Is There a Domain Wall Problem?

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We show that spontaneous breaking of discrete or continuous symmetries does not necessarily imply either symmetry restoration or the thermal production of defects at high temperature (at least up to $T \sim M_{\rm Planck}$). This may imply that there is no domain wall problem. As an example we show how this applies to the Peccei-Quinn scenario.

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We know from daily life that in the process of being heated physical systems normally undergo phase transitions becoming less ordered. By analogy one suspects the same of the field theory systems with spontaneous symmetry breaking; at sufficiently high temperature the order parameter or the vacuum expectation value (VEV) of the scalar field should vanish, leading to symmetry restoration. It turns out that this is precisely what happens in the case of a single scalar field [1]. If true, in general, this would lead in many cases to the production of topological defects during the phase transition [2]. Some of these defects, such as domain walls, are disastrous for cosmology, since in the context of the standard big bang scenario they carry too much energy. This is known as the domain wall problem [3].

Various remedies have been offered to this problem, the most celebrated one being inflation. However, this beautiful mechanism would be of no help in many interesting cases when the temperature is below the scale of inflation whose era is expected to end at very high scales (around 10^{16} GeV or so) [4].

In the present paper we address the question of whether the symmetry nonrestoration can provide a solution to the domain wall problem. The fact that symmetries may remain broken at high T was found long ago [5,6], however, to our knowledge, the possible role of the symmetry nonrestoration for the solution of the domain wall problem was, in fact, never studied.

This has prompted us to reconsider this important issue of the high temperature behavior of gauge theories. Much to our surprise, we find out that the nonrestoration of symmetry at high T seems to be a natural consequence in many minimal and realistic models (in particular, in the theories of spontaneously broken P, CP, and Peccei-Quinn symmetries). We shall present these findings in a separate publication; here we show how this happens in the invisible axion model. Furthermore, it turns out that the requirement that the dangerous domain walls (or strings) are not produced thermally, at least up to temperatures of order M_P, implies additional constraints on the parameters.

Let us first recall briefly the essential features of both symmetry restoration and nonrestoration mechanisms at high temperature. The symmetry restoration at high T can be illustrated on the example of real scalar field ϕ with the Lagrangian

$$L(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}(\phi^2 - \eta^2)^2. \tag{1}$$

This Lagrangian possesses a discrete Z_2 symmetry $\phi \to -\phi$ spontaneously broken by $\langle\phi\rangle = \pm\eta$. The dominant high temperature contribution to the effective potential for $T \gg \eta$ is given by

$$\Delta V(T) = \frac{\lambda}{8} T^2 \phi^2. \tag{2}$$

Since the boundedness of the potential demands $\lambda > 0$, for $T \gg T_C = 2\eta$, this leads inevitably to the restoration of the Z_2 symmetry.

Now, according to the standard Kibble [2] scenario, this fact leads to the production of domain walls during the phase transition when the Universe cools down below T_C.

However, the situation may change drastically in theories with more fields. To see this, take a simple example of two real scalar fields ϕ_1 and ϕ_2 with a potential

$$V = \frac{\lambda_1}{4}(\phi_1^2 - \eta_1^2)^2 + \frac{\lambda_2}{4}(\phi_2^2 - \eta_2^2)^2 - \frac{\lambda}{2}\phi_1^2\phi_2^2, \tag{3}$$

which has a $Z_2 \otimes Z_2$ symmetry: $\phi_1 \to -\phi_1, \phi_2 \to \phi_2$ and vice versa. Now the boundedness of the potential requires

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad \lambda_1\lambda_2 > \lambda^2. \tag{4}$$

At high T the potential receives the correction [5]

$$\Delta V(T) = \frac{1}{24}[(3\lambda_1 - \lambda)\phi_1^2 + (3\lambda_2 - \lambda)\phi_2^2]T^2. \tag{5}$$

One and *only* one of these mass terms can be negative without conflicting (4), meaning that one VEV can remain

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nonzero at high T. Thus the symmetry is only partially restored to a single Z_2.

