Resonant Photon-Graviton Conversion and Cosmic Microwave Background Fluctuations

Pisin Chen
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 7 July 1994; revised manuscript received 25 July 1994)

We point out that the coupling between the cosmic microwave background radiation (CMBR) and the primordial magnetic field can resonantly convert the photons into gravitons, which induces a frequency-independent fluctuation in the photon flux. Using the observed CMBR fluctuation, we derive a bound on the primordial field strength. The effect can also convert the relic gravitons into photons. For the non-string-based inflation theories it provides a direct test via measurement of long-wavelength electromagnetic waves. For the string cosmology it gives a new bound on the Hubble parameter at the big bang.

PACS numbers: 98.70.Vc, 98.80.Cq

The cosmic microwave background radiation (CMBR) is one of the few windows from which we can look back into the early history of our Universe. The physical origin of the CMBR temperature fluctuation at large scales detected by the Cosmic Background Explorer satellite (COBE) [1] has been much discussed. These fluctuations are generally attributed to the well-known Sachs-Wolfe effect [2]. Both density fluctuations (scalar modes) and relic gravitons (tensor modes) generated at earlier epochs, such as inflation [3], can contribute to perturbations of the lightlike geodesics, causing a redshift in the CMBR spectrum, and therefore its temperature fluctuation and anisotropy [4].

In this Letter we point out that, due to the coupling between the thermal CMBR photons and the background primordial magnetic field in the post-decoupling (or recombination) epoch, the thermal photons can convert into gravitons, causing a fluctuation in the number and energy flux. As we will see in the following, this resonant conversion probability is essentially the same for all frequencies that we consider. Using the observed CMBR fluctuation as a bound, we derive a constraint on the primordial field strength and show that, within the uncertainties and approximations, it is reasonably consistent with the bounds deduced from other astrophysical considerations. Since this effect also allows for the relic gravitons to convert into photons, we discuss the possibility of testing different models of cosmology.

Gertsenshtein [5] first pointed out that a propagating electromagnetic (EM) wave can couple its field-strength tensor $F_{\mu\nu}$ to that of a transverse background EM field to give rise to a nontrivial energy-momentum stress tensor, which serves as a source for the linearized Einstein equation to excite a gravitational wave [6]. In quantum language this corresponds to a mixing between the propagating photon and a graviton via a Yukawa-type coupling mediated by a virtual photon from the background field. In our discussion, we shall adopt the matrix formalism developed by Raffelt and Stodolsky [7].

For a mixed photon-graviton state traversing a magnetic field with strength B at an angle Θ, the wave equation can be linearized, using the expansion $\omega^2 + \partial_z^2 = (\omega + i\partial_z)(\omega - i\partial_z) = (\omega + k)(\omega - k) \approx 2\omega(\omega - i\partial_z)$, as

$$\left[\omega - i\partial_z + \begin{pmatrix} \Delta_\perp & \Delta_M & 0 & 0 \\ \Delta_M & 0 & 0 & 0 \\ 0 & 0 & \Delta_\parallel & \Delta_M \\ 0 & 0 & \Delta_M & 0 \end{pmatrix}\right] \begin{bmatrix} A_\perp \\ G_+ \\ A_\parallel \\ G_\times \end{bmatrix} = 0, \tag{1}$$

