Limiting Value for the Width Controlling the Coupling of Collective Vibrations to the Compound Nucleus

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We show that the damping of nuclear collective excitations in the regime of chaotic intrinsic dynamics is well described by the coupling to doorway states. Subsequent collision processes which eventually lead the system into the compound nucleus eigenstates do not increase the damping width. We also argue that the damping width of collective excitations in nuclei does not increase without bounds as the excitation of the nucleus is increased, but acquires a limiting value.

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The question concerning the variety of mechanisms at work in the damping of collective motion of many-body systems is a fascinating one, and central in a variety of fields ranging from condensed matter to atomic nuclei. An example is provided by the nuclear isovector giant dipole resonance (GDR), that is, the collective vibration of protons against neutrons. Once excited, the mode will relax its energy and angular momentum either by particle emission or by coupling to the complicated intrinsic motion of the nucleons, the compound nucleus. The relaxation of the resonance is recognized by the width of the associated bell-shaped strength function $P(E)$, which is the probability distribution for finding the resonance at the excitation energy E.

The experimental strength function, which has been measured in photoabsorption experiments essentially in all nuclei throughout the mass table [1], has a total width $\Gamma \approx \Gamma^{\downarrow} + \Gamma_p$. Here Γ_p is associated with particle emission, while $\Gamma^{\downarrow}$ measures the decay rate of the resonance into the many degrees of freedom associated with the compound nucleus. A part of the width $\Gamma^{\downarrow}$ arises from the direct coupling of the GDR to the quadrupole deformations of the nuclei. In fact, the experimental width is systematically larger for deformed nuclei than for spherical nuclei and the absorption spectra may develop a two-peak structure for strongly deformed systems. Here we shall not consider the highly coherent contribution to the width due to the nuclear deformation but only the width Γ_c arising from the uncorrelated decay of the resonance into the compound nucleus eigenstates.

In the present paper we shall estimate the value of the damping width Γ_c. It will be concluded that Γ_c is independent of the excitation energy of the system, and that its value is controlled by very simple couplings. We write the total Hamiltonian describing the system as

$$H = H_0 + V, \tag{1}$$

sum of a mean field Hamiltonian H_0, and a residual two-body interaction V. The mean field Hamiltonian H_0 gives rise to the shell structure observed in nuclei, the eigenstates being the independent many-particle–many-hole excitations of the fermionic system, that is, the one-particle–one-hole (1p-1h), 2p-2h, 3p-3h, etc. excitations.

Because giant resonances are strongly excited by a one-body external field, it is natural to describe these states as a linear combination of 1p-1h states. As a consequence, we shall start by diagonalizing the total Hamiltonian H in a restricted basis of 1p-1h states. For multipolarity $J = 1$ and negative parity, this diagonalization leads essentially to one collective state $|c\rangle$, the GDR, which collects about 100% of the energy weighted sum rule and lies at an energy which is approximately twice the average excitation energy of the 1p-1h basis states. For other multipolarities and parities, e.g., $J^{\pi} = 2^{+}$, the diagonalization of the total Hamiltonian in the space of 1p-1h excitations leads typically to a low-lying surface vibration aside from the corresponding giant (quadrupole) vibration. The other states obtained in the diagonalization of H within the 1p-1h space are, regardless of spin and parity, noncollective, being close in energy to the unperturbed 1p-1h excitations.

The diagonalized 1p-1h states, together with the set 2p-2h, 3p-3h, etc. states form a complete set of basis states which we shall denote by $|\mu\rangle$ and with energies $E_\mu = \langle\mu|H|\mu\rangle$. The exact stationary states of the total Hamiltonian obtained by diagonalizing H in the complete basis of states $|\mu\rangle$ will be denoted $|i\rangle$ with the energies E_i. The damping of the collective state $|c\rangle$ is controlled by its coupling to the states $|\mu\rangle$. Among these states, those from the 2p-2h class are the only states that couple strongly to the collective state $|c\rangle$. These states will be denoted the "doorway states" to the collective state $|c\rangle$. Among the doorway states, those leading to the largest matrix elements consist of a surface vibration plus an incoherent 1p-1h state (cf., e.g., Ref. [2]). We shall show

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that the damping of the GDR into the compound nucleus eigenstates can be well estimated just from its coupling to the set of doorway states.