Now it is time to ask: What about the domain wall problem in such theories? For simplicity let us return to the case with two Z_2 symmetries. First of all, the reader should not be confused by only partial nonrestoration, since in realistic cases at least one of the fields transforms under a continuous gauge SU(2) ⊗ U(1) symmetry. As a result one of the Z_2 factors automatically becomes a part of the SU(2) ⊗ U(1) group and obviously cannot lead to the domain wall formation. Thus we always need to "nonrestore" only one actual Z_2 at high T. Let us choose as such the $\phi_1 \to -\phi_1$ symmetry in our toy model, meaning that we will assume

$$3\lambda_1 - \lambda < 0. \tag{6}$$

Now at high T the effective potential is minimized by

$$\langle \phi_1 \rangle = \pm T\left(\frac{\lambda - 3\lambda_1}{12\lambda_1}\right)^{1/2} \tag{7}$$

and the symmetry is never restored. Although the order parameter (VEV of ϕ_1) is growing with temperature, the thermal production of domain walls will take place anyway at some (very high) T. That is, if we start heating the system in a homogeneous "initial condition," in which at $T = 0$ the vacuum is in, say, (+) phase everywhere, the thermal fluctuations will finally destroy this picture by pulling $\langle \phi_1 \rangle$ over the potential barrier and therefore creating domains of new (−) phase separated from the "old" ones by domain walls. However, such an effect could only take place at very high temperatures (in the very early Universe). Production of a spherically symmetric wall of radius R is equivalent to the creation of a vacuum bubble of the different phase.

The corresponding rate per unit time and unit volume is given by [7]

$$\Gamma = T^4\left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}, \tag{8}$$

where S_3 is the energy of the bubble (domain wall)

$$S_3 = 4\pi R^2 \sigma \tag{9}$$

and σ is the energy density per unit area [8] of a planar wall

$$\sigma = \frac{4}{3}\sqrt{\frac{\lambda_1}{2}}\,\langle \phi_1 \rangle^3. \tag{10}$$

Now, a spherical wall of thickness larger than its own radius would simply mean that no domain of the opposite phase is formed, since the Higgs field is in the false vacuum *everywhere* inside the given sphere. So, only walls whose size is bigger than their width should be considered as being "formed." The width of a domain wall is given by

$$\delta = \sqrt{\frac{2}{\lambda_1}}\,\langle \phi_1 \rangle^{-1} = 2\sqrt{\frac{6}{\lambda - 3\lambda_1}}\,T - 1\,, \tag{11}$$

and so we have $R > \delta$. If you assume for the moment $R \gg \delta$, from (7) and (9) you get

$$\frac{S_3}{T} \simeq \frac{2\pi}{9\sqrt{6}}\,\frac{(\lambda - 3\lambda_1)^{3/2}}{\lambda_1} R^2 T^2, \tag{12}$$

and thus

$$\frac{S_3}{T} \gg \frac{16\pi}{3\sqrt{6}}\,\frac{\sqrt{\lambda - 3\lambda_1}}{\lambda_1}\,. \tag{13}$$

One can see that, for λ_1 small, the above suppression factor can be as small as one wishes and the numerical factors work in our favor. Of course, as R gets closer to δ, the numerical factors become less certain. However, the qualitative feature remains: for small λ_1 one gets a large suppression.

Of course, this analysis breaks down for temperatures close to the Planck scale. Since we really do not know what might happen at such high temperatures, one may hope that the Planck scale corrections can modify our effective potential in such a way that domain wall production even there never happens. This is a subject on which we cannot speculate, but the important message from our analysis is that one can naturally raise the formation temperature (say, for the electroweak scale domain walls) by 16 to 17 orders of magnitude. Consequences are straightforward: (a) either the walls are never formed or (b) if formed, inflation, even if it takes place at scales close to M_P and with an arbitrary high reheating temperature, can cure any domain wall problem including those attributed to the electroweak symmetry breaking. Of course, as we said before, here we have assumed the Universe to be homogeneous on the scales of the comoving scale of the present horizon. We cannot justify this and it obviously cries for inflation. For the reason of space we leave the discussion of inflation and its impact on the production of domain walls for a longer paper now in preparation.