where $\Delta_M \approx B \sin\Theta / M_P$, M_P is the Planck mass, and $\Delta_j = (n_j - 1)\omega$, n_j are the refractive indices. $A_\perp$, $A_\parallel$ and G_+, $G_\times$ are the amplitudes of the photon and graviton states, respectively. For a less than perfect vacuum imbedded in a strong external field, there are two major contributions to Δ_j. The Lagrangian for the Euler-Heisenberg nonlinear QED effect due to the presence of a strong magnetic field gives rise to [8] $n_\perp^{\rm QED} = 1 + 2\xi$, $n_\parallel^{\rm QED} = 1 + 7\xi/2$, and $\xi = (\alpha/45\pi)(B\sin\Theta/B_c)^2$. $B_c = m^2/e \approx 4.4 \times 10^{13}$ G is the Schwinger critical field. In addition, the medium also introduces refractive index. So in principle we have $\Delta_j = \Delta_j^{\rm QED} + \Delta_j^{\rm m}$. For the plasma epoch prior to the decoupling, we have $\Delta_j^{\rm m} = -\omega_p^2/2\omega$, where ω_p is the plasma frequency. For the postrecombination era when the Universe was essentially in gas form, $\Delta_j^{\rm m}$ is induced by the Cotton-Mouton effect [9]: birefringence of the photon due to the presence of an external magnetic field in a medium. Note that $\Delta_j^{\rm QED} \propto \omega$, while $\Delta_j^{\rm m} \propto -1/\omega$.

Focused on the reduced 2×2 matrix, we can perform a rotation with angle θ for diagonalization. The strength of the mixing is characterized by the ratio of the off-diagonal term to the difference of the diagonal terms: $\frac{1}{2}\tan 2\theta = \Delta_M/\Delta_\parallel$. In the weak mixing case, $\frac{1}{2}\tan 2\theta \approx \theta \ll 1$, and the photon-graviton degeneracy is removed. In this case the transition probability is

$$P(\gamma_\parallel \to g_\times) = 4\theta^2 \sin^2(\Delta_\parallel z/2). \tag{2}$$

If the path is much longer than the oscillation length, $l_{\rm osc} = 2\pi/\Delta_\parallel$, then the probability $P \approx 4\theta^2 \ll 1$.

 0031-9007/95/74(5)/634(4)$06.00

On the other hand, the maximum mixing occurs when $\theta = 45°$, corresponding to the situation where $\Delta_\parallel = 0$. Here the degeneracy between the photon and the graviton states is reinstated, and the two are in resonance. Then,

$$P(\gamma_\parallel \to g_\times) = \sin^2(\Delta_M z). \tag{3}$$

In this case a complete transition is possible. In the typical situation, however, the coupling is so weak that for any physically realistic distance the argument can never reach $\pi/2$. So, practically, $P(\gamma \to g) \approx \Delta_M^2 z^2$. Note also that if $\Delta_\parallel \neq 0$, yet $\Delta_\parallel z \ll 1$, then Eq. (2) reduces to the same form. This is to say that for a given external field and distance z, there is a resonance frequency window which satisfies the condition $\Delta_\parallel(\omega_{\rm res} \pm \Delta\omega) \lesssim \pi/z$, and within this window the conversion probability is essentially $P(\gamma \to g) \approx \Delta_M^2 z^2$, independent of the photon frequency.

For the case of an inhomogeneous field, Raffelt and Stodolsky [7] show that

$$P(\gamma_\parallel \to g_\times) = \left| \int_0^z dz' \, \Delta_M(z') \exp\left[-i \int_0^{z'} \Delta_\parallel(z'') \, dz'' \right] \right|^2, \tag{4}$$

as long as the external field varies smoothly (in both strength and orientation) over the photon wavelength. Later, when we put in physically reasonable parameters at the recombination time, it can be shown that the value of $\Delta_\parallel$ is so small that the phase factor in Eq. (4) for any frequency is entirely negligible even when integrated up to the horizon radius. In this limit the transition probability is identical for both $\parallel$ and $\perp$ modes.

We now derive the probability for a photon to convert into a graviton by traversing one large magnetic domain, or "bubble," with size L and a uniform field strength B at an angle Θ with respect to the photon propagation direction. Let t be the time when the photon enters the bubble. As the photon propagates through this domain both L and B will evolve. Assuming the conservation of magnetic flux, we find $B(t) \propto 1/L^2(t)$. As the postdecoupling era is matter dominated, we have $L \propto t^{2/3}$ and thus $B \propto t^{-4/3}$. Neglecting the phase factor, we find from Eq. (4)

