Two-Dimensional Regimes of Self-Focusing, Wake Field Generation, and Induced Focusing of a Short Intense Laser Pulse in an Underdense Plasma

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The results of a two-dimensional particle-in-cell simulation and of an analytical description of the propagation in an underdense plasma of a short, relativistically intense, laser pulse are presented. Self-focusing is proven in an ultrarelativistic regime for moderately long pulses. Pulses shorter than the plasma wavelength, but wider than it, excite a wake wave with a regular electric field. The electron density in the wake has a "horseshoe" shape and focuses a long pulse locally. The excitation of stimulated Raman backward scattering is observed.

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The propagation of relativistically strong laser pulses in an underdense plasma is relevant to a wide range of physical problems [1]. In this context, it is now important to determine the effect of the transverse nonuniformity of the pulse, such as relativistic self-focusing [2–5], and its influence on the generation of a strong wake field [6,7] and on the latter's interaction with electromagnetic (EM) radiation [8]. These problems are of interest, e.g., for the laser-plasma acceleration of charged particles [9] and the development of high power sources of hard EM radiation [10].

Previous descriptions of the effects of the pulse transverse nonuniformity were mainly formulated within the framework of relativistic fluid equations for the plasma electrons and of the so-called "envelope" approximation for the EM radiation. These equations, which have been studied both analytically and numerically [2–4,6–8], require that the wavelength of the plasma waves be larger than that of the EM radiation but smaller than all transverse scale lengths. In the dimensionless form, for a circularly polarized wave, they read [2,8]

$$\partial|a|^2/\partial\tau + \nabla_\perp(|a|^2\nabla_\perp\theta) = 0\,, \tag{1}$$

$$\partial\theta/\partial\tau + (\nabla_\perp\theta)^2/2 = \phi/2(1+\phi) + \nabla_\perp^2|a|/2|a|\,, \tag{2}$$

$$\partial^2\phi/\partial\xi^2 = \tfrac{1}{2}[(1+|a|^2)/(1+\phi)^2 - 1]\,, \tag{3}$$

where $\phi = e\varphi/m_ec^2$ is the electrostatic potential, $a = |a|\exp(i\theta)$ the complex amplitude of the EM vector potential $a = eA/m_ec^2 = [a\exp(-i(\omega_0 t - k_0 x)) + \text{c.c.}]/2$, $\tau = \omega_{pe}^2 t/\omega_0$, $\xi = k_p(x - v_g t)$, with $v_g = k_0c^2/\omega_0 \approx c$ the linear group velocity, $\omega_0^2 = k_0^2c^2 + \omega_{pe}^2$ and $k_p = \omega_{pe}/v_g$. Terms of order $\alpha \equiv \omega_{pe}/\omega_0$ and α^2 are small in an underdense plasma and have been neglected. Linearizing Eqs. (1)–(3) along the lines of [2] leads to

$$4\Gamma^2 = Q_\perp^2[|a|^2/(1+|a|^2)^{3/2} - 2Q_\perp^2]\,, \tag{4}$$

which, for $|a|^2 < 1$, yields the threshold for the self-focusing instability in terms of the pulse power $P > P_{\text{cr}} = 15(\omega_0/\omega_{pe})^2$ GW obtained in [2]. Here Γ and $Q_\perp$ are the normalized growth rate and perpendicular wave number, respectively.

For short pulses, and for pulses with a sharp leading front in the region where $\xi \ll 1$, the electrostatic potential is small and we can write Eqs. (2) and (3) as $\partial\theta/\partial\tau + (\nabla_\perp\theta)^2/2 \approx \phi/2$, $\partial^2\phi/\partial\xi^2 \approx |a|^2/2$. A solution that describes the propagation of a pulse with a transverse parabolic amplitude is $|a|^2(\xi,y,\tau) = |a|^2(\xi,0,0)[1 - y^2/2S^2]/S$, $\theta(\xi,y,\tau) = [y^2/2S]\partial S/\partial\tau$, and $\phi(\xi,y,\tau) = \phi(\xi,\tau)y^2/2$, where $S = S(\xi,\tau)$ is determined by

$$S^3\frac{\partial^2}{\partial\xi^2}\left(\frac{1}{S}\frac{\partial^2 S}{\partial\tau^2}\right) = \frac{|a|^2(\xi,0)}{2}\,. \tag{5}$$

