Plasma-Maser Instability of Electromagnetic Radiation in the Presence of Lower Hybrid Turbulence

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(Received 12 May 1994)

The generation mechanism of the electromagnetic radiation is studied based on the plasma-maser interaction among the electrostatic lower hybrid turbulence, accelerated electrons, and extraordinary mode radiation. The theory agrees with the most striking new features of the observation of the auroral kilometric radiation (AKR) in that the AKR bursts are observed at frequencies well above the local electron gyrofrequency. The theory also explains the close correlation between the lower hybrid turbulence and the microwave emission reported based on recent computer simulations.

PACS numbers: 52.35.Mw, 52.35.Hr, 52.35.Ra

According to recent progress in plasma turbulence theory, there is a new mode-mode coupling process called the plasma maser which is effective for energy up-conversion from the low-frequency mode to the high-frequency mode [1]. The plasma-maser effect occurs when nonresonant as well as resonant plasma oscillations are present. The resonant waves are those for which the Cherenkov resonance condition $\omega - \mathbf{k} \cdot \mathbf{v} = 0$ is satisfied, while the nonresonant waves are those for which both the Cherenkov and the scattering conditions are not satisfied, i.e., $\Omega - \mathbf{K} \cdot \mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K} - \mathbf{k}) \cdot \mathbf{v} \neq 0$. Here, ω and Ω are frequencies of the resonant and nonresonant waves, respectively, and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers [2]. The plasma-maser process is quite important in strongly magnetized plasmas, where $\Omega_e > \omega_{pe}$ [3]; Ω_e and ω_{pe} represent the electron gyrofrequency and electron plasma frequency, respectively. The energy up-conversion is quite effective because the standard Manley-Rowe relation is violated [4].

Recent observations by the Viking satellite [5] have reported that the sporadic auroral kilometric radiation (AKR) occurs well above the electron gyrofrequency together with electrostatic lower hybrid turbulence [6–9] and upward-going electron beams with broad spectra ($\sim$100 eV to 1 keV). The most striking new features lie in the fact that the AKR bursts are observed at frequencies often well above the local electron gyrofrequency up to $1.2\Omega_e$. These events do not satisfy the usual AKR cyclotron maser theory [10,11], because it predicts AKR is generated in the *RX* mode at frequencies just above the cutoff frequency of this mode, which is very close to Ω_e for $\Omega_e > \omega_{pe}$. Accordingly, an alternative theory is necessary to explain the above Viking observations of AKR.

A recent numerical simulation [12] shows that the accelerated electrons by the electrostatic lower hybrid waves generate electromagnetic radiation at the electron gyrofrequency. Unlike the electron cyclotron maser instability [10,11], which is driven by a loss cone-type anisotropy, this electromagnetic radiation is driven by an electron distribution with a nonthermal tail along the parallel direction to the external magnetic field. The generation mechanism of this microwave emission is not yet clear.

In this Letter, we investigate the plasma-maser interaction among the electrostatic lower hybrid (LH) turbulence, the accelerated electrons, and the electromagnetic radiation. Electrons whose velocity is close to the phase velocity of the LH turbulence are strongly accelerated. The acceleration occurs in two different ways [13], namely direct current (dc) and alternating current (ac) ones. The dc acceleration corresponds to the standard quasilinear process between beam electrons and the LH waves. On the other hand, the ac acceleration causes plasma-maser interaction which generates the electromagnetic radiation. It should be stressed that both accelerations always occur simultaneously.

Consider a homogeneous plasma in the presence of an enhanced electrostatic LH turbulence and an external magnetic field $B_0\hat{Z}$. The interaction of the turbulence fields with an electron which leads to X mode radiation test wave is governed by the Vlasov-Maxwell equations,

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left[\mathbf{E}_l + \delta\mathbf{E} + \frac{\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B})}{c}\right] \cdot \frac{\partial}{\partial \mathbf{v}}\right) f_e = 0, \quad (1)$$

$$\nabla \times \delta\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{B}, \quad (2)$$

$$\nabla \times \delta\mathbf{B} = +\frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E} + \frac{4\pi}{c}\mathbf{J}, \quad (3)$$

$$\mathbf{J} = -n_0 e \int \mathbf{v} f_e(\mathbf{r}, \mathbf{v}, t)\, d\mathbf{v}, \quad (4)$$

where $\mathbf{E}_l$ is the electrostatic LH turbulence field which is assumed to be in the Z and X direction. Here, $\delta\mathbf{E}(\mathbf{r}, t)$ and $\delta\mathbf{B}(\mathbf{r}, t)$ are the perturbed electric and magnetic fields due to the electromagnetic test wave, respectively. The high frequency electromagnetic X mode propagates in the X direction, which is perpendicular to B_0.