$$P(t) \approx \begin{cases} L^2(t) B^2(t) \sin^2\Theta / M_P^2, & L(t) \lesssim H^{-1}(t), \\ 9t^2[1 - t/L] B^2(t) \sin^2\Theta / M_P^2, & L(t) \gtrsim H^{-1}(t), \end{cases} \tag{5}$$

where $H(t)$ is the Hubble parameter at time t. The upper expression is strictly true for $L_* \ll H_*^{-1}$, but is $\sim 20\%$ overestimation for $L_* \sim H_*^{-1}$. Note also that $P(t)$ is asymptotically independent of the bubble size. Starting from the recombination time t_* to the present time t_1, a photon will have to cross N such bubbles with similar size L_* at t_*:

$$N \sim \frac{1}{L_*} \int_{t_*}^{t_1} \left(\frac{t_*}{t} \right)^{2/3} dt \sim 3 \left(\frac{t_1}{t_*} \right)^{1/3} \frac{t_*}{L_*}. \tag{6}$$

Let us first examine the case where $L_* \lesssim H_*^{-1}$. If the bubbles have sharp domain walls, i.e., the change of field strength and orientation across the boundary is not adiabatic, and if these changes are entirely random from bubble to bubble, then the mean total probability is

$$P = \sum_{i=1}^N P(t_i) \approx \frac{1}{\pi} \int_0^\pi d\Theta \int_{t_*}^{t_1} \frac{dt}{L(t)} P(t) \sim \frac{1}{2} \frac{t_*}{L_*} P_*, \tag{7}$$

where $P_* \sim B_*^2 L_*^2 / M_P^2$. The rms fluctuation around the mean is

$$P_{\rm rms} \approx \left[\frac{1}{\pi} \int_0^\pi d\Theta \int_{t_*}^{t_1} \frac{dt}{L(t)} P^2(t) - \frac{1}{4N} \left(\frac{t_*}{L_*} \right)^2 P_*^2 \right]^{1/2} \sim \frac{3}{2\sqrt{14}} \left(\frac{t_*}{L_*} \right)^{1/2} P_*. \tag{8}$$

This "leakage" of photons into gravitons leads to a *frequency-independent* fluctuation in the CMBR flux, i.e.,

$$\langle \delta\rho_\gamma / \rho_\gamma \rangle \sim \frac{3}{2\sqrt{14}} \left(\frac{t_*}{L_*} \right)^{1/2} P_*, \qquad L_* \lesssim H_*^{-1}, \tag{9}$$

where $\rho_\gamma(x) = (T^4/\pi^2) x^3 / (e^x - 1)$, and $x \equiv \omega/T$.

If, on the other hand, the coherence scales are much larger that H_*^{-1}, the mean total conversion probability is obtained by integrating the lower expression of Eq. (5) over the angle, and we find $P \sim (\frac{9}{2}) B_*^2 t_*^2 / M_P^2$. In this limit, the rms fluctuation is primarily induced through the randomness of the field orientations in different bubbles, which gives a coefficient of $(\frac{3}{8} - \frac{1}{4})^{1/2} = \frac{1}{2}\sqrt{2}$. Thus the fluctuation reaches an asymptotic value

$$\langle \delta\rho_\gamma / \rho_\gamma \rangle \sim \frac{9}{2\sqrt{2}} \left(\frac{t_*}{L_*} \right)^2 P_*, \qquad L_* \gg H_*^{-1}, \tag{10}$$

independent of L_* (since $P_* \propto L_*^2$).

The anisotropy of such a fluctuation is associated with the only physical scale of the process, namely the bubble size L_* at t_*. Thermal photons arriving at our detector from different angles have crossed different sets of randomly oriented bubbles. So the flux varies at the scale of the bubble size across the sky. For an observer at present, this bubble size has been Hubble-expanded to $L_1 \sim (t_1/t_*)^{2/3} L_*$.

