivity along the length of the cell, are relatively small. The cell top was temperature controlled with a ^{4}He melting curve thermometer [14] and the bottom temperature was measured using a CAB magnetic susceptibility thermometer [12,15] and a germanium resistance thermometer.

The conductivity of the confined helium is given by $\lambda_f = (d/A)[1/(R - R_b) - C_w]$, where d is the cell thickness, A is the helium cross-sectional area, R is the measured thermal resistance, R_b is the boundary resistance, and C_w is the parallel conductance due to the sidewall, epoxy, and the glass in the GCA [16]. The boundary resistance R_b, which is only significant for temperatures near or below T_λ, was determined empirically by measuring the thermal resistance well below T_λ, where the helium resistance was negligible, as described below. The parallel conductance C_w and the geometric factor (d/A) were determined by doing a two-parameter fit to the known bulk helium conductivity for data 1.5 to 12 mK above T_λ. For this temperature range the correlation length is much smaller than the scale of the confining geometry and any finite-size effects should be tiny. The geometric factor, $d/A = 0.39\ \mathrm{cm}^{-1}$, determined from this fit agrees with the factor determined from room temperature measurements within the measurement accuracy.

Figure 1 shows a logarithmic plot of the thermal conductivity above T_λ versus reduced temperature, where the reduced temperature is taken relative to the bulk transition temperature T_λ. The open circles show the conductivity λ_f of the finite system. For reference the bulk conductivity data λ_b from cell F of Tam and Ahlers [17] is shown on the same plot as solid circles. Well above the transition λ_f is approximately equal to λ_b, but as T_λ is approached λ_f becomes smaller than λ_b due to the confining geometry. The confined helium conductivity does not diverge as T_λ is approached; instead it saturates at a finite value, indicating that T_λ is not a critical point for this system.

The thermal resistivity $r_f = 1/\lambda_f$ of the confined helium is plotted as open circles on a linear scale over two different temperature ranges in Fig. 2. The data for the bulk resistivity [17] r_b are shown as closed circles and the theoretical result for bulk helium by Dohm [18] (which had been fitted to the data of Ref. [17] for large t) is shown as a dashed line. Above T_λ, r_f is clearly higher than r_b. As the temperature is increased above T_λ,

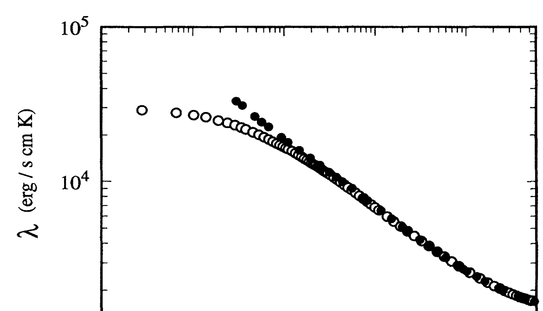

FIG. 1. Thermal conductivity versus reduced temperature on a log-log scale. The open circles are the data for helium confined in the GCA and the solid circles show the bulk helium data from cell F of Ref. [17]. For both data sets the reduced temperature is taken relative to the bulk transition point, T_λ.

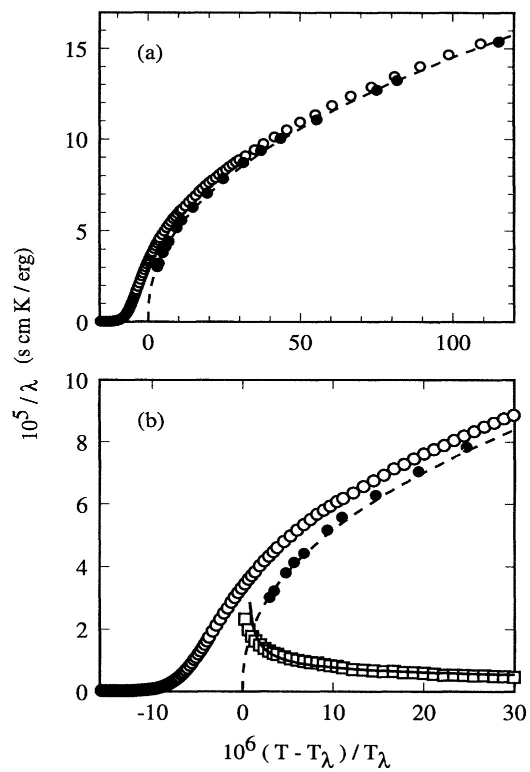

FIG. 2. Thermal resistivity versus reduced temperature on linear scales shown over two different temperature ranges. Open circles: data for helium confined in the GCA. Solid circles: bulk helium data from cell F of Ref. [17]. Dashed line: fit of the theoretical result for bulk helium by Dohm [18] to the data of Ref. [17] for large t. Squares in (b): the extra resistance due to confinement in the GCA, obtained by subtracting the dashed line from the open circles. Solid line in (b): the result of the model given by Eq. (1), with $a = 0.22$.

