Staged Z Pinch

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We consider a staged Z pinch in which an annular plasma implodes onto a coaxial wire. The physical model involves conservation of magnetic flux created by the current initially trapped in the wire plasma and adiabatic heating. It leads to magnetic-field and current amplification with a current rise time of 4×10^{15} A/sec. At peak current levels approaching 5 MA the wire plasma attains a density $n = 10^{24}$ cm^{-3}, a temperature $T = 10$ keV, and $n\tau > 10^{15}$ cm^{-3} sec.

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The purpose of this paper is to investigate how an imploding Z pinch may couple energy and amplify current in a central wire. The configuration is referred to as a "staged Z pinch" and is schematically illustrated in Fig. 1. An annular Z pinch is imploded so that it collides with several-hundred-micron-diameter wire (or fiber) located on the axis. Initially the wire supports a current of tens of kA, before the surrounding liner is fully ionized. This configuration was first employed on a low-voltage pinch for wavelength spectra calibrations of an argon (or alternately, a krypton) Z pinch using an Al on-axis wire [1]. In a high-voltage, pulse-line-generator arrangement [2–4] the wire current would be supplied by the usual prepulse, which is typically about 5% of the full voltage. Alternately, an auxiliary prepulse circuit may be used to provide more reproducible initial conditions for the on-axis plasma. The common characteristics of these experiments were improved pinch stability and a more energetic photon spectrum when imaged by time-integrated, x-ray pinhole photography.

Consider the path integral illustrated in Fig. 1. According to Maxwell's equation,

$$\oint \mathbf{E} \cdot d\mathbf{S} = -\frac{1}{c}\frac{\partial}{\partial t}\int \mathbf{B} \cdot d\mathbf{A} = 0, \tag{1}$$

and therefore the flux

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A} = \int_{r=a}^{r=r_i} \frac{1}{5}\frac{I_0}{r} h\, dr = \frac{hI_0}{5}\ln\frac{r_i(t)}{a(t)} \tag{2}$$

is conserved, where I_0 is the initial current through the fiber plasma, $r_i(t)$ and $a(t)$ are the inner liner and wire plasma radii, and h is the length of the pinch; initial means after the prepulse. At the beginning of the main-generator pulse both the wire plasma and Z-pinch plasma have negligible resistivity. As the implosion proceeds both $r_i(t)$ and $a(t)$ decrease. Since flux is conserved, the current in the wire plasma changes according to

$$I(t) = I_0\frac{\ln[r_i(0)/a(0)]}{\ln[r_i(t)/a(t)]}. \tag{3}$$

Initially $r_i(0)/a(0) = 18$. Since $r_i(t)/a(t)$ will be close to unity at maximum compression, the maximum value of $I(t)$ can be much greater than I_0.

To determine $r_i(t)$ we model the Z-pinch implosion with a thin-shell approximation. This assumes that $[r_0(t) - r_i(t)]/r_0(t) \ll 1$, where $r_0(t)$ is the radius of the outer liner; initially this ratio has the value 0.1. The equation of motion for a shell of arbitrary thickness (assuming no magnetic diffusion) is

$$\rho\frac{dV_r}{dt} = -\frac{\partial P}{\partial r}. \tag{4}$$

In order to simplify the equations for a compressible-plasma shell, compared to the more conventional Eulerian approach, it is convenient to introduce the Lagrange coordinate

$$\mu = \int_{r(0,t)}^{r(\mu,t)} \rho 2\pi r\, dr. \tag{5}$$

From Eq. (5) $\rho = (2\pi r \partial r/\partial\mu)^{-1}$ and, since $dV_r/dt = \partial^2 r(\mu,t)/\partial t^2$, Eq. (4) is transformed to

$$\frac{\partial^2 r}{\partial t^2} = -2\pi r\frac{\partial P}{\partial \mu}. \tag{6}$$

We integrate Eq. (6) over the Lagrange coordinate μ from $\mu = 0$ to $\mu = M$,

$$\frac{\partial^2\langle r\rangle}{\partial t^2} = -\frac{1}{M}\int_0^M d\mu 2\pi r\frac{\partial P}{\partial\mu} = -\frac{2\pi}{M}[r(M,t)P(M,t) - r(0,t)P(0,t)] + \frac{2\pi}{M}\int_{r_i(t)}^{r_0(t)} P\, dr. \tag{7}$$

In this equation, $\langle r\rangle = \frac{1}{M}\int_0^M d\mu r(\mu,t)$, $r(M,t) = r_0(t)$, and $r(0,t) = r_i(t)$. The boundary conditions are

$$P(M,t) = \frac{B_\theta^2}{8\pi}\bigg|_{r=r_0(t)} = \frac{1}{8\pi}\left[\frac{I_p}{5r_0(t)}\right]^2, \tag{8}$$