For this purpose, we shall use a model for the mixing of states developed within the context of nuclear rotational damping [3]. This model is based on two assumptions. First, the spreading of any state $|\mu\rangle$ into the compound nucleus eigenstates $|i\rangle$ associated with the intrinsic chaotic dynamics is characterized by random, uncorrelated amplitudes X_μ^i,

$$|\mu\rangle = \sum_i X_\mu^i |i\rangle . \tag{2}$$

This is fully consistent with the results from random matrix theories [4] predicting uncorrelated Gaussian distributions for the expansion coefficients X_μ^i in an arbitrary basis. Since the basis formed by the states $|\mu\rangle$ is very different from the eigenbasis formed by the compound nuclear states $|i\rangle$, we expect this to be valid in the present case. Realistic shell model calculations [5] also give support to this assumption.

The second assumption is that of the uniformity of the spectrum and of the structure of the states $|\mu\rangle$. To each of these states we can associate a strength function

$$P_\mu(E_i) = \rho(E_i)\,|\langle\mu|i\rangle|^2 = \rho(E_i)\,|X_\mu^i|^2, \tag{3}$$

where ρ denotes the total density of states with the same quantum numbers as those of the mode under consideration. The strength functions are normalized according to $\int dE_i\, P_\mu(E_i) = 1$. The assumption of uniformity asserts that $P_\mu(E_i) = P(E_i - E_\mu)$, where E_μ is the unperturbed energy of $|\mu\rangle$ and where we have implicitly assumed averaging over some nearby states $|\mu\rangle$. In other words, the states $|\mu\rangle$ are all characterized by a single strength function P centered around each unperturbed energy E_μ. Arguments in favor of such an assumption can be found in estimates of the damping width of the shell-model states (cf., e.g, Ref. [3], appendix A). Using Fermi's golden rule and constant interaction matrix elements, the damping widths are found to have only a rather weak dependence on the number of particle-hole excitations.

On the other hand, the strength function $P_c(E - E_c)$ associated with the collective state $|c\rangle$ can be exceptional because of its highly coherent character which may reduce its damping width below that of incoherent 1p-1h states [2]. As a result, we shall allow the strength function P_c of the collective resonance to be different from the strength function P.

The uniformity assumption also requires that the coupling between states $|\mu\rangle$ has a uniform distribution. For the associated average coupling strength per unit energy, we write

$$\rho_\mu(E_\nu)\,\overline{|H_{\mu\nu}|^2} = \rho_\mu(E_\nu)\,\overline{|\langle\mu|H|\nu\rangle|^2} = f(E_\nu - E_\mu). \tag{4}$$

Here ρ_μ denotes the level density of those states which couple, through the interaction V, to a given state $|\mu\rangle$. Similarly, for the GDR we write

$$\rho_c(E_\mu)\,\overline{|H_{c\mu}|^2} = \rho_c(E_\mu)\,\overline{|\langle c|H|\mu\rangle|^2} = f_c(E_\mu - E_c), \tag{5}$$

with ρ_c being the density of doorway states associated with the collective excitation $|c\rangle$.