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Since the electrostatic LH turbulence is present in the system, the Fourier component of f_{1e} which shows the linear response to the electrostatic LH field $\mathbf{E}_l(\mathbf{k},\omega)$ with wave vector $\mathbf{k} = (k_\perp, 0, k_\parallel)$ is given by

$$f_{1e}(\mathbf{k},\omega) = \frac{e}{m}\sum_{p,q=-\infty}^{\infty} \frac{J_p(k_\perp v_\perp/\Omega_e)J_q(k_\perp v_\perp/\Omega_e)\exp[i(p-q)\phi]}{-i(\omega - k_\parallel v_\parallel - q\Omega_e + i0)}\left[\frac{q\Omega_e}{k_\perp v_\perp}E_{l\perp}(\mathbf{k},\omega)\frac{\partial}{\partial v_\perp} + E_{l\parallel}(\mathbf{k},\omega)\frac{\partial}{\partial v_\parallel}\right]f_{0e}, \quad (5)$$

where J_p is the Bessel function, f_{0e} is the unperturbed electron distribution function, and $+i0$ in the denominator ensures causality. The symbols $\perp$ and $\parallel$ represent perpendicular and parallel to the external magnetic field, respectively.

To investigate the problem of the plasma maser from a quasisteady turbulent plasma with enhanced electrostatic LH wave, we perturb the steady state by introducing high-frequency electromagnetic X mode fields $\delta\mathbf{E}(\mathbf{K},\Omega)$ and $\delta\mathbf{B}(\mathbf{K},\Omega)$ in the system with $\mathbf{K} = (K_\perp, 0, 0)$. According to the well-established method, after a lengthy but straightforward calculation, we obtain the effective dielectric constant of the X mode $[D(\mathbf{K},\Omega)]$ in the presence of the electrostatic LH stationary turbulence as

$$D(\mathbf{K},\Omega) = D_0(\mathbf{K},\Omega) + D_p(\mathbf{K},\Omega), \quad (6)$$

where $D_0(\mathbf{K},\Omega)$ is the linear dielectric function of the X mode given by

$$D_0(\mathbf{K},\Omega) = K_\perp^2 - \left(\frac{\Omega}{c}\right)^2 + \frac{\omega_{pe}^2\Omega}{c^2}\sum_n \int d\mathbf{v}\,\frac{v_\perp (J_n')^2}{n\Omega_e - \Omega}\frac{\partial f_{0e}}{\partial v_\perp}. \quad (7)$$

Here we discuss the validity of Eq. (7). If we consider the wave propagates perpendicular to $B_0\hat{Z}$ with the isotropic Maxwell distribution for f_{0e}, the dispersion relation reduces to

$$\begin{vmatrix} D_{xx} & D_{xy} & 0 \\ -D_{xy} & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{vmatrix} = 0. \quad (8)$$

Although the energetic beam electrons give a current along the field lines, the above Eq. (8) is approximately valid if the current is weak. Thus, the disturbance can be split into two mixed longitudinal and transverse modes (generalized X mode) and one pure transverse 0 mode. Furthermore, when $c^2K_\perp^2 \gg \omega_{pe}^2$, D_{xy} is much less than D_{yy} and the dispersion relation for the X mode is simply given by $D_{yy} = 0$ [14].

The dominant nonlinear plasma-maser contribution comes from the polarization term $[D_p(\mathbf{K},\Omega)]$ which is given by

$$D_p(\mathbf{K},\Omega) = \frac{\omega_{pe}^2\Omega}{c^4}\left[\frac{e^2}{m^3}\right]\sum_{\mathbf{k},\omega}\frac{4\pi e^2(\Omega-\omega)}{D_0(\mathbf{K}-\mathbf{k},\Omega-\omega)}[(A+B)(C+D)], \quad (9)$$

