Conversion of dc Fields in a Capacitor Array to Radiation by a Relativistic Ionization Front

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A mechanism for generating coherent radiation of high power, variable duration, and broad tunability over several orders of magnitude from a laser-ionized gas-filled capacitor array is described. The scheme directly converts a dc electric field "wave" into a coherent electromagnetic wave train when a relativistic ionization front passes between the plates. The frequency and duration of the radiation is controlled by the gas pressure and/or capacitor spacing. Output frequency and power are calculated and compared to 2D particle-in-cell simulations.

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Most high power radiation sources that exist today are either free electron sources—such as free electron lasers (FEL's), gyrotrons, or synchrotrons that use high power electron beams—or laser or maser sources that are based on photon emission due to transitions between quantum states. Recently, alternate sources based on direct conversion of electric fields to light have been successfully pioneered in vacuum devices [1] and in photoswitched semiconductors [2]. In addition, the use of laser-produced ionization fronts [3–5] have been successfully employed to up-shift existing microwave radiation from 30 to over 150 GHz [6] by a mechanism described alternatively as phase modulation in a time-varying medium [7] or photon acceleration [8] in a plasma. However, the vacuum devices appear to be limited to microwave frequencies [1]. In the semiconductor devices the electron-hole carriers play a role similar to that of a gaseous plasma, but the carrier concentration and frequency are not as directly controllable as is density in a gas or plasma [2]. The plasma devices, on the other hand, require both a high power laser (to produce an ionization front) and a lower frequency radiation source of high power (to be up-shifted).

In this Letter, we describe an approach to generating radiation that combines some aspects of each of the alternate approaches just described. We consider the radiation produced when an ionization front moves through a gas-filled capacitor array that is biased to produce a static electric field of wave number k_0 and zero frequency (Fig. 1). Each time the ionization front crosses a capacitor it creates a burst of current and consequently a half-cycle pulse of radiation. The pulses from each capacitor add up coherently to produce a wave train in a particular direction and at a certain frequency that depends on the distance between capacitors and the density of ionized gas. Since the radiated wave train is similar to the dc wave form of the static electric field (e.g., it has approximately the same number of cycles), we refer to this as a dc to ac converter for radiation. The energy for the radiation comes directly from the dc electric field. This is unlike a FEL in which the kinetic energy of an electron beam is converted to radiation via a dc wiggler field. In that case the dc field (e.g., from wiggler magnets) does no work and provides none of the energy.

The geometry of the radiation source is shown in Fig. 1. In order to illustrate the basic mechanism, we begin with a simple 1D description of the field structure between the capacitors. Later we will take into account the 2D field structure in computing the amplitude of the radiation. In the 1D description the alternately biased capacitors produce a static electric field of the form $\mathbf{E} \sim (E_0 \sin k_0 x)\mathbf{y}$ in a working gas of density n_0, where $k_0 = \pi/d$ and d is the spacing between adjacent capacitor plates. An ionization front (e.g., created by a short laser pulse) moves between the plates in the $+\mathbf{x}$ direction with velocity v_f. For a front created by a laser of frequency ω_L, the front moves at the group velocity of the laser in the plasma, so the front velocity $v_f = c(1 - \omega_p^2/\omega_L)^{1/2} \approx c$ and $\gamma_f = (1 - v_f^2/c^2)^{-1/2} \approx \omega_L/\omega_p$, where $\omega_p = 4n_0e^2/m$ is the plasma frequency of the ionized gas.

To describe the radiation generated we begin by considering the situation in a reference frame moving with the ionization front. Since $v_f \approx c$, the Lorentz transformed electric field approximates an electromagnetic wave in the moving frame (i.e., $\omega' \approx k_0'c$, $B_0' \approx E_0'$, the primes denote front frame quantities) of Doppler shifted frequency $\omega' = \gamma_f k_0 v_f$. In this frame the front is static, and the incident wave moves in the $-x$ direction (see Fig. 1) and gives rise to reflected ($+x$ direction) and transmitted ($-x$ direction) waves all at the same frequency ω'. The

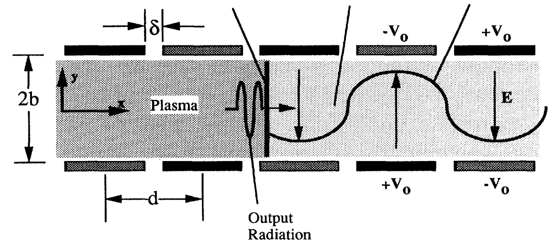

FIG. 1. Schematic of dc to ac light converter.