This fluctuation is different in character from that generated by the Sachs-Wolfe effect, which is frequency dependent. Since the number of photons per mode in blackbody radiation is an adiabatic invariant, a frequency variation is equivalent to a temperature variation: $\delta\omega/\omega = \delta T/T$. So for the Sachs-Wolfe effect we have

$$\langle \delta\rho_\gamma / \rho_\gamma \rangle_{\rm sw} = \frac{x}{1 - e^{-x}} \langle \delta T/T \rangle. \tag{11}$$

Note that for $x \gg 1$, $\langle \delta\rho_\gamma/\rho_\gamma \rangle_{\rm sw} \approx x \langle \delta T/T \rangle$; while for $x \ll 1$, $\langle \delta\rho_\gamma/\rho_\gamma \rangle_{\rm sw} \approx \langle \delta T/T \rangle$, independent of frequency.

Observations of CMBR fluctuations at various scales and frequency ranges fit reasonably well with the above scaling law [10]. Nevertheless, due to uncertainties in the measurements, noise in the signals, and possible foreground contamination (e.g., from the galaxy or radio sources at low frequencies), a frequency-independent contribution to $\langle \delta\rho_\gamma/\rho_\gamma \rangle$ in addition to the frequency dependent one cannot be ruled out. It is clear that the maximum allowed photon-graviton conversion induced fluctuation can never exceed the observed CMBR fluctuation. Since our effect is frequency independent, the constraint should be set by the measurements at low frequencies. From Eq. (9), this means

$$\frac{B_*}{B_c} \lesssim 0.14 \frac{M_P}{m} \frac{\lambda_c}{t_*^{1/4} L_*^{3/4}} \sqrt{\langle \delta T/T \rangle}, \qquad L_* \lesssim H_*^{-1}. \quad (12)$$

Note that the anisotropy scale $L_1 \sim (t_1/t_*)^{2/3} H_*^{-1} \sim$ 280 Mpc, i.e., the Hubble-expanded horizon size at t_*, corresponds to a coherence angle $\theta_c \sim 1.5°$. From the Saskatoon experiment at this scale [11], which gives $\langle \delta T/T \rangle \sim 1 \times 10^{-5}$, we find $B_* \lesssim 0.03$ G.

At the recombination time, the typical photon energy is $T_* \sim 0.3$ eV, and the gas density is $n_* \sim 10^3$ cm^{-3}. With $B_* \sim 0.03$ G, the corresponding changes in the refractive index are $\Delta_{j*}^{\mathrm{QED}} \sim 10^{-38}$ cm^{-1} and $\Delta_j^{\mathrm{m}} \sim -10^{-34}$ cm^{-1} [12]. These values are so small that the corresponding oscillation length $l_{\mathrm{osc}}^*(\omega = T_*) = 2\pi/|\Delta_{\parallel,\perp}^*| \sim 10^{35}$ cm $\gg H_1^{-1} \sim 10^{28}$ cm. It is clear that the resonance window covers all possible frequencies. This confirms our assumption that this fluctuation is essentially frequency independent.

There are several arguments for the existence of an intergalactic magnetic field. For example, to obtain the observed high energy cosmic rays ($E > 10^{20}$ eV), one would need an intergalactic magnetic field with strength of the order $\sim 10^{-7}$–10^{-9} G at scales $L_1 \sim 100$ Mpc to confine the accelerated particles [13]. There have been many proposals regarding the origin of this magnetic field [14–16], as well as efforts to look for its constraints. In Ref. [17] it was found that the maximum strength of the primordial magnetic field at the BBN epoch ($t \sim 1$ min, $\sim 2 \times 10^{12}$ cm) is $B \lesssim 10^{11}$ G on scales $H_{\mathrm{BBN}}^{-1} \gtrsim L \gtrsim 10^4$ cm. By assuming magnetic flux conservation, the authors of Ref. [17] deduced that these bounds evolve into $B_* \lesssim 0.1$ G on scales $10^{18} \gtrsim L_* \gtrsim 10^{11}$ cm at t_*. Note that although this field strength at t_* is an upper bound, it was argued [17] based on Hogan's theory [18] that it corresponds to an intergalactic field of $\lesssim 7 \times 10^{-9}$ G at present. On the other hand, the bounds on the coherence scales appear to be conservative. These are the Hubble-expanded values of the bounds at the BBN epoch, with the implicit assumption that the magnetic bubbles have been "frozen" in time without interactions. However, as demonstrated by Tajima *et al.* [16,19], during the plasma epoch magnetic bubbles, once in contact, tend quickly to "polymerize" into larger bubbles. For example, near the recombination time, it takes only $\sim 10^8$ sec ($\ll t_* \sim 10^{13}$ sec) before the polymer extends to the event horizon. Under this scenario of polymerization, the bounds deduced from BBN can in principle be extended to the scale $L_* \lesssim H_*^{-1}$, the largest possible causally connected scale at t_*. This bound is then reasonably consistent with ours.