In some cases, for example, in the famous Peccei-Quinn scenario for the solution of the strong CP problem, the domain wall problem is a consequence of the existence of cosmic strings in the model. Thus in the context of symmetry nonrestoration the solution of the axionic domain wall problem reduces to the elimination of the string producing phase transition. In complete analogy with the domain wall case above, we can estimate the string production rate. For this, assume that in our toy model one of the Higgs fields (say, ϕ_1) is transforming under a global U(1) symmetry. Since U(1) is spontaneously broken, this model admits a topologically stable global string solution [8].

Now much in the way of the Z_2 symmetry, the high T correction to the ϕ_1-dependent part of the potential is

$$\Delta V(T, \phi_1) = \frac{4\lambda_1 - \lambda}{24} T^2 |\phi_1|^2, \tag{14}$$

so that for $4\lambda_1 - \lambda < 0$, the U_1 symmetry is never restored, with

$$|\langle \phi_1(T) \rangle| = \left(\frac{\lambda - 4\lambda_1}{12\lambda_1}\right)^{1/2} T \tag{15}$$

at high T.

Again, the rate of the thermal production of a closed string with radius R is given by (8), where $S_3 = 2\pi R\mu$ and μ is the energy per unit length

$$\mu = \int [|\nabla\phi_1|^2 + V(\phi_1) - V(\langle\phi_1\rangle)]\, d^2x\,. \tag{16}$$

To estimate μ it is useful to separate the contributions inside and outside of the core of the string, i.e., to assume for the straight string (in cylindrical coordinates ρ, θ, z)

$$\phi_1(x) = \langle\phi_1\rangle f(\rho) e^{i\theta}, \tag{17}$$

where $f(\rho)$ is some (monotonic) function such that $f(0) = 0$ and $f(\rho) = 1$ for any $\rho > \delta$. Here δ is the thickness of the string, and obviously we assume $R \gg \delta$. (In reality f will approach 1 exponentially as one goes to infinity.) Then the general form of μ is

$$\mu = 2\pi\langle\phi_1\rangle^2 \left[K(\delta) + \ln\frac{R}{\delta} \right], \tag{18}$$

where K comes from the core contribution and depends on the explicit form of f. For any given ansatz f, the thickness is determined through minimization of μ as $\delta^{-1} = dK/d\delta$, giving K a number of order 1. For example, for the linear dependence inside the core $f = \rho/\delta$ we have $K = 3/2$ and $\delta = \sqrt{12/\lambda_1}\langle\phi_1\rangle^{-1}$ and thus from $\mu > 2\pi\langle\phi_1\rangle^2$ we find the suppression factor for the string production

$$\frac{S_3}{T} > 4\pi^2\langle\phi_1\rangle^2 \frac{R}{T} \tag{19}$$

which for $R \gg \delta$ gives

$$\frac{S_3}{T} \gg 4\pi^2 \frac{\sqrt{\lambda - 4\lambda_1}}{\lambda_1}\,. \tag{20}$$

As in the case of walls, the thermal production rate for strings is suppressed for small λ_1. Notice that the numerical factor in front is already large. However, again for R close to δ it cannot be trusted.

Here we wish to discuss how the possibility of symmetry nonrestoration may solve the infamous domain wall problem of the Peccei-Quinn mechanism [9]. This mechanism, commonly accepted as the solution to the strong CP problem, is based on the concept of the continuous anomalous symmetry $U(1)_{PQ}$, whose explicit breaking by instantons fixes $\overline{\theta}$ to be naturally small. The instanton effects in the Higgs sector can be mimicked by the effective phase dependent term in the potential [10]

$$\Delta V + \Lambda_{QCD}^4 (1 - \cos N\theta), \tag{21}$$

where N is an integer and $a = \theta M_{PQ}$ is the axion field. For example, in the invisible axion extension [11] of the original Peccei-Quinn model N is the number of quark flavors. Thus instantons preserve a discrete subgroup of $U(1)_{PQ}$ characterized by $\theta \rightarrow \theta + 2\pi/N$. The eventual spontaneous breaking of this discrete symmetry leads to the formation of domain walls [12]. The dynamics of the domain wall formation goes as follows [13]. At the scale M_{PQ} when $U(1)_{PQ}$ is broken spontaneously the network of global axionic cosmic strings is formed and θ winds by 2π around each minimal string. Later on, at the temperature $T \sim \Lambda_{QCD}$ the instanton effects are switched on and it becomes energetically favorable to choose one out of the discrete set of values $2\pi k/N$ ($k = 1, 2, \ldots, N$). Since $\Delta\theta = 2\pi$ around the string, this results in the formation of N domain walls attached to the string. The domain walls are topologically stable and thus cosmologically troublesome for $N > 1$.