where

$$A = \sum_{\substack{s,t,a\\ b=-\infty}}^{\infty}\int \frac{v_y J_s(K_\perp v_\perp/\Omega_e)\exp[i(s-t)\phi]}{i(t\Omega_e-\Omega)}\hat{M}_t(\mathbf{k},\omega,K_\perp v_\perp/\Omega_e)$$
$$\times \frac{J_a(K'_\perp v_\perp/\Omega_e)\exp[i(a-b)\phi]}{b\Omega_e - k_\parallel v_\parallel - (\Omega-\omega)}\,\frac{J_{b+1}(K'_\perp v_\perp/\Omega_e) - J_{b-1}(K'_\perp v_\perp/\Omega_e)}{2}$$
$$\times\left\{\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{\partial}{\partial v_\perp} - \frac{k_\parallel v_\perp}{\Omega-\omega}\frac{\partial}{\partial v_\parallel}\right\}f_{0e}\,d\boldsymbol{v}, \quad (10)$$

$$B = \sum_{\substack{s,t,p\\ q=-\infty}}^{\infty}\int \frac{v_y J_s(K_\perp v_\perp/\Omega_e)\exp[i(s-t)\phi]}{i(t\Omega_e-\Omega)}\Bigg[\frac{J_{t+1}(K_\perp v_\perp/\Omega_e) - J_{t-1}(K_\perp v_\perp/\Omega_e)}{2}$$
$$\times\left\{\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{\partial}{\partial v_\perp} - \frac{k_\parallel v_\perp}{\Omega-\omega}\frac{\partial}{\partial v_\parallel}\right\} + i\left\{\frac{J_{t+1}(K_\perp v_\perp/\Omega_e) + J_{t-1}(K_\perp v_\perp/\Omega_e)}{2}\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{1}{v_\perp}\right.$$
$$\left. - \frac{K'_\perp J_t(K_\perp v_\perp/\Omega_e)}{\Omega-\omega}\right\}\frac{\partial}{\partial\phi}\Bigg]\left[\frac{im}{e}f_{1e}(\mathbf{k},\omega)\right], \quad (11)$$

$$C = A[\Omega \longleftrightarrow \Omega-\omega,\quad \mathbf{K}\longleftrightarrow \mathbf{K}-\mathbf{k};\quad \omega\longleftrightarrow -\omega,\quad \mathbf{k}\longleftrightarrow -\mathbf{k}], \quad (12)$$

$$D = B[\Omega \longleftrightarrow \Omega-\omega,\quad \mathbf{K}\longleftrightarrow \mathbf{K}-\mathbf{k};\quad \omega\longleftrightarrow -\omega,\quad \mathbf{k}\longleftrightarrow -\mathbf{k}], \quad (13)$$

and

$$\hat{M}_t(\mathbf{k},\omega,K_\perp v_\perp/\Omega_e) = \Bigg\{E_{l\perp}(\mathbf{k},\omega)\,\frac{J_{t+1}(K_\perp v_\perp/\Omega_e) + J_{t-1}(K_\perp v_\perp/\Omega_e)}{2}\,\frac{\partial}{\partial v_\perp} + E_{l\parallel}(\mathbf{k},\omega)J_t(K_\perp v_\perp/\Omega_e)\,\frac{\partial}{\partial v_\parallel} - \frac{1}{v_\perp}\,E_{l\perp}(\mathbf{k},\omega)\,\frac{J_{t+1}(K_\perp v_\perp/\Omega_e) - J_{t-1}(K_\perp v_\perp/\Omega_e)}{2i}\,\frac{\partial}{\partial\phi}\Bigg\}, \tag{14}$$

where $K'_\perp = K_\perp - k_\perp$. In deriving Eq. (6), we use integration along the unperturbed orbit, that is $v_x = v_\perp\cos\phi$, $v_y = v_\perp\sin\phi$, and $v_z = v_\parallel$, where ϕ is the azimuthal angle.

Now we consider the generation mechanism of the AKR [5]. The electron distribution function is modeled by

$$f_{0e}(\mathbf{v}) = n_c\left(\frac{m}{2\pi T_c}\right)^{3/2}\exp\left[-\frac{m(v_\perp^2 + v_\parallel^2)}{2T_c}\right] + n_h\left(\frac{m}{2\pi T_h}\right)^{3/2}\exp\left[-\frac{m\{v_\perp^2 + (v_\parallel - v_0)^2\}}{2T_h}\right]. \tag{15}$$

Here n_c, n_h, T_c, and T_h are the number densities and temperatures for the background cold plasma and hot component with a shifted Maxwellian which drives the electrostatic LH waves [6–9].

