Vortex Mutual Friction in Rotating Superfluid ^{3}He-*B*

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We have measured the resistive and reactive mutual friction coefficients B and B' in rotating superfluid ^{3}He-B at pressures of 1.6, 10, 20, and 29.3 bars. Near T_c, $B\rho_n/\rho$ diverges as $(T_c - T)^a$, with $a \approx -0.7$, and $B'\rho_n/\rho - 2$ tends to zero. Coefficients $d_\parallel$ and $d_\perp$ that relate the force to $\mathbf{v}_n - \mathbf{v}_L$, the relative velocity of normal fluid and vortex lines show relaxation-dissipation behavior tending slightly towards resonant dissipation. Within our experimental error there is no clear indication of the expected vortex core transition in our data at 20 and 29.3 bars.

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In a rotating superfluid in the interaction of quantized vortices with the normal fluid (or thermal excitations) manifests itself as a force of mutual friction between the two fluids. In equilibrium at angular velocity Ω the force $\mathbf{F}_{ns}$ on unit volume of normal fluid is

$$\mathbf{F}_{ns} = B\frac{\rho_n \rho_s}{\rho}\Omega(\mathbf{v}_s - \mathbf{v}_n)_\perp - B'\frac{\rho_n \rho_s}{\rho}\Omega \times (\mathbf{v}_s - \mathbf{v}_n), \qquad (1)$$

where the suffix $\perp$ denotes a component perpendicular to Ω. In a type II superconductor there is a similar mutual friction force mediated by quantized flux lines; in this case the resistive and reactive forces represented by the dimensionless coefficients B and B' contribute to the longitudinal resistivity and Hall effect, respectively, in the flux flow state. There has recently been considerable interest in the change of sign of the Hall effect for high-T_c superconductors, observed on cooling through T_c in an applied field [1]. Mutual friction gives a contribution to the Hall effect of the appropriate sign to explain this if $B'\rho_n/\rho - 2 > 0$; we show below that the opposite inequality holds in ^{3}He-B, as predicted by Kopnin, Ivlev, and Kalatsky [2] for superconductors with an isotropic Fermi surface.

Mutual friction has been extensively studied in ^{4}He [3], but so far there are only limited measurements of B in superfluid ^{3}He [4,5]. The present experiments are the first in ^{3}He designed to measure B' as well, and thus obtain complete information. We report here the results of an extensive series of measurements on the B-phase; measurements so far in the A phase are markedly less reproducible, presumably because of textural problems. Theory [6] predicts that for singular vortices the friction should be dominated by the interaction between excitations bound to the vortex core and free excitations. Consequently, the mutual friction is expected to provide valuable information on vortex core structures.

Our experimental cell, which is mounted on the nuclear refrigeration stage of a rotating cryostat [7], is shown very schematically in Fig. 1. A circular Kapton diaphragm separated two disk-shaped regions of liquid, each nominally 100 μm thick. The roof of the cell has six electrodes set into it by means of which the modes of the diaphragm may be driven and detected electrostatically. The modes that are of interest are those with a single nodal line along the diameter. Motion of the diaphragm then displaces superfluid as indicated in the figure, while the normal fluid is held at rest by its viscosity to a very good approximation. The frequency of these modes is of order 50 Hz at $T = 0$ and varies with temperature and pressure as $\rho_s^{1/2}/\rho$. We have shown elsewhere [8,9] that the mode frequencies and the dissipation in the nonrotating state are well understood. Ideally, there are two degenerate modes with a single nodal line; in practice anisotropy of tension produces two well-defined orthogonal modes, with nodal lines along the x and y axes, say, and with frequencies ω_x and ω_y differing by 3.2%. The essential idea of our experiment is that, in the presence of a vortex lattice, the B force produces extra damping (i.e., increased bandwidth) of the resonances and the B' force, perpendicular to $\mathbf{v}_s - \mathbf{v}_n$, produces coupling between the x and y modes. Solution of the equations of motion of the fluid and diaphragm [9] shows that the response of a particular electrode to forces F_x and F_y driving the two modes at angular frequency ω may be

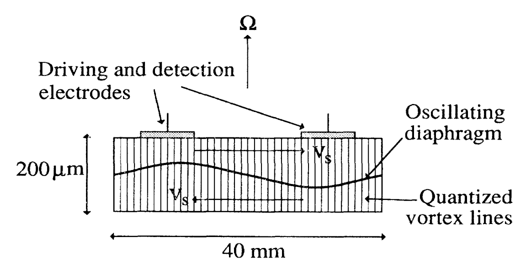

FIG. 1. Schematic diagram of the experimental cell, height greatly exaggerated. The aluminized Kapton film is clamped between two machined copper plates, each of which has a small hole on axis allowing the experimental helium to make thermal contact with the nuclear stage. Since these holes are on the nodal lines, they do not perturb the experiment.