The diagonalization of the full Hamiltonian H now proceeds in two steps. First H is diagonalized among all the states $|\mu\rangle$ except for the single state $|c\rangle$, thus producing a new set of eigenstates $|\alpha\rangle$. These states $|\alpha\rangle$ have no mutual interactions but couple only to the state $|c\rangle$ with typical matrix elements

$$\begin{aligned}\overline{|H_{c\alpha}|^2} = \overline{\Big|\sum_\mu H_{c\mu}\langle\mu|\alpha\rangle\Big|^2} &\approx \sum_\mu \overline{|H_{c\mu}|^2}\,\overline{|\langle\mu|\alpha\rangle|^2} \\ &\approx \sum_\mu \overline{|H_{c\mu}|^2}\,\frac{1}{\rho(E_\alpha)}\,P_\mu(E_\alpha),\end{aligned} \tag{6}$$

where the strength function P_μ is defined in Eq. (3). In writing the above equation, the following assumptions have been used: (1) that the amplitudes are uncorrelated and (2) that the omission of couplings between the states $|\mu\rangle$ and the collective state $|c\rangle$ does not affect the strength function $P_\mu(E)$ describing a typical state $|\mu\rangle$.

The coupling matrix elements $H_{c\alpha}$ control the last stages of the relaxation process leading to extremely complicated (chaotic) compound nucleus states $|i\rangle$, each carrying tiny fractions of the collective strength. If N is the number of principal shell model components in the compound nucleus states, then $\rho \sim N$ and, from Eq. (6) $H_{c\alpha} \sim N^{-1/2}$. Such arguments have been used before [6] to explain the dynamical enhancement of parity nonconservation in compound nuclear states (see also Ref. [7]).

The second part of the diagonalization can be carried out exactly, with the strength function given by [8]

$$\begin{aligned}P_c(E - E_c) &= \frac{1}{\pi}\,\mathrm{Im}[E - E_c - \Sigma_c(E - E_c)]^{-1}, \\ \Sigma_c(E - E_c) &= \sum_\alpha \frac{|H_{c\alpha}|^2}{E - E_\alpha - i0}.\end{aligned} \tag{7}$$

The self-energy part $\Sigma_c(E)$ of the collective excitation is expressed in terms of mixing matrix elements given in Eq. (6). Since we average over nearby shell model states, we may write

$$\begin{aligned}\Sigma_c(E - E_c) &\approx \sum_\alpha \sum_\mu \frac{1}{E - E_\alpha - i0}\,\overline{|H_{c\mu}|^2}\,\frac{1}{\rho(E_\alpha)}\,P(E_\alpha - E_\mu) \\ &\approx \int dE_\alpha \int dE_\mu \Big[\rho_c(E_\mu)\,\overline{|H_{c\mu}|^2}\Big]\frac{P(E_\alpha - E_\mu)}{E - E_\alpha - i0} \approx \int dx \int dy\, f_c(x - y)\,\frac{P(y)}{E - E_c - x - i0}.\end{aligned} \tag{8}$$

Equations (7) and (8) relate the strength function P_c of the collective state to the strength function P of states $|\mu\rangle$.

One can use the same procedure of two-step diagonalization for any state $|\mu\rangle$. In keeping with Eq. (7), the strength function P_μ is written in terms of the corresponding self-energy Σ_μ which includes coupling to other states $|\nu\rangle$. Using the assumption of uniformity, the infinite set of equations for P_μ is substituted by the integral equation for the common strength function $P(E)$,

$$P(z) = \frac{1}{\pi}\,\mathrm{Im}[z - \Sigma(z)]^{-1},$$
$$\Sigma(z) \approx \int dx \int dy\, f(x - y)\,\frac{P(y)}{z - x - i0}. \qquad (9)$$

The set of coupled equations (7)–(9) can be solved by iterating and defining the collective strength function in terms of average coupling matrix elements introduced in Eqs. (4) and (5). The relations (7)–(9) implicitly contain the coupling of the collective excitation to a hierarchy of ever more complicated many-particle–many-hole states $|\mu\rangle$. In ordinary shell model calculations the set of equations is truncated at a maximum number of particle-hole excitations. Here, instead, we have imposed uniformity in the equations so that strength functions for high seniority states, i.e., large number of particle-hole excitations, are not neglected but rather retained at a limiting form.