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reflected wave will be an extremely short pulse of hard x rays and is discussed later. The transmitted wave will turn out to be the tunable radiation that is the primary focus of this Letter. Although quite different in the laboratory frame, in the front frame the present scheme becomes similar to the up-shift of light waves considered in Refs. [3–5].

The transmitted wave must satisfy the dispersion relation in the plasma, i.e., $k'^2c^2 = \omega'^2 - \omega_p^2$. Lorentz transforming ω' and k' back to the laboratory frame gives the emitted frequency

$$\omega = \gamma_f^2 k_0 v_f \left[1 - \frac{v_f}{c} \left(1 - \frac{\omega_p^2}{\gamma_f^2 k_0^2 v_f^2} \right)^{1/2} \right]. \quad (1)$$

When $\omega' \gg \omega_p$, namely $\omega_L \gg \omega_p^2/k_0 v_f$, this can be approximated as

$$\omega \approx k_0 v_f/2 + \omega_p^2/2k_0 v_f. \quad (2)$$

From this expression we see that (for a fixed gas density) ω has a minimum value of ω_p when $k_0 = \omega_p/v_f$. High frequency can be obtained then by employing capacitor arrays with either large k_0 (i.e., a microstructure) or small k_0 (a macrostructure) compared to this value. For the macrostructure, tunability is achieved by varying the gas pressure since the output frequency is nearly linear in the density. For the macrostructure, the upper limit on the frequency is approximately the laser frequency ω_L and occurs for $\omega_p^2/2k_0 c = \omega_L$. For larger ω_p the static field is reflected at the front.

The frequency of the transmitted radiation can also be obtained directly in the laboratory frame. The frequency follows from two conditions: (i) the plasma dispersion relation, and (ii) continuity conditions at the front boundary. The dispersion relation is $\omega^2 = \omega_p^2 + c^2k^2$. For any of the fields to be continuous across the boundary, their phases must be the same at the front. The phase of the incident "wave" is $\pm k_0 x$, while the phase of the transmitted wave is $\omega t + kx$. Equating these and using $x = v_f t$ at the position of the front gives the condition for phase continuity: $\omega + kv_f = k_0 v_f$ for the mode of interest. Substituting for k from the latter equation into the dispersion equation and rearranging gives the result in Eq. (1).

It is instructive to construct graphical solutions to the above two equations [9]. The dispersion equation and the continuous phase condition are plotted in Fig. 2(a); their intersection ($T1$) gives the output frequency and wavelength. Figure 2(a) illustrates the case of $k_0 < \omega_p/v_f$ (macrostructure). We see that ω/k and $\partial\omega/\partial k$ are negative at the intersection which indicates that the output (transmitted) radiation moves in the same direction as the front, the $+\mathbf{x}$ direction. The output frequency is approximately $\omega_p^2/2k_0 c$ for this case. We point out that if $k_0 > \omega_p/v_f$ (microstructure), the constant phase line would intersect the dispersion curve in the other quadrant, indicating that the output (transmitted) radiation is in the opposite direction—in the $-\mathbf{x}$ direction or opposite to the laser front. The output frequency in this case is approximately $k_0 c/2$ and is nearly independent of plasma density.

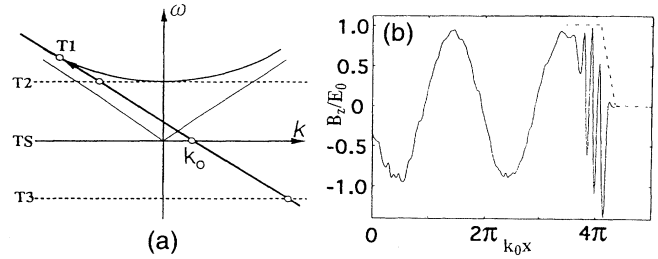

FIG. 2. Dispersion diagram (a) for electromagnetic, electrostatic, and free streaming plasma modes intersected by line of constant phase; and 2D particle-in-cell simulation (b) of B_z/E_0 (solid) and plasma density $n/n_{\max}$ (dashed) vs $k_0 x$ for a continuous front located at position $k_0 x = 13.8$ and length $0.4/k_0$.