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written as

$$V = \frac{A_x}{\omega_x - \omega + i\omega_{Bx}/2}\left(F_x - \frac{i(B'\rho_n/\rho - 2)\mu\Omega F_y}{\omega_y - \omega + i\omega_{By}/2}\right) + \frac{A_y}{\omega_y - \omega + i\omega_{By}/2} \times \left(F_y + \frac{i(B'\rho_n/\rho - 2)\mu\Omega F_x}{\omega_x - \omega + i\omega_{Bx}/2}\right), \quad (2)$$

where the bandwidths are given by

$$\omega_{Bi} = \omega_{B0i} + (B\rho_n/\rho)\Omega \quad (i = x, y), \quad (3)$$

with ω_{B0i} the bandwidth in the absence of rotation; A_x and A_y denote the coupling strengths of the modes to the detection electrodes, and $\mu = 0.350\,684$ is a combination of Bessel functions expressing the efficiency of mode coupling. Thus even if only the x mode is driven ($F_y = 0$), motion in the y mode appears on rotation due to the cross coupling. Rotation produces coupling of the modes even if $B' = 0$ through the Coriolis force represented by the –2 in $B'\rho_n/\rho - 2$.

Our experimental procedure is to observe, on separate lock-in amplifiers, the response of two different electrodes as the frequency is swept through the x and y resonances. This is done for two different drive configurations in which the same two drive electrodes are used but the ratio of voltages applied to them is varied so as to produce two very different admixtures of F_x and F_y. By fitting with Eq. (2) the data obtained with the cryostat rotating and with it stationary at the same temperature, all the electrode sensitivity parameters implicit in this equation together with the mutual friction constants can be obtained with good precision. For rotating measurements the cryostat was first rotated briefly at a speed greater than that used for measurement, to ensure an equilibrium array of vortices. Further details of the experimental procedure and data analysis together with the theory of the diaphragm modes are being published elsewhere [9].

We have made measurements at pressures of 1.6, 10, 20, and 29.3 bars. Thermometry was by LCMN susceptibility, calibrated against $T_c(p)$ using Greywall's [10] temperature scale. The dependence on reduced temperature of the measured friction parameters $B\rho_n/\rho$ and $B'\rho_n/\rho - 2$ is very similar at all pressures; the data for 1.6 bars are shown in Fig. 2. At low temperatures (where $\rho_n/\rho \to 0$) $B\rho_n/\rho \to 0$ and $B'\rho_n/\rho - 2 \to -2$, the Coriolis value, as expected. If the temperature dependence associated with ρ_n/ρ is removed, then at our lowest temperatures ($T/T_c \lesssim 0.4$) B and B' are both of order unity; B is roughly constant but B' is still probably decreasing with decreasing temperature. Our measurements of B extend closest to T_c at 1.6 bars pressure, and the inset to Fig. 2(a) shows the asymptotic behavior near T_c, $B\rho_n/\rho \propto (T_c - T)^a$, with $a = -0.76 \pm 0.02$. The small value of the error, obtained from the scatter of the data about the straight line fit,