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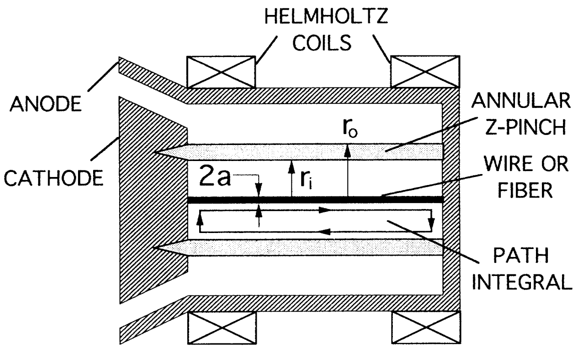

FIG. 1. Schematic illustration of the staged Z-pinch configuration.

$$P(0,t) = \frac{B_\theta^2}{8\pi}\bigg|_{r=r_i(t)} = \frac{1}{8\pi}\left[\frac{I_f}{5r_i(t)}\right]^2, \tag{9}$$

where I_p is the Z-pinch current and I_f is the current in the wire plasma. The last term can be compared with the first two terms on the right-hand side of Eq. (7). The ratios are

$$\frac{\langle P\rangle}{P(M,t)}\frac{r_0(t)-r_i(t)}{r_0(t)} \quad \text{and} \quad \frac{\langle P\rangle}{P(0,t)}\frac{r_0(t)-r_i(t)}{r_i(t)}.$$

The last term can be neglected if $[r_0(t) - r_i(t)]/r_0(t) \ll 1$, which we assume is satisfied initially. For a conventional pinch without the central wire plasma $r_i(t) \to 0$, and the inequality must break down. However, if $r_i(t) > a$, where a is some finite value, the thin-shell model can still apply, since the Z-pinch plasma is compressible. As $r_0(t)$ decreases the pressure increases, and the thickness $\Delta r(t) = r_0 - r_i$ decreases.

Assume, for example, the adiabatic equation of state $P = P_0(\rho/\rho_0)^\gamma$ with $\gamma = 5/3$. The mass $M = \pi\rho(r_0^2 - r_i^2)$ is conserved. Initially $\Delta r \ll r_0, r_i$ so that $M \cong 2\pi\rho_0 r_0(0)\Delta r(0)$. For maximum compression,

$$r_0(t)^2 = r_i^2(t) + (M/\pi\rho_0)(P_0 P)^{1/\gamma} \tag{10}$$

with a minimum value of $a(t) = a = 10\ \mu$m and $r_i \cong a$, Eq. (9) provides an estimate of the pressure so that

$$r_0^2 = a^2\{1 + [M/\pi\rho_0 r_i^2(0)]\,[I_0/I_f]^{6/5}\,[r_i(0)/a]^{4/5}\}. \tag{11}$$

For the parameters $I_0 = 200$ kA, $I_f = 3$ MA, $r_0(0) = 2$ cm, $r_i(0) = 1.8$ cm, $\rho_0 = 9 \times 10^{-6}$ g/cm^3, and $\Delta r/r_0 = 0.1$, at maximum compression $\Delta r/r_0 \cong (r_0 - a)/r_0 \cong 0.2$ and $\rho = 11$ g/cm^3. For these assumptions the thin-shell approximation remains valid up to and including maximum compression, since r_0 has a finite lower bound. Of course, the adiabatic equation of state is not accurate when the density ρ becomes very large. Even when ρ is not large the adiabatic assumption cannot usually be justified because of Ohmic heating radiation.

The above argument justifies the standard model with the thin-shell approximation only when the net energy gain from Ohmic heating and radiation of the Z pinch can be neglected during the implosion. A plasma where this is a good approximation could be obtained by appropriately seeding a low-Z plasma with high-Z ions. Experimental data are consistent with this assumption for Z pinches of high-atomic-number plasmas or seeded, low-atomic-number plasmas; for example, a Z pinch of hydrogen does not implode as well as a high-atomic-number pinch of argon or krypton [5]. The present calculations will be carried out assuming the adiabatic equation of state and adiabatic compressional heating.

