Fock State Generation by the Methods of Nonlinear Optics

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We have found a Hamiltonian for generating a pure Fock state of the single-mode field and described an interaction scheme for its realization.

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The possible quantum states of the single-mode field, such as Fock states, coherent states, and squeezed states, play a central role in quantum optics, and are widely studied, especially since the discovery of nonclassical states of the electromagnetic field [1]. It is very useful for both mathematical calculations and experimental realization to have a Hamiltonian H creating a given state $|\psi\rangle$ from the vacuum state: $\exp\{itH\}|0\rangle = |\psi\rangle$. The following two Hamiltonians are well known and widely used: one for generating a coherent state $H_{\text{coh}} = \alpha a^\dagger + \alpha^* a$ (a and $a^\dagger$ are the photon annihilation and creation operators correspondingly), which describes the radiation of a classical current [2], and another one for generating a squeezed vacuum state $H_{\text{sq}} = \beta(a^\dagger)^2 + \beta^* a^2$ [3], which was realized in degenerate parametric amplification and in degenerate four-wave mixing [1]. The Hamiltonian $H_{\text{cat}} = \omega a^\dagger a + C(a^\dagger a^k)$, where k is an integer, was proposed recently [4] for transforming a coherent state into a quantum superposition of coherent states, known as "Schrödinger cat" state. But, as we know, the Hamiltonian for the Fock state generation is still not found. In our Letter we obtain such a Hamiltonian as a function of the operators a and $a^\dagger$ and describe a physical process in which the field in a Fock state can be generated. Our proposal for generating Fock states differs principally from that based on cavity quantum electrodynamics, widely discussed in recent years [5].

We proceed from the fact that a transformation of the Hilbert space of a quantum system is unitary if and only if it transforms an orthonormal basis into an orthonormal one. Choosing $|\psi_1\rangle$ to be the first vector of the first basis and $|\psi_2\rangle$ to be the first vector of the second basis we see that there is an infinite number of unitary transformations, transforming $|\psi_1\rangle$ into $|\psi_2\rangle$, and therefore an infinite number of Hamiltonians H such that for some t we have $\exp\{itH\}|\psi_1\rangle = |\psi_2\rangle$. We are interested in generating the n-photon Fock state $|n\rangle$ of the single-mode field from the vacuum state $|0\rangle$, but we restrict ourselves to a subclass of transformations $\exp\{it_nH_n\}|0\rangle = |n\rangle$, namely, to that with $t_n = \pi'2 + 2\pi m$, where m is an integer, and with H_n transforming the vacuum state into the n-photon Fock state and vice versa:

$$H_n|0\rangle = |n\rangle, \qquad H_n|n\rangle = |0\rangle. \tag{1}$$

Generally H_n is a power series in a and $a^\dagger$, and therefore it describes nonlinear processes of different order. Our aim is to find a Hamiltonian H_n of the lowest possible power in a and $a^\dagger$ and the corresponding value of the parameter t_n.

The Hermitian operator satisfying Eq. (1) has the general form

$$H_n = |0\rangle\langle n| + |n\rangle\langle 0| + PFP, \tag{2}$$

where

$$P = 1 - |0\rangle\langle 0| - |n\rangle\langle n|, \tag{3}$$

and F is any Hermitian operator. The representation [6]

$$|0\rangle\langle 0| = \lim_{s\to\infty} e^{-sa^\dagger a} = Ne^{-a^\dagger a}, \tag{4}$$

where N is the operator of normal ordering, shows that the operators

$$|n\rangle\langle 0| = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle\langle 0|, \qquad |0\rangle\langle n| = |0\rangle\langle 0|\frac{a^n}{\sqrt{n!}} \tag{5}$$

are the infinite power series in a and $a^\dagger$. Now we try to choose the operator F in such a manner as to make the expansion of H_n a finite one. To do it, note that any operator in the Hilbert space of harmonical oscillator is a function of a and $a^\dagger$, and therefore it can be uniquely represented in the following form:

$$g(a,a^\dagger) = g_0(a^\dagger a) + \sum_{m=1}^{+\infty}[(a^\dagger)^m g_m(a^\dagger a) + g'_m(a^\dagger a)a^m], \tag{6}$$

where $g_m(x)$ and $g'_m(x)$ are some functions. Equations (4) and (5) show that the operators $|n\rangle\langle 0|$ and $|0\rangle\langle n|$ are of the types $(a^\dagger)^m g_m(a^\dagger a)$ and $g'_m(a^\dagger a)a^m$ correspondingly. Therefore we may leave only the same terms in the expansion of F. As we shall see later, the term $g_0(a^\dagger a)$ is necessary for the physical interpretation of the Hamiltonian, so we write

$$F = \frac{(a^\dagger)^n}{\sqrt{n!}}f^*(a^\dagger a) + f_0(a^\dagger a) + f(a^\dagger a)\frac{a^n}{\sqrt{n!}}, \tag{7}$$

where $f_0(x)$ is some real function, $f(x)$ and $f^*(x)$ are some complex function and its conjugation. Now Eq. (2) can be rewritten in the form

$$\begin{aligned} H_n = {} & f_0(a^\dagger a) - f_0(0)|0\rangle\langle 0| - f_0(n)|n\rangle\langle n| \\ & + \Bigg[f(a^\dagger a)\frac{a_n}{\sqrt{n!}} + |0\rangle\langle n| \\ & - f(n)\frac{\sqrt{(2n)!}}{n!}|0\rangle\langle 2n| - f(0)|0\rangle\langle n| + \text{H.c.}\Bigg]. \end{aligned} \tag{8}$$