Implicit in this result is the assumption that the plasma density is high enough to fully short out the capacitors' electric field. This requires $n_0 \gg E_0/8\pi eb$ and is easily satisfied. A more exact treatment of the second case should include the effect of the capacitors on the dispersion relation of the output radiation.

Next we estimate the output power of the radiation by finding the transmission and reflection coefficients at the ionization front boundary. To begin we must determine the field structure for the "incident," reflected, and transmitted fields in 2D (a 1D model for the capacitor field does not satisfy Maxwell's equations and leads to a factor of 2 error for the transmission coefficient and greatly overestimates the reflection coefficient). For capacitors of half separation b and small gaps $\delta \ll k_0^{-1}$ (see Fig. 1), the fields between the plates are given by

$$E_y = \sum_{n=0}^{\infty} \frac{(-1)^n 4k_0 V_0}{\pi \sinh(2n+1)k_0 b} e^{i(2n+1)k_0 x} \cosh(2n+1)k_0 y,$$

$$E_x = \sum_{n=0}^{\infty} \frac{i(-1)^n 4k_0 V_0}{\pi \sinh(2n+1)k_0 b} e^{i(2n+1)k_0 x} \sinh(2n+1)k_0 y.$$

Near the axis, the first term ($n = 0$) in the sum is always the largest term by a factor of 3 or more, so we keep only the first term in the following analysis.

Next we need the mode structure for the transmitted waves in the plasma. In 2D the well known plasma dispersion relations are $\omega^2 = \omega_p^2 + c^2k_x^2 + c^2k_y^2$ for transverse modes, $\omega^2 = \omega_p^2$ for longitudinal modes, and $\omega = 0$ for the free streaming mode. To assure continuity everywhere along the boundary, we take each transmitted mode to have fields with the same transverse dependence as the incident fields: $E_x = E_x e^{(i\omega t + kx)} \sinh(k_0 y)$ and $E_y = E_y e^{(i\omega t + kx)} \cosh(k_0 y)$. The sinh and cosh terms Fourier decompose into an infinite number of k_y components, and because of dispersion for the transverse modes, each Fourier k_y component would lead to a different ω and k_x. However, when $\omega_p \gg ck_0$ (large up-shifts), then the $c^2k_y^2$ term (of order $c^2k_0^2$) in the dispersion relation can be neglected, and the transmitted mode can be considered as having a single frequency.

We notice that the capacitor field has a longitudinal component (E_x), so we expect to couple to the longitudinal modes in the plasma. Adding these to our dispersion diagram in Fig. 2(a) we see that we expect to couple to one transverse mode ($T1$) as well as two longitudinal modes ($T2, T3$) and a free streaming mode (static magnetic field TS). The form of the reflected and *four* transmitted modes are given in Table I, where we make use of the fact that $\nabla \cdot \mathbf{E} = 0$ for transverse modes, $\nabla \times \mathbf{E} = 0$ for the longitudinal mode, and $\partial/\partial t = 0$ for the streaming mode.

The determination of the coefficients T_1, T_2, T_3, T_s, and R requires five boundary conditions. In addition to the usual conditions that E_{tang} and B_{tang} be continuous, three more conditions follow from the fact that electrons are "born" at rest with no velocity at the moment they are ionized. As a result $j_y = 0$, $j_x = 0$, and $\rho_s = 0$ at the front. The complete set of boundary conditions that follow from these and Faraday's and Gauss's laws are continuity of (1) E_y, (2) B_z, (3) $\partial B_z/\partial x + (1/c)\partial E_y/\partial t\ (= 0)$, (4) $\partial B_z/\partial y - (1/c)\partial E_x/\partial t\ (= 0)$, and (5) E_x. Applying these to the fields in Table I yields five equations that can be solved for the five unknown coefficients [10] for arbitrary β and ω_p/k_0c. For relativistic fronts ($\beta \approx 1$) and large up-shifts ($\omega_p/k_0c \gg 1$), the reflection and transmission coefficients can be approximated as $R \approx 4\omega_p^2/\gamma_f^2\beta^2k_0^2c^2$, $T_1 \approx 1 + 2(k_0c/\omega_p)^2$, $T_2 \approx -k_0c/2\omega_p(1 + 2k_0c/\omega_p)$, $T_3 \approx k_0c/2\omega_p \times (1 - 2k_0c/\omega_p)$, and $T_s \approx -1$. Thus the output radiation amplitude (T_1) is approximately equal to the dc capacitor field E_0.