For a pulse with a sharp leading front [$|a|^2(\xi) = \text{const}$ for $\xi < 0$, and $|a|^2(\xi) = 0$ for $\xi > 0$], Eq. (5) has a self-similar solution which behaves as $S \approx [\Lambda\ln(\Lambda)]^{2/3}$ for $\Lambda \equiv \xi\tau \approx 0$, and as $S \approx [\ln(\Lambda)]^{4/3}$ for $\Lambda \approx 1$. These dependences describe either the pulse instantaneous self-focusing ($\tau \to 0$) (a similar regime was considered in [7]) or the "horn-shaped" self-focusing of the pulse on its axis [3,4,11] ($S \to 0$ for $\Lambda \to 1$). The pulse leading front is nonfocused since $\phi \to 0$ for $\xi = 0$. This approximation neglects the effect of the excitation of both forward (FSRS) and backward (BSRS) stimulated Raman scattering [12] (see also [4]) which causes modulation of the pulse amplitude and changes the shape of its leading front [13]. Moreover, one can expect that, for a pulse with an ultrarelativistic amplitude, nonlocal nonlinearities arising from the excitation of wake waves lead, in a 2D geometry, to wave breaking.

Numerical simulations with particle-in-cell codes provide an appropriate method for investigating these phenomena. The evolution of the self-focusing of a long pulse with moderately relativistic amplitude in a plasma was studied numerically in [14]. Here we address the case of a pulse with an ultrarelativistic amplitude and focus our attention on the wake field generation. We use a $2(\frac{3}{3})$D fully self-consistent, particle-in-cell, relativistic EM code. All physical variables depend on two spatial coordinates, x and y. Ions are treated as a fixed charge neutralizing background. A 128×256 grid is used with approximately 2×10^5 particles.

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The boundary conditions for the EM fields are periodic in the y direction. In the x direction they consist of an EM wave incoming into the region $x > 0$ and of wave absorption, without reflection, on the left and right boundaries. The outgoing particles are replaced by particles with zero temperature: This corresponds to inelastic collisions with the boundaries. The plasma starts at $x = 5$ and is preceded by a vacuum region. The pulse is initialized outside the plasma at $x = 0$, $t = -5$. At $t = 0$ the pulse is at $x = 5$. The radiation is circularly polarized in all runs presented in this Letter.

First, we consider self-focusing in an underdense plasma of a finite length ($l_p = 15\lambda$), relativistically strong (maximum amplitude $a = 3$) pulse with transverse width $l_\perp = 5\lambda$. In terms of the self-focusing threshold (4), we estimate that the pulse power exceeds its critical value by a factor of 2. The plasma density corresponds to $\omega_{pe}/\omega_0 = 0.45$, i.e., to $n \approx 0.1 n_{\rm cr}$ if we account for the relativistic corrections to the electron plasma frequency $\omega_{pe}(1 + |a|^2)^{-1/4}$. The pulse is chosen sufficiently long and wide so that the excitation of wake plasma waves can be initially discarded. The self-focusing of this laser pulse is shown in Fig. 1 for $\omega_0 t/2\pi = 25$. Spatial coordinates are normalized on $\lambda \equiv 2\pi c/\omega_0$. At $t = 0$ the pulse is at $x = 5$. Figure 1(a) shows the isoenergetic contours $W \equiv E^2 + B^2 = \text{const}$; Fig. 1(b) gives $W = W(x)$, while Figs. 1(c) and 1(d) give the y-averaged phase plane (P_x, x) and x,y distribution of the electron density. These results prove the self-focusing of a finite length laser pulse on a time scale close to the inverse growth rate of the transverse instability given by Eq. (4). The pulse is focused in a narrow region with a transverse width of one or two wavelengths and strong amplitude magnification. Its leading part has comparable transverse and longitudinal scales with no indication of a horn-shaped front. This can be explained as due to the excitation in the pulse front of BSRS, with growth rate $\approx (\omega_0 \omega_{pe}^2)^{1/3}$, which triggers an ultrafast depletion of the pulse [13]. The steepening of the pulse front is seen in Fig. 1(b). The breaking of the nonlinear plasma waves is seen in Fig. 1(c). The development of transverse and longitudinal modulations of the pulse lead to the excitation of strong plasma turbulence: The electron density is distributed irregularly, Fig. 1(d), and the electron component is heated to energies 2–4 times larger than their quiver energy in the initial phase of the pulse propagation ($\approx$4.5), but less than that of the self-focused phase ($\approx$30). We do not see a complete radial expulsion of the electrons from the center of the pulse due to the action of the ponderomotive force. This can be explained by the break of the plasma wave just in the vicinity of the leading edge and by the consequent fast heating of the electrons to energies larger than the ponderomotive potential energy, as well as by the pinching of the electrons under the action of the magnetic field [15].