the effect of the finite geometry becomes smaller and r_f approaches r_b, as shown in Fig. 2(a).

The data in Fig. 2(b) show that r_f is finite at the bulk transition temperature and that there is no sharp phase transition. At a reduced temperature of approximately $t = -1 \times 10^{-5}$, r_f approaches zero on this scale and the slope approaches zero as well. This is in contrast to bulk helium for which the slope diverges at the critical point. The difference $\Delta r = r_f - r_b$ above T_λ is plotted as the open squares and represents the extra thermal resistance due to the confining geometry.

Since there are no theoretical calculations for the thermal resistance of helium in a confined geometry, we consider a simple model. We approximate the conductivity above T_λ in each capillary as the sum of two parts: a central core which has the bulk conductivity, and a layer along the walls of thickness proportional to ξ for which the conductivity is given by the nondivergent part of the bulk helium conductivity. This assumes that the order parameter vanishes at the walls, and that the effect of the walls scales with the correlation length [19]. The conductivity of the confined helium is then modeled as

$$\lambda_f = \frac{(R_c - a\xi)^2\lambda_b + [R_c^2 - (R_c - a\xi)^2]\lambda_{\infty 0}}{R_c^2}, \quad (1)$$

where R_c is the capillary radius, a is fitting parameter expected to be of order unity, and $\lambda_{\infty 0} = 1222$ ergs/s cm K [17] is the nondivergent part of the bulk conductivity. The solid line in Fig. 2(b) shows the resulting Δr obtained from this model using $a = 0.22$. Far enough above the transition, it fits the data well, but, as expected, this simple model breaks down as T_λ is approached and ξ becomes comparable to R_c.

Figure 3 shows the measured conductivity plotted on a log-linear scale. The open circles show the total measured conductivity, including the Kapitza resistance at the end plates and the resistance in the copper end pieces. As the temperature is decreased below T_λ, the measured conductivity increases dramatically and then saturates, presumably limited by the boundary resistance. While there has been work done on the temperature and power dependence of the Kapitza resistance for bulk helium [13], we do not know how this would be affected by the confining geometry. We therefore used a purely empirical approach to remove the boundary resistance [20], extrapolating a linear fit to the data for $-3.0 \times 10^{-5} < t < -1.8 \times 10^{-5}$ to estimate the boundary resistance for $t > -1.6 \times 10^{-5}$. Note that this boundary resistance is only significant compared to the confined helium resistance over a very limited range of temperatures below T_λ; at $t = 0$ the estimated boundary resistance is about 0.7% of the helium resistance.

The conductivity with the boundary resistance subtracted is shown as filled circles in Fig. 3. The scatter in the data increases at low temperatures as the helium resistance becomes small compared to the boundary resistance. The data suggest that, at low temperatures, the conductivity increases exponentially with decreasing temperature:

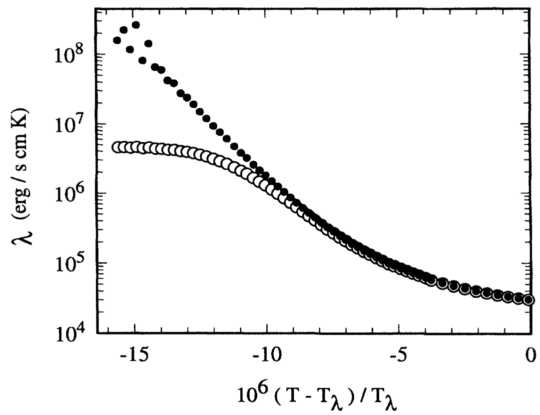

FIG. 3. Thermal conductivity versus reduced temperature on a log-linear scale. Open circles: measured thermal conductivity across the cell, including boundary resistances. Solid circles: effective conductivity with the boundary resistance subtracted out.

$$\lambda_f = \lambda_0 \exp(-t/t_0). \quad (2)$$

A fit to the data, shown in Fig. 3, for $-1.43 \times 10^{-5} < t < -1.08 \times 10^{-5}$ yields $\lambda_0 = 190$ ergs/s cm K and $t_0 = 1.11 \times 10^{-6}$. The correlation length corresponding to t_0 is 3.3 μm, which is of the scale of the confining holes in the GCA. We have tried fitting the boundary resistance using several different functional forms, and determined that the resulting value of t_0 varies by only a few percent.