The strength functions obtained by solving the set of coupled equations (7)–(9) depend on both the strength as well as the energy range of the residual interaction. In order to discuss some of the general properties of the solutions and eventually find analytic solutions to the equations, we parametrize the couplings given in Eqs. (4) and (5) as

$$f_c(E) = \frac{\beta_c v_c^2}{2\pi}\,\frac{W_c}{E^2 + (W_c/2)^2} \qquad (10)$$

and

$$f(E) = \frac{\beta v^2}{2\pi}\,\frac{W}{E^2 + (W/2)^2}. \qquad (11)$$

The quantity W_c measures the range in energies E_μ over which the collective state $|c\rangle$ couples to the doorway states. The strength of the residual interaction $\beta_c v_c^2$ has been written in terms of a typical rms matrix element v_c and the number of doorway states β_c. If the collective state couples to an unbounded set of doorway states, we may let both β_c and W_c go to infinity while keeping their ratio β_c/W_c constant, the ratio being proportional to the density ρ_c of doorway states. With this parametrization, the second moment of the strength function P_c of the collective state $|c\rangle$ is given by

$$M_c^{(2)} = \langle c|(H - \langle H\rangle)^2|c\rangle = \sum_\mu |\langle c|H|\mu\rangle|^2$$
$$\approx \int dE_\mu\, f_c(E_\mu - E_c) = \beta_c v_c^2 = \sum_{\mu=1}^{\beta_c} v_c^2, \qquad (12)$$

and a similar expression holds for the second moment $M_\mu^{(2)}$ $(= \sum_{\nu=1}^{\beta} v^2)$ of any state $|\mu\rangle$.

With the above parametrization, the self-energies acquire the form

$$\Sigma_c(z) = \int dx\,\frac{\beta_c v_c^2}{z - x - iW_c/2}\,P(x),$$
$$\Sigma(z) = \int dx\,\frac{\beta v^2}{z - x - iW/2}\,P(x). \qquad (13)$$

In the weak coupling limit, $\sqrt{\beta_c}v_c \ll W_c$ and $\sqrt{\beta}v \ll W$, the self-energy $\Sigma_c(z)$ is a slowly changing function of energy within the spreading width. Consequently, the collective strength function will not depend on the details of $P(x)$ and eventually attain a Breit-Wigner form (exponential decay),

$$P_c(E - E_c) = \frac{1}{2\pi}\,\frac{\Gamma_c}{(E - E_c)^2 + (\Gamma_c/2)^2}, \qquad (14)$$

where the spreading width of the collective resonance is given by

$$\Gamma_c = 2\,\mathrm{Im}\Sigma_c(0) = \frac{4\beta_c v_c^2}{W_c}, \quad \sqrt{\beta_c}v_c \ll W_c. \qquad (15)$$

This result also could have been obtained directly from Eqs. (5) and (10) using Fermi's golden rule, that is,

$$\Gamma_c = 2\pi v_c^2 \rho_c(E_\mu \approx E_c) = 2\pi f_c(0). \qquad (16)$$

In keeping with the assumptions made above, this relation applies in those cases in which the damping width Γ_c is much smaller than the range of the interaction W_c.

In the strong coupling limit for the intrinsic states, that is, when $\sqrt{\beta}v \gg W$, the exact shape of the coupling parametrized in Eq. (10) is not important and the Hamiltonian can be modeled by a banded matrix with random matrix elements of magnitude v in a band of size β surrounding the diagonal. The resulting self-consistent strength function of generic background states is a semicircle [3] of radius R, that is,

$$P(z) = \frac{2}{\pi R^2}\sqrt{R^2 - z^2}, \quad R = 2\sqrt{\beta}v, \quad \sqrt{\beta}v \gg W. \qquad (17)$$