An advantage of the dc capacitor array over schemes based on up-shifting existing radiation is that it may be possible to achieve higher output power by pulsing the dc bias voltage on a nanosecond time scale. For such short bias pulses much higher "incident wave" fields can be established without suffering breakdown than are possible by propagating microwaves through the gas.

Since the number of cycles of output radiation is roughly equal to the number of cycles of the static field, the pulse length, bandwidth, and efficiency can be estimated from the geometry. The pulse length is $\tau_{\text{pulse}} \approx N\lambda/c = 2\pi N/\omega_1$, where N is the number of capacitor periods (half the number of capacitors) and λ is the output wavelength. Control of the number of cycles and even the creation of wave trains encoded with missing peaks can be readily accomplished by connecting or disconnecting some of the capacitors from the dc bias supply. The bandwidth scales as $\Delta\omega/\omega \approx 1/N$, while the efficiency is $\eta = \lambda/d = 4k_0^2c^2/\omega_p^2$, where η is the ratio of the ac energy in the output (transmitted) pulse to the dc electric field energy in the ionized volume and does not include the energy used in the switching process (e.g., the laser energy).

The above analysis is strictly valid only for sharp fronts. The condition for a sharp front is that the scale length of the front L_f be much less than $(\gamma_f^2 k_0)^{-1}$ [5]. However, the frequency of the output radiation is unchanged as long as the front is shorter than $c\tau_{\text{pulse}}$, the duration of the output pulse [10]. We now show that the transmission coefficient is unchanged for continuous fronts. We introduce the spatial-temporal analog of a WKB approximation. We assume that the wave's amplitude depends on the distance that it has propagated through the front (i.e., it is a function of $x - v_f t$) so that

$$E_y(x, y, t) = A(x - v_f t)e^{i\int(k\,dx + \omega\,dt)}\cosh k_0 y\,,$$

where $\omega \approx \omega_p^2(x - v_f t)/2k_0c$ is the local up-shifted frequency and $k = k(x - v_f t) \approx ck_0 - \omega_p^2(x - v_f t)/2k_0c$. Substituting this form of the solution into the laboratory frame wave equation $(\partial^2/\partial x^2 + \partial^2/\partial y^2 - 1/c^2\,\partial^2/\partial t^2) \times \mathbf{E} = \omega_p^2(x - v_f t)/c^2\mathbf{E}$ and neglecting terms of order $k' + \beta\omega'/c$ compared to $(k + \beta\omega/c)^2$ where a prime denotes derivatives with respect to $x - v_f t$ yields the first order differential equation $A'2(k + \beta\omega/c) + A(k' + \beta\omega'/c) \approx 0$, with the solution

$$A = E_0\frac{\sqrt{k_0}}{\sqrt{k + \beta\omega/c}} = E_0\left(\frac{\beta}{1 - \omega/\gamma_f^2 k_0 c}\right)^{1/2} \approx E_0\,. \tag{3}$$

Thus the continuous front transmission coefficient ($T_1 = A/E_0$) is approximately 1 just as for a sharp front. We

TABLE I. Mode structure of fields.

Mode	E_x	E_y	B_z
Incident (static) mode	$ie^{ik_0x}\sinh k_0y$	$e^{ik_0x}\cosh k_0y$	0
Reflected mode	$-i\dfrac{k_0}{k_r}Re^{i(\omega_rt - k_rx)}\sinh k_0y$	$Re^{i(\omega_rt - k_rx)}\cosh k_0y$	$\left(k_r - \dfrac{k_0^2}{k_r}\right)\dfrac{c}{\omega_r}Re^{i(\omega_rt - k_rx)}\cosh k_0y$
$T1$ mode	$i\dfrac{k_0}{k_1}T_1e^{i(k_1x + \omega_1t)}\sinh k_0y$	$T_1e^{i(k_1x + \omega_1t)}\cosh k_0y$	$\left(\dfrac{k_0^2}{k_1} - k_1\right)\dfrac{c}{\omega_1}T_1e^{i(k_1x + \omega_1t)}\cosh k_0y$
$T2, T3$ mode	$i\dfrac{k_{2,3}}{k_0}e^{i(\omega_{2,3}t + k_{2,3}x)}\sinh k_0y$	$T_{2,3}e^{i(\omega_{2,3}t + k_{2,3}x)}\cosh k_0y$	0
TS (free streaming) mode	0	0	$T_se^{ik_sx}\cosh k_0y$

comment that the reflected mode, although up-shifted to even higher frequencies and more pulse compressed than the transmitted mode ($\omega \approx 2\gamma_f^2 k_0 c$, possibly yielding hard x rays), has an extremely small amplitude coefficient for continuous fronts [4,10]. On the other hand, the amplitude of the free streaming mode T_s is unchanged unless $L_f > k_0^{-1}$ [4,10].