The resonant interaction between the LH turbulence and electrons occurs for the Cherenkov resonance condition $\omega - k_\parallel v_\parallel = 0$, because $\omega \ll \Omega_e$. Thus, we keep only terms with $p = q = 0$ in the summation of Eqs. (11) and (13). The most dominant contribution of Eqs. (10)–(13) comes from $t = \pm 1$ and $b = \pm 1$ terms in the summation. After a lengthy but straightforward calculation, we get, from Eq. (9), the dominant plasma-maser contribution as

$$\mathrm{Im}\,D_p(\mathbf{K},\Omega) = \sum_{\mathbf{k},\omega} -\frac{\pi^{1/2}\delta\,\omega_{pe}^4\Omega^3(\Omega-\omega)e^2K_\perp|E_{l\parallel}(\mathbf{k},\omega)|^2}{mc^2(\Omega^2-\Omega_e^2)^4\,|k_\parallel|T_h}\left(\frac{\omega - k_\parallel v_0}{k_\parallel v_h}\right)\exp\left\{-\frac{m(\omega/k_\parallel - v_0)^2}{2T_h}\right\}\left[\frac{E_{l\perp}(-\mathbf{k},-\omega)}{E_{l\parallel}(-\mathbf{k},-\omega)} \times\left\{1 - \frac{K'_\perp(\Omega^2+\Omega_e^2)T_c}{\Omega\,\Omega_e^2 m}\left(\frac{k_\parallel}{\omega}\right)\frac{E_{l\parallel}(-\mathbf{k},-\omega)}{E_{l\perp}(-\mathbf{k},-\omega)}\right\} - \frac{K'_\perp(\Omega^2+\Omega_e^2)E_{l\perp}(\mathbf{k},\omega)}{2K_\perp\Omega^2E_{l\parallel}(\mathbf{k},\omega)}\right], \tag{16}$$

where Im is the imaginary part, $T_h = mv_h^2/2$, and $\delta = n_h/n_c < 1$.

For the long wavelength mode, all terms in the summation except $n = \pm 1$ are small in Eq. (7). Taking the small argument expansion of the Bessel function, we get

$$D_0(K,\Omega) = K_\perp^2 - \left(\frac{\Omega}{c}\right)^2 - \frac{\omega_{pe}^2\Omega^2}{c^2(\Omega_e+\Omega)(\Omega_e-\Omega)} = 0. \tag{17}$$

For $\Omega \geqq cK_\perp > \omega_{pe}$ and $\Omega_e > \omega_{pe}$ which are satisfied for the AKR, we obtain from Eq. (17) in the vicinity of $\Omega = \Omega_e$ ($|\Delta| \ll 1$)

$$\Omega = \Omega_e(1+\Delta), \tag{18}$$

with

$$\Delta = \frac{\omega_{pe}^2}{2(\Omega_e^2 - c^2K_\perp^2)},$$

and

$$\frac{\partial D_0(\mathbf{K},\Omega)}{\partial\Omega} = -\frac{2\omega_{pe}^2\Omega^3}{c^2(\Omega^2-\Omega_e^2)^2} - \frac{2\Omega}{c^2}. \tag{19}$$

The growth rate of the X mode (γ) is given by

$$\gamma = -\frac{\mathrm{Im}D_p(\mathbf{K},\Omega)}{\partial D_0/\partial\Omega|_{\Omega=\Omega_r}}. \tag{20}$$

Inserting Eqs. (16), (18), and (19) with $|\omega/k_\parallel - v_0| \simeq v_h$ into Eq. (20), we obtain

$$\frac{\gamma}{\Omega_e} \simeq 2\pi^{1/2}\delta\left[1 - \left(\frac{cK_\perp}{\Omega_e}\right)^2\right]^2 \times\frac{1+\Delta}{(2+\Delta)^2(1+[(2+\Delta)^2\Delta^2/(1+\Delta)^2](\Omega_e/\omega_{pe})^2)} \times\sum_{\mathbf{k},\omega}\frac{|\mathbf{E}_{l\perp}(\mathbf{k},\omega)|^2}{4\pi n_cT_h}\left(\frac{K_\perp}{k_\perp}\right). \tag{21}$$