The collective strength function, solution of Eqs. (13) and (17), becomes

$$P_c(z) = \frac{2}{\pi}\,\frac{\sqrt{R^2 - z^2}}{R_c^2 + 4z^2(R^2/R_c^2 - 1)}, \qquad (18)$$

where $R_c = 2\sqrt{\beta_c}v_c$, that is, a hybrid of the semicircle and Breit-Wigner shapes. The effective spreading width (FWHM) of the collective resonance varies from $\Gamma_c = \sqrt{3}R_c$ at $R_c = R$ to $\Gamma_c = R_c^2/R$ at $R \gg R_c$. Again, the upper boundary for the width is determined solely by the coupling to the doorway states. Subsequent scattering into the compound nucleus eigenstates may change the

shape of the strength function but does not add to the damping width, which may actually decrease due to the effect similar to motional narrowing [3,9]. Note that the result given by Eq. (18) (strong coupling limit) could not have been obtained in perturbation theory.

It is important to emphasize that our limiting solutions given by Eqs. (14) and (18) are valid only for the central part of the strength function. The wings of the distribution depend on the details of the energy dependence of the coupling matrix elements given in Eqs. (10) and (11) and cannot be determined in general form.

The estimates of P_c collected in Eqs. (14) and (18) give an upper limit for the damping width of the collective state. These results are expected to remain valid also in the case where the compound nucleus has a very high excitation energy and the collective state is thermally excited, provided a thermal averaging of Eqs. (7)–(9) is carried out.

In the case of the GDR, the result of microscopic calculations [10], which can be understood in terms of general arguments, shows that as the excitation energy (temperature) of the compound nucleus increases, the number of doorway states increases, while the average coupling v_c decreases so that the combination $R_c = 2\sqrt{\beta_c} v_c$ remains essentially constant. In fact, as the temperature T of the system is increased, the collectivity of the low-lying surface vibrations, to which the GDR couples, is decreased. This phenomenon takes place through a fragmentation of the surface vibrations over an energy range of the order of T. The effect of each doorway state is therefore reduced as compared to the $T = 0$ situation. In the cases studied [10] this effect more than upsets the increase in the number of doorway states taking place as T increases.

In keeping with the above discussion, it may be useful to remember [cf. Eq. (12)] that R_c is proportional to the square root of the expectation value of H^2 in the collective state and thus does not contain any exponentially growing parameters. Since $\sqrt{3} R_c$ in all cases provides an upper bound for the damping width, we conclude that the width does not increase indefinitely with temperature but saturates at a modest value, determined by the properties of the residual interaction between simple states.

Experimental studies of the properties of the GDR in hot nuclei seem to give support to these results. In fact, recent measurements of the giant dipole resonance strength function and of the anisotropy of the emitted gamma rays [11] have allowed us to separate the effects that intrinsic excitation energy (temperature) and angular momentum have on the GDR strength function. The results indicate that the observed increase in the experimentally measured width is accounted for almost entirely by the increased deformation of the nuclear shape, driven by angular momentum (cf. also Ref. [12]).

Another example of this constancy is provided by the isobaric analog resonance. Experimental studies show that the variations of the spreading width of this resonance at different excitation energies, spins, parities, and isospins in different nuclei are small [13]. In this case the residual interaction mixes the resonance with the background states of lower isospin. The ratio $\Gamma^{\downarrow}/v_c$ gives a measure of the isospin violation by the residual forces which is a smooth function of the mass number A [14]. Recent experimental data for the isospin mixing in light nuclei ($A \approx 60$) [15] show that the width $\Gamma^{\downarrow}$ remains constant even at excitation energies of the order of 100 MeV, where the evaporation width is much larger than $\Gamma^{\downarrow}$.

We conclude that the damping width of a collective vibration does not depend on the detailed coupling to the compound nucleus eigenstates. For chaotic intrinsic dynamics, the width is not sensitive to the temperature and at most is equal to few times a typical matrix element of the residual effective interaction mixing different mean field configurations.

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