At this point, we give a qualitative microscopic description of the radiation mechanism. The ionization of a capacitor produces free electrons that are accelerated by the dc electric field that exists when they are born. The radiation we describe ($T1$) is due to this initial acceleration. The displacement of the electrons shorts out the electric field leaving them to drift with a residual velocity that sustains the dc magnetic field of the free streaming (TS) mode. Inertia of the electrons causes them to overshoot the displacement that would exactly short the electric field; this results in the plasma oscillations ($T2, T3$).

Particle-in-cell simulations [10] with sharp and continuous fronts bear out the general conclusions of our analysis. The simulations were done on a 2D grid of length $5\pi/k_0$ and half-width $k_0 b = 1.38$, with the front moving to the right at $\beta = 0.99999$. The gas density was chosen such that $\omega_p/k_0 c = 5.64$, giving a predicted up-shift factor of $\omega_p^2/2k_0^2c^2 = 16$. Figure 2(b) shows a snapshot of the magnetic field B_z on the x axis. The short wavelength oscillations are the up-shifted and pulse compressed radiation ($T1$) following the front to the right. The longer wavelength oscillation is the free streaming mode (TS). The amplitude coefficients of the transverse mode and free streaming mode are approximately 1 in agreement with our model, and the wavelength (and pulse length) is shortened by a factor of 16 in excellent agreement with the theoretical prediction.

To illustrate the potential of the scheme, we consider two examples. For an array of capacitors with plate separation $d \approx b \approx 2$ cm and a dc bias voltage of 30 kV, Eqs. (3) and (2) predict that radiation with peak power on the order of 1 MW (in a round spot) could be tuned from a wavelength of 1 cm to 1 μm by varying the neutral gas pressure from 10^{-4} to 1 Torr (for a doubly ionized gas, corresponding to $n_0 = 6 \times 10^{12}-6 \times 10^{16}$ cm^{-3}). For a device 40 cm long, the radiation would have a fractional bandwidth $\Delta\lambda/\lambda \approx 10\%$. At the highest pressures, the laser energy required to ionize such a large volume may be quite high (tens of joules). Thus we consider a second example with a fairly small structure that could be more readily ionized with a modest laser (mJ) and that could be designed to operate in the 10–100 μm wavelength regime where conventional lasers cannot. Such a device would have $d \approx 300$ μm and a length of about 1 cm to produce bursts of radiation in the range of 500–5 μm lasting 50–0.5 psec (or less) for gas pressures of 0.1 to 10 Torr, respectively.

We conclude by pointing out that the various frequency up-shift schemes involving ionization fronts in unbounded plasmas [3–5], plasmas in fast wave structures [6], and slow wave structures (the present work) with either counterpropagating [3–6,9] or copropagating [7,8,11] incident fields can be unified by the single equation of continuity of phase at the front: $\omega_0 \pm k_0 v_f = \omega \pm k v_f$ or

$$\frac{\omega}{\omega_0} = \frac{1 \pm v_f/v_{\phi 0}}{1 \pm v_f/v_{\phi \mathrm{up}}},$$

where $v_{\phi 0} = \omega_0/k_0$ and $v_{\phi \mathrm{up}} = \omega/k$ and the + (−) sign corresponds to radiation moving opposite (toward) the front. From this we see that very large up-shifts are possible in two ways: (1) the numerator will be large if $v_{\phi 0} \ll v_f$, i.e., a very slow wave structure [12], and (2) the denominator can be large if v_f is equal to $v_{\phi \mathrm{up}}$, that is, if the phase velocity of the up-shifted wave is matched to the front velocity. The latter condition would require some way of slowing down the up-shifted light's phase velocity (typically greater than c) or superluminous fronts ($v_f > c$) such as may be created by sweeping the ionizing laser from the side.

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