Note that a remarkably simple way out would be not to have strings formed at all, which as we have just seen could result from the nonexistence of the first phase transition. In such a case, above $T \simeq \Lambda_{QCD}$, θ would be aligned having some typical value θ_0 which after the QCD phase transition would relax to the nearest minimum (unless, of course, by some miracle θ_0 would turn out to lie at one of the local maxima). Now, the minimal realistic PQ model is based on the introduction of a singlet field on top of two Higgs doublets [11]. As the reader knows by now, it is perfectly natural to keep the VEV of the singlet nonvanishing at high T, thus avoiding the formation of cosmic strings and the subsequent troublesome domain walls.

Let us discuss this briefly. The potential for the PQ model with the doublets ϕ_i ($i = 1, 2$) both having $Y = 1$ and a $SU(2) \times U(1)$ singlet S may be written as

$$\begin{aligned} V_{PQ} = &-\sum_i \frac{\mu_i^2}{2}\phi_i^\dagger\phi_i + \sum_i \frac{\lambda_i}{4}(\phi_i^\dagger\phi_i)^2 + \frac{\lambda}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \frac{\lambda'}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) - \frac{\mu_s^2}{2}S^*S + \frac{\lambda_s}{4}(S^*S)^2 \\ &+ \alpha(\phi_1^\dagger\phi_2 S + \phi_2^\dagger\phi_1 S^*) + \frac{1}{2}\left(\sum_i \beta_i\phi_i^\dagger\phi_i\right)S^*S\,. \end{aligned} \tag{22}$$

Besides the $SU(2)_L \times U(1)_Y$ local gauge symmetry, V_{PQ} has a chiral $U(1)_{PQ}$ symmetry (ϕ_1 couples to, say, down quarks, and ϕ_2 to up quarks)

$$\phi_1 \rightarrow e^{i\alpha}\phi_1, \qquad \phi_2 \rightarrow e^{-i\alpha}\phi_2, \qquad S \rightarrow e^{2i\alpha}S\,. \tag{23}$$

Now, among other terms at high T, we will get

$$\Delta V(T) = \cdots - \frac{1}{6}(\lambda_s + \beta_1 + \beta_2)T^2 S^*S\,. \tag{24}$$

Since we can take $\lambda_s + \beta_1 + \beta_2 < 0$, the temperature dependent mass term for S remains negative and thus $\langle S\rangle \neq 0$ at high T.

Notice that the nonrestoration of $U(1)_{PQ}$ at high T is impossible without the singlet. Namely, with two doublets only one of the VEVs can remain nonzero at high T [since $U(1)_{PQ}$ forbids terms linear in the fields] implying a necessary restoration of $U(1)_{PQ}$. We find it remarkable that the singlet is essential for the solution of the domain wall problem, since it is needed for completely different reasons, both phenomenological and astrophysical [10].

In summary, we have shown here that even in the minimal models there may not be a domain wall problem. We must stress *may not*, since one does not know what can happen when one approaches the Planck scale. The crucial point is that the usual scenario of Kibble may not hold in general, and whether or not there is a phase transition depends on as of yet unknown parameters in the theory in question.

Furthermore, one can show that for a large range of parameters, the thermal production of domain walls and strings never takes place (except possibly for $T \simeq M_P$). This means that, even in the case that these objects do get produced for T near the Planck scale, there is plenty of time for inflation to dilute the density of their remnants. This provides a solution to the domain wall problem even for the case of electroweak scale breaking of the discrete symmetry in question. Finally, this would also imply a way out of the infamous domain wall problem in the Peccei-Quinn scenario with the invisible axion.

As we mentioned before, the details of the high temperature phase diagrams and the production rates of topological defects in the early Universe will be presented in a longer paper [14].

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