In the models where the magnetic field "seeds" are generated during inflation [14], the coherence scale can in principle be larger then H_*^{-1}. In this case, our fluctuation reaches an asymptotic value, yet the CMBR constraint scales as $L_1^{-2/3}$. At large scales, we deduce from the COBE result [20] a scaling law: $\langle \delta T/T \rangle \sim 1 \times 10^{-5} (10°/\theta_c)^{2/3}$. Combining with Eq. (10), we find

$$\frac{B_*}{B_c} \lesssim 2.9 \times 10^{-4} \frac{M_P}{m} \frac{\lambda_c}{t_*} (H_* L_*)^{-1/3}, \qquad L_* \gg H_*^{-1}. \quad (13)$$

This effect can in principle also convert relic gravitons [21,22] into photons. It can be shown that prior to the decoupling, e.g., during the e-p plasma epoch, the magnetic field and the plasma density are both so high that the resonance window is very narrow around the resonance frequency at any given time: $\omega_{\mathrm{res}}(t) = \sqrt{90\pi/7\alpha}\,[B_c/B(t)]\omega_p(t)$. In turn, the time for a photon to remain in resonance, or the so-called level crossing, $\Delta t \sim \{\sqrt{90\pi/7\alpha}\, B_c/B(t) [\pi t/\omega_p(t)]\}^{1/2}$, is very short. As a result the resonant conversion is negligible. Thus the relic graviton spectrum is well preserved until the decoupling time.

Non-string-based inflation theories predict a flat or decreasing graviton spectrum (in frequency) [3]. For scales $L_* \sim H_*^{-1}$, the lower limit of the resonant frequency set by $\Delta_j^{\mathrm{m}}(\omega_{*l}) = 2\pi H_*$ allows for resonant conversion for frequencies $\omega_* \gtrsim \omega_{*l} \sim 3 \times 10^{-10}$ eV, or $\lambda_* \lesssim 3 \times 10^6$ cm. In terms of the value at present, $\lambda_{\mathrm{res}} \sim (t_1/t_*)^{2/3} \lambda_* \lesssim 3 \times 10^9$ cm. We see that the lower limit of the Harrison-Zel'dovich scale-invariant spectrum ($\lambda_{\mathrm{HZ}}^{\mathrm{min}} \sim 10^7$ cm) lies inside the resonance window. Here the wavelength is $\sim$7–9 orders of magnitude larger than the CMBR wavelength, which is way out in the Planckian tail. Any measured EM wave at this wavelength and scale may be a signal of $g \to \gamma$ conversion. Constraint on the graviton density at the maximum wavelength ($\lambda_1 \sim H_1^{-1}$) gives the maximum possible energy density $\Omega_{\mathrm{HZ}} \sim 10^{-14}$ at present [21]. This gives the density fluctuation $\sim$8 orders of magnitude above the CMBR spectrum at $\lambda_{\mathrm{HZ}}^{\mathrm{min}}$. A direct measurement of the EM waves with such wavelength at large scales would be a test of the inflation theories.