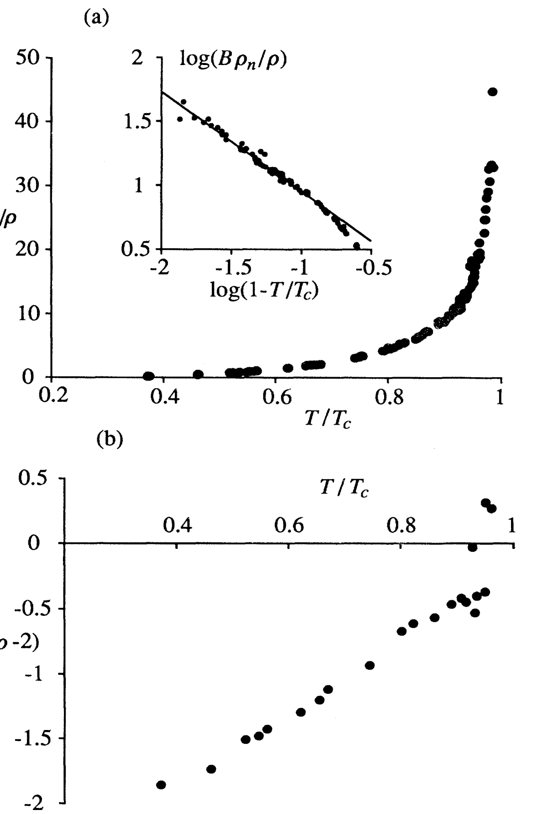

FIG. 2. Measured values of (a) $B\rho_n/\rho$ and (b) $B'\rho_n/\rho - 2$ at 1.6 bars. The inset in (a) shows the temperature dependence near to T_c; the line has a slope of -0.76.

should not be taken too seriously. Our 10 and 20 bars measurements, although not extending so close to T_c, give values of a of -0.62 and -0.67, respectively; also there is some evidence for smaller (i.e., less negative) values of a for data taken at low drive levels. The large dissipation near T_c means that measurements of the reactive coefficient cannot be made very close to T_c; however, Fig. 2(b) shows that $B'\rho_n/\rho - 2$ is certainly small close to T_c and probably tends to zero at T_c. Note that this is very different from the behavior observed in ^{4}He, where B and B' both diverge approximately as $(T_\lambda - T)^{-1/3}$ [11]. Sonin [12] has explained the behavior of ^{4}He near T_λ using an order parameter relaxation theory with appropriate critical exponents. A corresponding theory for ^{3}He near T_c has not been given; indeed, in contrast to our observations, Pitaevskii [13] claims that both B and B' remain finite at T_c in Ginzburg-Landau theory.

B-phase measurements at 29.3 bars are shown in Fig. 3; the values of $B\rho_n/\rho$ are slightly larger on average than those in Ref. [5] but are essentially within the large scatter of the Helsinki data. We observe no significant indication of the expected vortex core transition [14] at $T \sim 0.6T_c$,

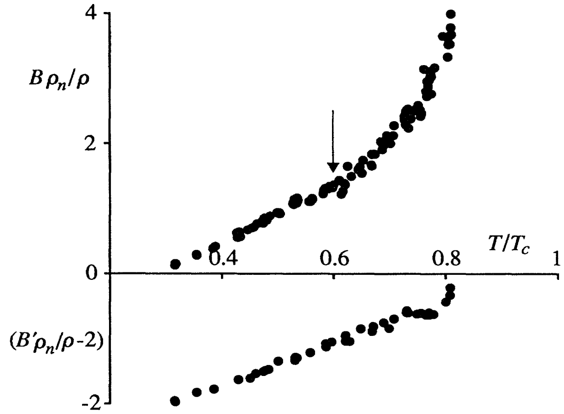

FIG. 3. Measured values of $B\rho_n/\rho$ and $B'\rho_n/\rho - 2$ at 29.3 bars. The arrow indicates the temperature at which the vortex core transition was expected.

although there could be a discontinuity in $B\rho_n/\rho$ of about 10% at $T \sim 0.55T_c$. At this pressure we have also made continuous measurements of resonance amplitude during a temperature sweep while rotating; we saw no indication of a sudden change in amplitude on either falling or rising temperature. Either the mutual friction is not sensitive to this transition or the transition is modified or suppressed in a 100 μm thick slab; we plan NMR experiments in a comparable geometry to check this point. We also see no sign of the transition in our 20 bars data.

At 29.3 bars we applied magnetic fields up to about 35 mT parallel and anitparallel to the rotation and found no effect on the B-phase data in contrast with the interpretation of some of the experiments in Ref. [5]. Measured values of B and B' were unaffected by variation of the oscillation amplitude by a factor of 20 at 20 bars except for a small (<10%) reduction in B at the lowest drive near T_c; we take this as an indication that the effects of vortex pinning are unimportant, since pinning forces are likely to be nonlinear. Measurements on the A phase at 29.3 bars are so far not reproducible, presumably because of textural problems; we can say only that $B\rho_n/\rho$ is of order 9 and not strongly temperature dependent.