The equations that describe the staged Z pinch are then

$$\frac{d^2r}{dt^2} = -\frac{I_p^2(t)}{100rM} + \frac{I_0^2}{100rM}\left[\frac{\ln(r_0/a_0)}{\ln(r/a)}\right]^2 - \frac{B_{z0}^2 r}{4M}\left[1 - \left(\frac{r_0^2 - a_0^2}{r^2 - a^2}\right)^2\right], \tag{12}$$

$$\frac{d^2a}{dt^2} = \frac{2}{1.6m_i}\frac{T}{a} - \frac{1}{a}\frac{I_0^2}{100M_c}\left[\frac{\ln(r_0/a_0)}{\ln(r/a)}\right]^2 - \frac{B_{z0}^2 a}{4M_c}\left(\frac{r_0^2 - a_0^2}{r^2 - a^2}\right)^2, \tag{13}$$

and

$$\frac{dT}{dt} = -\frac{4}{3}\frac{T}{a}\frac{da}{dt}. \tag{14}$$

In these equations $r \cong r_0(t)$, $r_i(t)$, $\langle r(t)\rangle$ because of the thin-shell approximation. $I_p(t) = I_m \sin(\pi t/2t_m)$. The inclusion of an initial axial magnetic field B_{z0} has been treated previously [6]. M, M_c are mass per unit length of the Z-pinch and wire plasmas. r_0 and a_0 are initial radii for the Z-pinch and wire plasmas. T is the temperature. The units are as follows: M,M_c are in μg/cm; I_f,I_0 are in MA; B_{z0} is in MG; t,t_0 are in nsec; r,a are in cm; m_i is the mass in μg; and T is the temperature of the central plasma in eV. The parameters used are $I_0 = 200$ kA, $r(0) = 2$ cm, $a_0 = 10^{-2}$ cm, $t_m = 1\ \mu$sec, $I_m = 2$ MA, $B_{z0} = 2 \times 10^{-4}$ MG, $T_0 = 200$ eV, and $M_c = 26\ \mu$g/cm.

Figure 2 displays the current profile and radial trajectory for the outer pinch. As typical, during current buildup the radius of the outer pinch remains relatively constant and decreases near peak current. On an expanded time scale Fig. 3 illustrates that the liner radius and fiber radius are separated by a few microns at peak compression. The compression of the magnetic field in this volume generates induction currents in the fiber plasma. At early times in Fig. 4 the fiber current has already amplified the 1 MA level and eventually increases to an average value of 3.5 MA (peak current 5 MA) with a rise time of 0.5×10^{-9} sec; thus, $dI_f/dt \approx 4 \times 10^{15}$ A/sec. Variations in the fiber radius (Fig. 3) and current (Fig. 4) are associated with the compressibility of the fiber plasma and increase or decrease with appropriate changes in the value of the adiabatic coefficient γ. Previously the period of such pinch oscillations was found to be of the order of a few acoustic-transient times [7], in agreement with the present results. Figure 5 displays the fiber-plasma density and temperature. At peak compression the density n is approximately

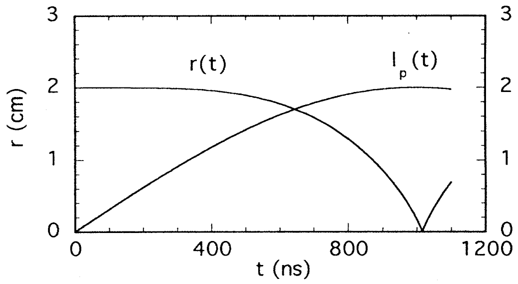

FIG. 2. Outer pinch current and outer pinch radius as a function of time.

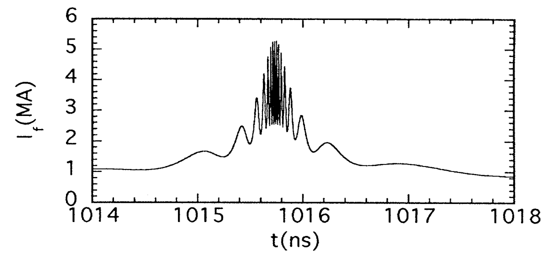

FIG. 4. Plot of the current through the fiber as a function of time.

4×10^{24} cm^{-3} and the plasma temperature is 10 keV, sustained for several hundred picoseconds. If we assume the same value of γ for a solid, deuterium-tritium fiber plasma as for a metal-wire plasma, then the Lawson "breakeven" criterion for thermonuclear fusion, $n\tau > 10^{14}$ cm^{-3} sec, would be surpassed by 1 order of magnitude. Reference [8] provides further details and justifies our initial assumption that adiabatic heating dominates in this configuration.