String cosmology allows for an increasing relic graviton spectrum [22]. In this case the constraint is fixed at the maximum frequency: $\omega_0 \sim 10^{29} (H_0/M_P)^{1/2} \omega_1$, where H_0, the Hubble parameter at $t = 0$, is a free parameter in the theory, and $\omega_1 \sim H_1 \sim 10^{-18}$ Hz is the minimal frequency inside the present Hubble radius. With the bounds $10^2 \gtrsim H_0/M_P \gtrsim 10^{-4}$ for an increasing spectrum, we see that $0.03 \lesssim \lambda_0 \lesssim 30$ cm at present, which covers the range of CMBR.

Let us introduce the magnetic energy density in units of the critical energy densit y ρ_c^* at t_*:

$$\delta \Omega_{\mathrm{EM}}^* = B_*^2/8\pi\rho_c^*. \quad (14)$$

For the curvature signature $k = 0$ and the isotropic pressure $p = 0$ we have, from the Friedmann equation, $H_*^2 = (8\pi/3) G \rho_c^*$. Inserting this and Eq. (14) into Eq. (8), we get

$$P_{\rm rms}(g \to \gamma) \sim (9/4\sqrt{7})\delta\Omega^*_{\rm EM}(H_*L_*)^{3/2}, \qquad L_* \lesssim H_*^{-1}. \tag{15}$$

Here the relation $H_*^{-1} \simeq 2t_*$ has been used.

Using Eq. (15) and the graviton spectrum from Gasperini and Veneziano [22], we find a graviton-induced CMBR fluctuation at present:

$$\frac{\delta\rho_{\rm GV}(x)}{\rho_\gamma(x)} \sim \delta\Omega^*_{\rm EM}(H_*L_*)^{3/2}\left(\frac{H_0}{M_P}\right)^2 \frac{\rho_\gamma}{\rho_\gamma(x)}\frac{x^2}{x_0^3}, \tag{16}$$

where $x_0 = \omega_0/T \sim (10^{29}H_1/T)(H_0/M_P)^{1/2}$, $T = 2.7\ ^\circ$K, and $\rho_\gamma = \int_0^\infty \rho_\gamma(x)\,dx$. Note that this fluctuation is frequency independent at small x. Since x_0 is not *a priori* determined in the string cosmology (because of H_0), we apply the general expression in Eq. (11) for the bound $\delta\rho_{\rm GV}(x_0)/\rho_\gamma(x_0) \lesssim x_0/(1 - e^{-x_0})\langle\delta T/T\rangle$. After some algebra, we obtain the following constraint:

$$\frac{\sinh^2(x_0/2)}{x_0} \lesssim \frac{15}{4\pi^4}\left(10^{29}\frac{H_1}{T}\right)^4 (H_*L_*)^{-3/2}\frac{\langle\delta T/T\rangle}{\delta\Omega^*_{\rm EM}}. \tag{17}$$

If the primordial field strength can be independently determined, then x_0, and therefore H_0, is constrained by the CMBR fluctuation. Within our scenario, however, $\delta\Omega^*_{\rm EM}$ is itself bounded by the CMBR fluctuation. As we discussed earlier, the primordial field so deduced, though an upper bound, is consistent with the field necessary to explain the high energy cosmic rays. We thus assume [cf. Eq. (15)] that $\delta\Omega^*_{\rm EM} \sim \langle\delta T/T\rangle$, or $B_* \sim 0.03$ G, at $L_* \sim H_*^{-1}$. Inserting into Eq. (17), we find an order-of-magnitude estimate for a bound on H_0:

$$H_0/M_P \lesssim 1. \tag{18}$$

This lies inside the previously deduced bounds [22].

In this Letter, the resonant conversion mediated by the primordial magnetic field was treated as unrelated to the Sachs-Wolfe effect. This may not necessarily be so. Prior to the decoupling time the Universe was in a plasma state. It is known in plasma physics that a local concentration of plasma density tends to expel the magnetic flux. In this regard the matter perturbation and the primordial magnetic bubbles may complement each other spatially. Indeed, we know that it takes $\delta\Omega^*_m = \delta\rho^*_m/\rho^*_c \sim 10^{-5}$ matter perturbation to give rise to a temperature fluctuation $\delta T/T \sim 10^{-5}$. Miraculously, from Eq. (15) we find that to attain the same level of fluctuation it also requires $\delta\Omega^*_{\rm EM} \sim 10^{-5}$ at the scale $L_* \sim H_*^{-1}$. This suggests that a certain balance between the density pressure and the magnetic pressure may have been attained at this scale prior to the decoupling. This scenario may even provide a physical basis for the isothermal picture of the predecoupling Universe.