To compare our measurements most directly with theory it is convenient to relate the mutual friction to $\mathbf{v}_n - \mathbf{v}_L$, where $\mathbf{v}_L$ is the velocity of the vortex lines. For a vortex line to be in equilibrium it must move in such a way that the Magnus force due to its motion through the superfluid just balances the force on it due to interaction with the normal fluid. This balance of forces can be written, for vortices of circulation κ along $\hat{\mathbf{z}}$ as

$$D(\mathbf{v}_n - \mathbf{v}_L)_\perp + D'\hat{\mathbf{z}}(\mathbf{v}_n - \mathbf{v}_L) + \rho_s\kappa\hat{\mathbf{z}}(\mathbf{v}_n - \mathbf{v}_L) = 0\,, \tag{4}$$

in which the last term is the Magnus force per unit length of line and the constants D and D' describe the interaction with the normal fluid [15]. Note that the large viscosity of ^{3}He means that $\mathbf{v}_n$ at a vortex is the same as the average value, so that the complications of inhomogeneous $\mathbf{v}_n$ that arise in ^{4}He are absent in ^{3}He. If we define dimensionless parameters $d_\parallel = D/\rho_s\kappa$ and $d_\perp = D'/\rho_s\kappa$ and use the fact that in rotational equilibrium there are $2\Omega/\kappa$ lines per unit area, comparison with Eq. (1) yields the relations

$$d_\parallel = \frac{B\rho_n/2\rho}{(B\rho_n/2\rho)^2 + (B'\rho_n/2\rho - 1)^2}\,, \tag{5a}$$

$$d_\perp - 1 = \frac{B'\rho_n/2\rho - 1}{(B\rho_n/2\rho)^2 + (B'\rho_n/2\rho - 1)^2}\,, \tag{5b}$$

thus showing that $d_\parallel$ and $d_\perp$ can be determined directly from our measured quantities.

Figure 4 shows values of $d_\parallel$ and $d_\perp - 1$ calculated from Eqs. (5) for all pressures. The maximum value of $d_\parallel$ is weakly pressure dependent, but the general shape at all pressures is of a maximum in dissipation at $T \sim 0.6T_c$ accomplished by a rapid change in the reactive coefficient; this is reminiscent of a relaxation process. The calculations of Kopnin and Salomaa [6] for ^{3}He-B as modified and extended by Kopnin [16] give

$$d_\parallel = \left\langle \frac{\omega_0\tau}{\omega_0^2\tau^2 + 1} \right\rangle, \quad d_\perp - 1 = \left\langle \frac{-\omega_0^2\tau^2}{\omega_0^2\tau^2 + 1} \right\rangle, \tag{6}$$

where the average is over a spectrum of values of ω_0, the level spacing of core excitations. Since the quasiparticle relaxation time τ is large at low temperatures and small near T_c, our data in Fig. 4 resemble this prediction qualitatively. However, in contrast with our observations, Eqs. (6) give a maximum $d_\parallel \le 0.5$ and $-1 < d_\perp - 1 < 0$. Scattering of free excitations by the long range potential is predicted to be usually negligible, but it does give a contribution $D' = -\rho_n\kappa$ in the low-temperature limit; there is perhaps an indication of this in our data, which show $d_\perp < 0$ for $T < 0.48T_c$. Near T_c our data give $|d_\perp - 1| \ll d_\parallel$, referred to by Kopnin [16] as the

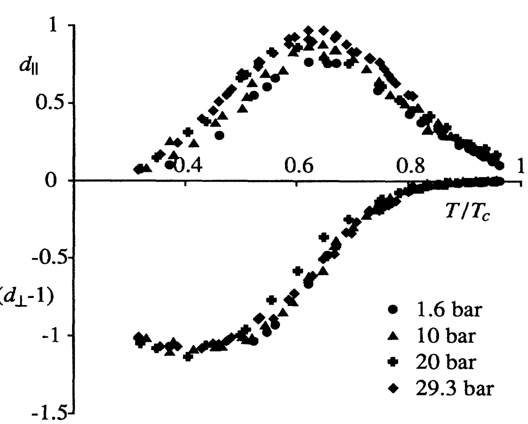

FIG. 4. Temperature dependence of $d_\parallel$ and $d_\perp$.