In obtaining Eq. (21), the relation $E_{l\perp}(\mathbf{k},\omega)/E_{l\parallel}(\mathbf{k},\omega) = k_\perp/k_\parallel$ is used. According to the observations in space [5], we have $n_c \approx 8\ \mathrm{cm}^{-3}$, $T_h \approx 100$ eV, $|\mathbf{E}_{l\perp}(\mathbf{k},\omega)| \approx 50$ mV/m, $K_\perp \approx 2\pi\times 10^{-5}\ \mathrm{cm}^{-1}$, $k_\perp \approx k_\parallel\,(M/m)^{1/2} \simeq (\omega/v_e)(M/m)^{1/2} \approx 2\pi\times10^{-5}$, and $\delta \approx 10^{-1}$. Inserting these values in Eq. (21) with $\Delta \ll 1$, we find $\gamma/\Omega_e \approx 10^{-5}$, which is strong enough to generate the AKR. Although the maximum growth rate occurs for $\Omega \approx \Omega_e$, the growth of the AKR is possible at frequencies above Ω_e, because the plasma maser does not require any matching conditions among different wave numbers and frequencies. However, we must acknowledge that both expressions for the real frequency [Eq. (18)] and growth rate [Eq. (21)] are not valid for values of δ as high as 0.2. The evaluation of the growth rate using exact solutions of Eq. (17) is left for a future study.

We now discuss other AKR generation mechanisms. The cyclotron maser mechanism works if and only if relativistic electrons satisfy $\Omega - K_\parallel v_\parallel - \Omega_e/\gamma = 0$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. The relativistic correction term provides a mechanism which couples the perpendicular electron motion $v_\perp^2$ with the waves and allows the free energy perpendicular to the magnetic field to be transferred to AKR. In contrast to the cyclotron maser, the plasma maser works even for nonrelativistic electrons, and the free energy to be transferred to AKR comes from electrons accelerated by LH turbulence along the magnetic field. According to the observations [5], the very strong ($\geqq 100$ mV/m) amplitudes of LH emissions are often reported. Under such conditions, AKR generation due to solitons is attractive. The generation mechanisms of AKR based on the whistler soliton, upper hybrid wave [15] and upper hybrid wave, lower hybrid soliton [16] interactions have been proposed.

Here we consider the numerical simulation [12]. The LH turbulence are driven by ion ring distribution. The electron distribution function accelerated by the LH waves can be modeled by the bi-Maxwellian

$$f_{0e}(\mathbf{v}) = N\left(\frac{m}{2\pi T_\parallel}\right)^{1/2}\left(\frac{m}{2\pi T_\perp}\right) \times \exp\left[-\frac{mv_\parallel^2}{2T_\parallel}\right]\exp\left[-\frac{mv_\perp^2}{2T_\perp}\right], \tag{22}$$

in which $T_\parallel$ and $T_\perp$ ($\ll T_\parallel$) are temperatures for parallel and perpendicular directions. Associated with the dc acceleration by the waves, the electron distribution function is highly anisotropic. Inserting Eq. (22) with $\Omega_e/\omega_{pe} \simeq 2$ into Eqs. (7), (9), and (20), we obtain

$$\frac{\gamma}{\Omega_e} \simeq \sum_{\mathbf{k},\omega} \frac{|\mathbf{E}_{l\perp}(\mathbf{k},\omega)|^2}{4\pi N T_\parallel}\left(\frac{K_\perp}{k_\perp}\right). \tag{23}$$

Accordingly, the extraordinary mode grows and the growth rate is much enhanced because all the accelerated background electrons take part in the wave emission. It should be mentioned that the electromagnetic emission without electron population inversion [Eq. (22)] is the most important characteristic of the plasma maser. Strictly speaking, the analysis including the relativistic effect can be necessary for the detailed comparison between theory and simulation results which is left for future study.

The plasma-maser mechanism can successfully explain most of the characteristics of the observation by the Viking satellite and the recent numerical simulation. First, the theory predicts that the AKR can be generated by the LH turbulence which causes the effective electron ac acceleration along the magnetic field. Second, the plasma maser is effective well above the local electron gyrofrequency, because it does not require any matching conditions between the AKR and LH waves. On the other hand, the standard cyclotron maser mechanism [10,11] predicts that the generation is effective only at frequencies just above the cutoff frequency of the X mode which is very close to Ω_e. Third, the plasma maser predicts that the accelerated electrons along the parallel direction produce bursts of high frequency radiation near the electron gyrofrequency even for the monotonic decreasing electron distribution function. The free energy for the radiation exists in the parallel direction, in contrast to the cyclotron maser mechanism. The plasma-maser instability has a potential importance to interpret numerous observations in space [1], in the laboratory [17], and in numerical simulations [18].

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