We thank J. Maynard for providing us with the GCA sample, Melora Larson and Tim Sullivan for characterizing the sample at room temperature, and Feng-Chuan Liu for helpful discussions. This research was supported by the National Science Foundation through Grant No. DMR 92-17577.

[1] See, for example, G. Ahlers, in *The Physics of Liquid and Solid Helium,* edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Pt. I, Chap. 2; G. Ahlers and R. V. Duncan, in *Frontiers of Physics, Proceedings of the Landau Memorial Conference,* edited by E. Gotsman, Y. Ne'eman, and A. Voronel (Pergamon, Oxford, England, 1990).
[2] F. M. Gasparini and I. Rhee, in *Progress in Low Temperature Physics,* edited by D. F. Brewer (North-Holland, Amsterdam, 1990), Vol. XIII, p. 1.
[3] V. Dohm, Phys. Scrp. **T49**, 16 (1993).
[4] G. Ahlers, Phys. Rev. **171**, 275 (1968).
[5] G. Ahlers, Phys. Rev. Lett. **21**, 1159 (1968).
[6] T. P. Chen and F. M. Gasparini, Phys. Rev. Lett. **40**, 331 (1978); F. M. Gasparini, G. Agnolet, and J. D. Reppy, Phys. Rev. B **29**, 138 (1984).

[7] J. A. Nissen, T. C. P. Chui, and J. A. Lipa, J. Low Temp. Phys. **92**, 353 (1993); M. Coleman and J. A. Lipa (to be published).
[8] I. Rhee, F. M. Gasparini, and D. J. Bishop, Phys. Rev. Lett. **63**, 410 (1989).
[9] V. Dohm (private communication).
[10] L. S. Goldner and G. Ahlers, Phys. Rev. B **45**, 13 129 (1992).
[11] A. Singsaas and G. Ahlers, Phys. Rev. B **30**, 5103 (1984).
[12] M. Larson, Ph.D. thesis, U.C.S.B. Department of Physics, 1993.
[13] R. V. Duncan and G. Ahlers, Phys. Rev. B **43**, 7707 (1991).
[14] L S. Goldner, N. Mulders, and G. Ahlers, in *Temperature: Its Measurement and Control in Science and Industry,* edited by J. F. Schooly (American Institute of Physics, New York, 1992), Vol. 6.
[15] J. A. Lipa, B. C. Leslie, and T. C. Wallstrom, Physica (Amsterdam) **107B**, 331 (1981).
[16] This formula is based on the approximation that the entire heat current (including the part conducted by the sidewalls, epoxy, and glass in the GCA) encounters the boundary resistance R_b. This is an excellent approximation because at temperatures where R is sufficiently small for R_b to matter, the wall conduction C_w is negligible.
[17] W. Y. Tam and G. Ahlers, Phys. Rev. B **32**, 5932 (1985).
[18] V. Dohm, Z. Phys. B **61**, 193 (1985); Phys. Rev. B **44**, 2697 (1991).
[19] One other possible effect on the thermal conductivity near the bulk λ transition may arise from the finite height of our cell. This effect has been previously considered by G. Ahlers and R. V. Duncan [Phys. Rev. Lett. **61**, 846 (1988)], and by D. Frank and V. Dohm [Physica (Amsterdam) **165B**, 543 (1990)]. We expect this effect to be much smaller than that due to the lateral confinement of the GCA. In addition, it is unclear how the effect of the finite cell height may be altered due to the additional confinement in the lateral dimension imposed by the GCA. Hence, for the simple model that we consider here we neglect any additional finite-size effects due to the finite cell height.
[20] This correction also includes the resistance due to viscous drag at the walls of the GCA. Near T_λ we estimate this to result in a resistivity of 1.7×10^{-9} s cm K/erg, which is only approximately 1% of the total boundary correction we are making.