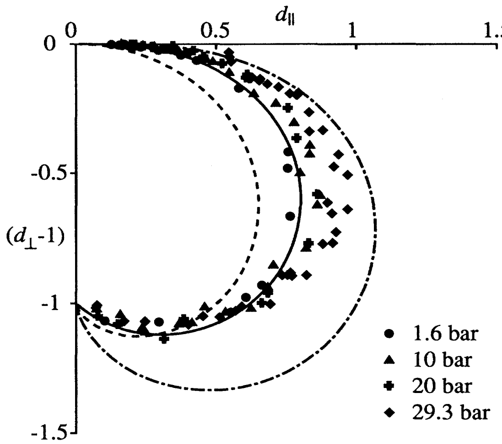

FIG. 5. Relation between $d_{\parallel}$ and $d_{\perp}$ compared with Eq. (7). Chain curve, $\alpha = 1$; dashed curve, $\alpha = 0.5$; full curve, interpolation $\alpha^2 = (1 + x)^{-1}$.

strong friction regime. This contrasts with the behavior of ^{4}He where $1 - d_{\perp} \propto d_{\parallel} \propto (T_{\lambda} - T)^{1/3}$ near T_{λ}.

Figure 5 shows the relationship between $d_{\parallel}$ and $d_{\perp}$, with temperature as a dummy variable. This type of plot gives a semicircle for a simple relaxation process and a large circle for a sharp resonance. Note that our experimental points lie entirely outside the relaxation semicircle, whereas Eqs. (6) give a curve inside it. We therefore attempt to model our data by the formula

$$d_{\perp} + id_{\parallel} = 1/(1 - \alpha^2 x^2 - ix), \tag{7}$$

where x corresponds to $\omega_0 \tau$ in Refs. [6] and [16]. Equation (7) has the same form as the response of a forced simple harmonic oscillator, and our approach is motivated by an analogy with the transition of an oscillator from relaxation to resonant behavior as inertia is introduced. But since ω_0 in Ref. [16] is the energy spacing of the excitation states in the vortex core, not a driving frequency, the physical interpretation of Eq. (7) remains to be clarified. Nevertheless, it provides a convenient way to summarize the departure of our data from the simple relaxation model $\alpha = 0$. The outermost curve in Fig. 5 is for $\alpha = 1$, which is the largest value of α for which $d_{\perp} \leq 1$ always. It is interesting to note that all our data lie inside this curve, though they do suggest that $\alpha \rightarrow 1$ at T_c; indeed the 29.3 bars data suggest that $\alpha \rightarrow 1$ at T_{AB} at this pressure. The curve for $\alpha = 0.5$ makes it clear that our data correspond to small α at low temperatures. We have therefore tried as an interpolation $\alpha^2 = (1 + x)^{-1}$; this gives the full curve in Fig. 5, which fits the 1.6 bars data remarkably well. Clearly, the dependence of our data on pressure can be described by small changes in the function $\alpha^2(x)$. Note that this interpolation agrees with the theoretical prediction [16] that in the low-temperature limit $d_{\parallel} \sim -d_{\perp} = (\rho_n/\rho) \sim (\omega_0 \tau)^{-1}$.

Our data thus agree qualitatively with Ref. [16] both in the low-temperature limit and in the approach to a "strong friction" regime near T_c, but existing theory gives no hint of how it is possible to have $d_{\parallel} > 0.5$ at intermediate temperatures. The intriguing observation that $\alpha \rightarrow 1$ at T_c shows that $|d_{\perp} - 1|$ goes to zero at T_c more rapidly than $d_{\parallel}^2$; the strong friction regime is thus approached very rapidly. This is a consequence of the limiting behavior $B\rho_n/\rho \rightarrow \infty$ and $B'\rho_n/\rho \rightarrow 2$ as $T \rightarrow T_c$. Why this is so remains a challenge to theorists, as do the value of the critical exponent for the divergence of B and the explanation of why the vortex core transition is not seen in our measurements; the theory [16] suggests that the mutual friction should be very sensitive to the core structure.

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