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To delete the infinite series in the expansion of H_n, we must let $f(0) = 1$, $f(n) = 0$, $f_0(0) = 0$, and $f_0(n) = 0$, the simplest forms of such functions being

$$f(a^\dagger a) = 1 - \frac{a^\dagger a}{n}, \qquad f_0(a^\dagger a) = \mu a^\dagger a \left(1 - \frac{a^\dagger a}{n}\right),$$

where μ is a real parameter. Substituting these functions into Eq. (8) we obtain

$$H_n = \mu a^\dagger a - \mu \frac{(a^\dagger a)^2}{n} + \left[\left(1 - \frac{a^\dagger a}{n}\right)\frac{a^n}{\sqrt{n!}} + \text{H.c.}\right]. \tag{9}$$

When $\mu = 0$ the Hamiltonian given by Eq. (9) describes the process in which a pump photon of frequency Ω is converted into n photons of frequency $\omega = \Omega/n$ in two ways, $\Omega \rightarrow n\omega$ and $\Omega + \omega \rightarrow (n+1)\omega$ simultaneously:

$$H_{\text{int}} = \chi^{(n)}(a^n E^* + \text{H.c.}) + \chi^{(n+2)}(a^\dagger a^{n+1} E^* + \text{H.c.}), \tag{10}$$

$E_p(t) = Ee^{-i\Omega t} + E^* e^{i\Omega t}$ being the pump field and the constants of nonlinear coupling satisfying the condition

$$\chi^{(n+2)} = -\chi^{(n)}/n. \tag{11}$$

In this process the vacuum fluctuation of the signal wave is amplified to the n-photon state of the output field, provided that the traveling time of the signal wave through the medium is $\tau = \pi(m + 1/2)/\chi^{(n)}|E|\sqrt{n!}$, m is an integer, and $\hbar = 1$.

In conclusion, we analyze in detail the generation of the one-photon Fock state, which seems to be the easiest way for the proposed method demonstration. The Hamiltonian given by Eq. (9) with $n = 1$ corresponds to linear and third-order nonlinear coupling between the signal and pump waves having the same frequency $\omega = \Omega$. According to the phase-matching condition the wave vectors must coincide too; hence the two waves may differ only by the polarization direction. We accept that the signal and pump waves propagate along the z axis, and are polarized in the x and y directions correspondingly. The linear coupling between the waves can be produced by the element $\chi_{xy}^{(1)}$ of the medium linear susceptibility, which can be modified by a constant electric E_0 or magnetic field applied to the sample (Pockel's, Kerr, Faraday, or Cotton-Mouton effect). The Hamiltonian of the linear coupling is

$$H_L = \chi_{xx}^{(1)} a^\dagger a + \chi_{yy}^{(1)} E^* E + (\chi_{xy}^{(1)} a^\dagger E + \text{H.c.}).$$

The third-order nonlinear coupling can be achieved when 2ω approaches the frequency of two-photon transition of the medium. Such a coupling is described by the following Hamiltonian:

$$\begin{aligned} H_{NL} = {} & \chi_{xxxx} a^\dagger a^\dagger aa + \chi_{xyxy} a^\dagger E^* aE + \chi_{yyyy} E^* E^* EE \\ & + (\chi_{xxyy} a^\dagger a^\dagger EE + \chi_{xxxy} a^\dagger a^\dagger aE \\ & + \chi_{xyyy} a^\dagger E^* EE + \text{H.c.}). \end{aligned}$$

Here χ_{ijkl} is a sum over the corresponding elements of the nonlinear susceptibility tenser $\chi_{ijkl}^{(3)}(\omega = -\omega + \omega + \omega)$ with permutated Cartesian indices and frequencies [7]. The structure of H_{NL} and H_L shows that for generating the pure n-photon Fock state it is necessary to satisfy the following relations: $\chi_{xxyy} = 0$, $\chi_{xx}^{(1)}(E_0) + \chi_{xyxy}|E|^2 = -\chi_{xxxx}$, $\chi_{xy}^{(1)}(E_0) + \chi_{xyyy}|E|^2 E = -\chi_{xxxy}E$, where the first condition is the most important while the other can be achieved by choosing the intensities of pump wave and external field.

To summarize, we have shown that the Hamiltonian given by Eq. (9) transforms the vacuum state of the single-mode field into a pure Fock state and that this Hamiltonian can be realized by the methods of nonlinear optics.

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