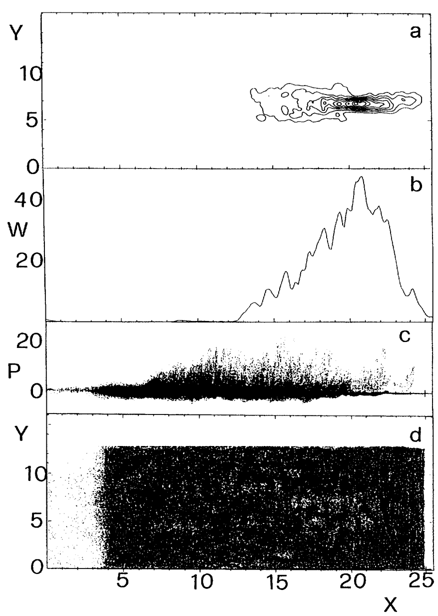

FIG. 1. Self-focusing of a short laser pulse at $\omega_0 t/2\pi = 25$; x, y are in units of λ: (a) contours of equal EM energy density; (b) EM energy density distribution along x for $y = 0$; (c) y-averaged phase space (P_x, x); and (d) electron distribution in the (x, y) plane.

Increasing the pulse width and/or decreasing its length or the plasma density leads to a slower transverse evolution of the pulse. However, the transverse nonuniformity still determines the structure of the wake field. This is shown for an ultrashort pulse in Fig. 2 for $\omega_0 t/2\pi = 35$. The pulse initial width and length are $5\lambda \times 5\lambda$, its maximum amplitude $a = 3$, and the plasma frequency $\omega_{pe} = 0.15\omega_0$. At the time shown the pulse is only marginally affected by self-focusing: It maintains its initial width and length [Fig. 2(a)], but the effect of linear diffraction is suppressed. In vacuum, in a 2D configuration the pulse energy would decrease as $1/x$. For the considered parameters the Rayleigh length $l_\perp^2/\lambda$ is $\approx 25\lambda$. For $|a| \gg 1$ the amplitude of the wake field is [13,16] $\phi \approx |a|^2/2$ and its wavelength is $\approx 2^{3/2}|a|$. Thus, through to the dependence of $|a|$ on y due to the pulse transverse nonuniformity, both the amplitude and the wavelength of the wake field depend on y. The resulting "horseshoe" shape is evident in Fig. 2(b). The condition for the plasma wave breaking is exceeded, as seen in Fig. 2(d), where the y-averaged (P_x, x) phase space is shown. In the wake, electrons are accelerated above their quiver energy. The wake field structure below the wave breaking threshold, a regime which is expected to be more convenient for particle acceleration, will be presented elsewhere. Nevertheless, since multistream motion occurs only in a finite region in the transverse direction, wave breaking does not strongly affect the shape and amplitude of the wake electric field which shows a quite regular x dependence [Fig. 2(c)]. Its increase with x could be related either to self-focusing of the pulse or to its frequency downshift, as discussed in [13].

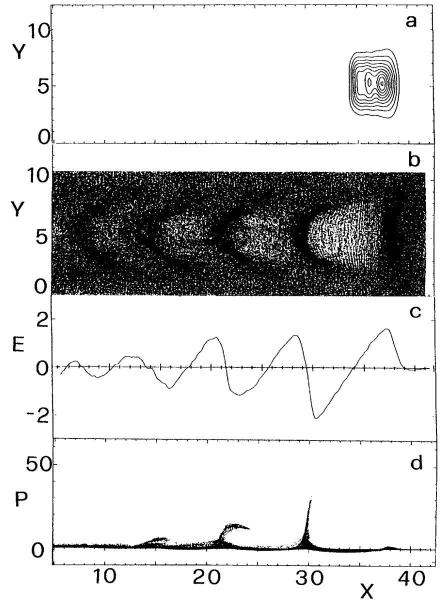

FIG. 2. Wake field excitation behind a short laser pulse for $\omega_0 t/2\pi = 35$: (a) as in Fig. 1; (b) electron distribution in the (x,y) plane; (c) electric wake field at $y = 0$; and (d) y-averaged phase space (P_x, x).

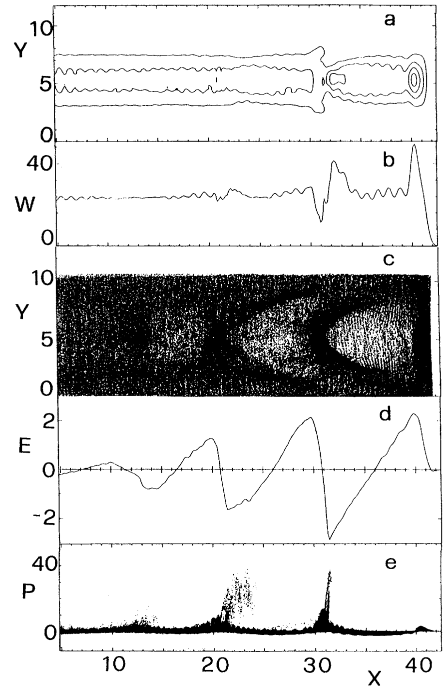

FIG. 3. Induced focusing of the EM radiation by the wake plasma wave at $\omega_0 t/2\pi = 40$: (a) contours of equal EM energy density; (b) EM energy density along x for $y = 0$; (c) electron distribution in the (x,y) plane; (d) electric wake field at $y = 0$; and (e) y-averaged phase space (P_x, x).