At maximum compression the peak magnetic field is in the range of hundreds of MG, trapped in a several-micron-thick annulus. With such dimensions it is essential to consider magnetic-diffusion losses. The collisionless skin depth is

$$\delta_0 = \frac{c}{\omega_p}, \tag{15}$$

and the collisional skin depth is

$$\delta = \frac{c}{\omega_p}\left(\frac{\Delta t}{\tau_{ei}}\right). \tag{16}$$

The larger of the two is applicable. Δt is the rise time of the magnetic field or current, which from Fig. 4 is about 0.2 nsec. For the interior (wire) plasma, the final plasma (electron) density is about 10^{24} cm^{-3} so that $\delta_0 = 0.56 \times 10^{-6}$ cm. τ_{ei} is the electron-ion collision time,

$$\tau_{ei} = \frac{\sqrt{m}\,T_e^{3/2}}{2\pi e^4 \ln\Lambda \sum_i n_i Z_i^2} \cong \frac{10^9}{n} T^{3/2}, \tag{17}$$

for a hydrogenlike plasma with $n = 10^{24}$ cm^{-3} and $T = 10$ keV at the final stage $\tau_{ei} = 3.2 \times 10^{-14}$ sec. Therefore, $\delta = 0.44$ μm, which is small compared to the interspace of 3 μm between the outer liner and the fiber, and a final fiber radius of $a = 10$ μm.

The final state of the liner plasma is determined assuming adiabatic compression. With an initial temperature of about 10 eV, the final temperature would be about 10 keV. If the ions are tenfold ionized, $n_e = 2.44 \times 10^{24}$ cm^{-3} and $c/\omega_p = 0.36 \times 10^{-6}$ cm, then $\tau_{ei} = 0.13 \times 10^{-14}$ sec. Thus, $\delta = 1.4$ μm and field penetration into the outer Z-pinch plasma is not negligible. For a seeded-plasma liner the resulting δ would be about the same as for the central (wire) plasma and would be negligible. Although flux diffusion may set an upper limit on the compressed-magnetic-field intensity, proper inclusion of field diffusion does not change the present results by more than a factor of 2, as confirmed in recent 2D code calculations [9].

Flux diffusion might be enhanced in a turbulent plasma. A necessary criteria for the onset of turbulence is when the electron-drift velocity exceeds the ion-thermal velocity, i.e.,

$$\frac{I}{2\pi a \delta n e} > \sqrt{\frac{2T}{m}} \cong 10^8 \text{ cm/sec}. \tag{18}$$

Assuming values of I, a, δ, and n in the compressed state, this criterion is satisfied when $\delta = \delta_0 \cong 10^{-6}$ cm, but not when $\delta \cong 10^{-4}$ cm so that the above classical calculations should still apply. Initially turbulence would be expected but would not be maintained if the current spreads out over the classical skin depth.

In this paper we have computed current coupling and amplification in a coaxial wire pinch (staged Z

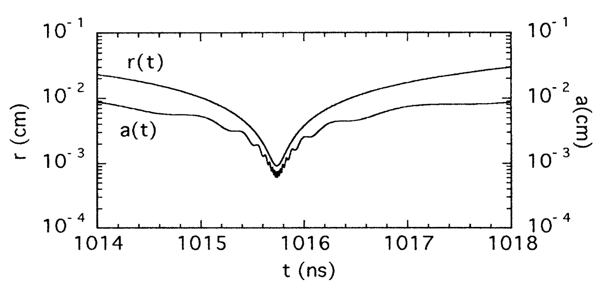

FIG. 3. Plots of the outer pinch radius and the inner fiber radius as a function of time during the last phase of compression.

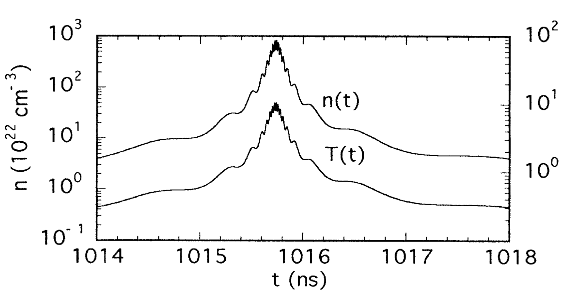

FIG. 5. Plots of the density and temperature of the fiber plasma as a function of time.

pinch) due to flux conservation and adiabatic compression. Previously we considered flux compression [6,8] of an axial magnetic field described by the last term of Eq. (12). In that case we obtained $dB_z/dt \cong 10^{16}$ G/sec. For similar parameters the present paper reports $dB_\theta/dt \cong 10^{17}$ G/sec, with the improvement due to the different geometric factors r^{-2} vs $1/\ln r$. The inclusion of B_z would produce magnetic shear, due to the combination of B_z and B_θ fields, and would be expected to control Rayleigh-Taylor instabilities. For the parameters considered here, Rayleigh-Taylor instabilities with a wavelength of the order of the wire radius (namely, tens of μm) may limit the peak value of magnetic field, since the field volume would be effectively increased. Thus, attaining maximum values of fiber current with the shortest rise times would require a suitable choice of the initial values of B_z and B_θ.

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