I appreciate helpful discussions with S. Ben-Menahem, P. Huet, M. Peskin, J. Primack, S. Rajagopalan, K. Thompson, G. Veneziano, R. Wagoner, and E. Wright. This work was supported by Department of Energy Contract No. DE-AC03-76SF00515.

Note added.—After the submission of this paper, the author's attention was called to a recent paper by J. C. R. Magueijo [Phys. Rev. D **49**, 671 (1994)], which investigates a similar effect. The generic expressions for the conversion probability in the two papers are similar. However, when related to the CMBR fluctuations, the bounds on the primordial magnetic field differ both qualitatively and numerically. In addition, the conversion of relic gravitons to photons and its implication on cosmology was not covered by Magueijo.

[1] G. F. Smoot *et al.*, Astrophys. J. Lett. **396**, L1 (1992).

[2] R. K. Sachs and A. M. Wolfe, Astrophys. J. **147**, 73 (1967).

[3] A. A. Starobinskii, JETP Lett. **30**, 682 (1979); V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. **115B**, 189 (1982); R. Fabbri and M. D. Pollack, Phys. Lett. **125B**, 445 (1983); L. F. Abbott and M. B. Wise, Nucl. Phys. **B244**, 541 (1984); A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).

[4] For a review, see, for example, M. White, D. Scott, and J. Silk, Ann. Rev. Astron. Astrophys. **32**, 319 (1994).

[5] M. E. Gertsenshtein, Sov. Phys. JETP **14**, 84 (1962).

[6] See, for example, P. Chen, Mod. Phys. Lett. **6**, 1069 (1991) and references therein.

[7] G. Raffelt and L. Stodolsky, Phys. Rev. D **37**, 1237 (1988).

[8] S. Adler, Ann. Phys. (N.Y.) **67**, 599 (1971).

[9] *American Institute of Physics Handbook,* 3rd ed., edited by D. E. Gray (McGraw-Hill, NY, 1972).

[10] See Ref. [4] and references therein for observations.

[11] E. J. Wollack *et al.*, Astrophys. J. Lett. **419**, L49 (1993).

[12] The Cotton-Mouton effect gives only the difference of the refractive indices: $n_\parallel - n_\perp = -CB^2/\omega$, where C is the Cotton-Mouton constant. The exact value for C at different physical conditions is hard to find. Here we assume that $n_\parallel - 1 \sim n_\perp - 1 \sim n_\parallel - n_\perp$, and $C \sim 10^{-14}(p/p_0)$ G^{-2} cm^{-1}, with the reference pressure $p_0 = 10^8$ dyne/cm^2.

[13] A. M. Hillas, Ann. Rev. Astron. Astrophys. **22**, 425 (1984).

[14] B. Ratra, Astrophys. J. Lett. **391**, L1 (1992).

[15] T. Vachaspati, Phys. Lett. **B 265**, 258 (1991).

[16] T. Tajima, S. Cable, K. Shibata, and R. M. Kulsrud, Astrophys. J. **390**, 309 (1992).

[17] B. Cheng, D. N. Schramm, and J. W. Truran, Phys. Rev. D **49**, 5006 (1994).

[18] C. J. Hogan, Phys. Rev. Lett. **51**, 1488 (1983).

[19] T. Tajima, S. Cable, and R. M. Kulsrud, Phys. Fluids B **4**, 2338 (1992).

[20] E. L. Wright *et al.*, Astrophys. J. Lett. **396**, L13 (1992).

[21] See, for example, M. S. Turner and F. Wilczek, Phys. Rev. Lett. **65**, 3080 (1990).

[22] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993), and references therein.