The transverse horseshoe structure of the plasma wake affects significantly the amplitude of a long laser pulse. It acts as a lens moving at relativistic speed and leads to induced focusing of the pulse [8]. To evaluate this effect in the geometrical optics approximation we assume the dependence of the potential ϕ in Eq. (2) on $\xi - 2^{3/2}|a_0|[1 - y^2/2Y^2]$, due to the dependence of the relativistic plasma wake on its amplitude, to be given. Here $|a_0|$ is the pulse amplitude on the x axis and Y its width in the y direction. Evaluating the maximum value of the transverse gradient of $\phi/(1 + \phi)$, we can write Eq. (2) for small y as $d^2y/d\tau^2 = -\Omega_f^2 y[\xi - y^2/2Y^2]$, with $2\pi/\Omega_f \approx Y/|a_0|^3$ the focusing time. For ultrarelativistic pulses this time can be shorter than the self-focusing time which scales as $1/|a|$. The nonlinear term in y leads to aberration.

The results of the 2D simulation of induced focusing is shown in Fig. 3 for a semi-infinite pulse with $l_\perp = 5\lambda$, $|a| = 3$, and $\omega_{pe}/\omega_0 = 0.15$. A strong wake plasma wave is excited due to the fast rise $(2 - 3\lambda)$ of the pulse front. Self-focusing suppresses diffraction spreading $(l_\perp^2/\lambda \approx 25\lambda)$ of the pulse [Figs. 3(a) and 3(b)] and even a strong increase of the pulse amplitude is seen near its front. One plasma wavelength behind, the horseshoe structure is formed [Fig. 3(c)] and the plasma wave interacts strongly with the EM radiation, locally increasing its amplitude and reducing its transverse localization scale. This in turn causes the amplitude of the plasma wave to grow and charged particles are accelerated more effectively [Fig. 3(e)]. The longitudinal electric field along the pulse axis is very regular [Fig. 3(d)]. The induced focusing starts near the second maximum of the wake [Figs. 3(a) and 3(b)].

An interesting feature is seen in the region between the pulse leading edge and the first wake field maximum: In both Figs. 2(b) and 3(c) a fine scale periodic modulation with wavelength $\approx\lambda/2$ of the electron density is apparent. From the modulation of the electron density it causes and from the curvature of its wave fronts, we conclude that it is a plasma wave with wave vector $2k_0$, copropagating with the EM pulse in the x direction excited by BSRS. This instability requires $\omega_0 = \omega_1 + \omega_{pe}$, $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_{ps}$, where $\mathbf{k}_{1,ps}$ and $\omega_{1,pe}$ are the wave numbers and frequencies of the backscattered EM wave and of the plasma wave. The latter is excited at the front of the pulse, in the region where $|a| \approx 1$ and the growth rate is maximum, but stays behind the pulse since its group velocity vanishes in a cold plasma. The modulation of the pulse with scale $\approx|k_1^{-1}|$ is also seen in Fig. 3(b). Because of the convective nature of the instability, the growth of the backscattered radiation is saturated.

We summarize our results as follows: An ultrashort, relativistically strong pulse can be self-focused in a plasma with strong magnification of its amplitude and channeling in a narrow channel shaped like a "bullet." A detailed discussion of the relativistic self-focusing will be published separately, for a wide range of parameters and including the case of a semi-infinite pulse with power exceeding the filamentation threshold. Plasma turbulence occurs in the region occupied by the pulse and behind it and leads to electron heating. No regular electric field is found to be excited in this regime. The enhanced acceleration in a self-modulated wake field, discussed in [6,7], requires that the pulse power P be greater than the critical value $P_{\rm cr}$ for the transverse instability to develop. It seems to us that to achieve regimes with a regular electric field pattern, the development of this self-modulation close to the instability threshold must be thoroughly investigated, if we are to gain control of the properties of the plasma wake wave. We think that a regular structure of the self-modulated laser pulse would be best obtained for $P \approx P_{\rm cr}$ rather than for $P \gg P_{\rm cr}$, as in this limit our simulations do not show a regular pattern.

A regular longitudinal electric field is produced in the wake of a wide pulse shorter than the plasma wave period and behind the sharp edge of a long pulse. The transverse nonuniformity of the pulse causes the formation of horseshoe structures that can be used to focus and accelerate electrons and photons. The induced focusing of the EM radiation leads to fast and strong modulation of the pulse.

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