Coulomb Gap and Correlated Vortex Pinning in Superconductors

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The positions of columnar pins and magnetic flux lines determined from a decoration experiment on $Bi_2Sr_2CaCu_2O_8$ were used to calculate the distribution of pinning energies in the Bose glass phase. A wide Coulomb gap is found, with effective gap exponent $s_{\text{eff}} \approx 1.2$, as a result of interactions between the vortices. As a consequence, the variable-range hopping transport of flux lines is considerably reduced with respect to the noninteracting case, the effective Mott exponent being enhanced from $p_0 = 1/3$ to $p_{\text{eff}} \approx 0.5$ for this sample.

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The remarkably rich phase diagram of magnetic flux lines in high-T_c superconductors, especially when subject to point and/or extended disorder, has attracted considerable experimental and theoretical interest [1]. Understanding the interaction of vortices with defects is especially important since flux lines must be pinned to minimize dissipative losses from flux creep. A promising pinning strategy involves the creation of linear damage tracks in materials by heavy-ion irradiation. These columnar defects effectively increase the critical current, and shift the irreversibility line significantly upwards [2]. Intrinsic *point* disorder (oxygen vacancies) is believed to be negligible under these conditions, especially *above* the irreversibility line of the unirradiated material [3].

A theory of flux pinning by correlated disorder has been developed to explain these results, exploiting a formal mapping of the statistical mechanics of directed lines onto the quantum mechanics of bosons [3]. In this study, the intervortex repulsion, whose range is determined by the London penetration depth λ, was only treated using approximate, order of magnitude estimates. However, if $\lambda \geq a_0$, where $a_0 = (4/3)^{1/4}(\phi_0/B)^{1/2}$ is the average distance between vortices ($\phi_0 = hc/2e$ is the elementary flux quantum), the interactions become strong, and may lead to important correlation effects.

A central concept here is $g(\epsilon)$, the distribution of pinning energies ϵ with interactions taken into account; by suitably "tailoring" the width and shape of $g(\epsilon)$, the vortex transport properties may be optimized. Indeed, the analogy of flux lines at low temperatures pinned to columnar defects (Bose glass) with localized carriers in doped semiconductors (Coulomb glass) [4] suggests that interactions may lead to a soft "Coulomb" gap in $g(\epsilon)$. When $\lambda \gg a_0$, the intervortex potential becomes a logarithmic repulsion, which replaces the $1/r$ interaction between electrons or holes in a semiconductor. Because such a gap will affect significantly the current-voltage characteristics in a variable-range hopping approach [3], it is important to estimate its size in the Bose glass phase and to understand the correlations induced by the intervortex repulsion. Recent successes, moreover, in simultaneously measuring both the columnar defect and flux line positions [5,6] allow for detailed comparison of the spatial correlations found in experiment and theory. Such a comparison is not feasible for semiconductors.

The distribution of pinning energies may be obtained by using a variant of the Monte Carlo algorithm described by Shklovskii and Efros [4,7]. Using the experimentally determined columnar defect and flux line positions shown in Fig. 1, we can predict the distribution of pinning energies and the transport characteristics for this specific sample in the variable-range hopping regime, at temperatures slightly below the irreversibility line. We find that the ensuing Coulomb gap is remarkably large, even in the case $\lambda \approx a_0$, i.e., when the interactions are relatively short range. Vortex repulsions raise the effective Mott exponent from the noninteracting result $p_0 = 1/3$ to a value $p_{\text{eff}} \approx 0.5$ for this specific sample. These results imply that correlation effects strongly enhance the pinning of flux lines to columnar defects.

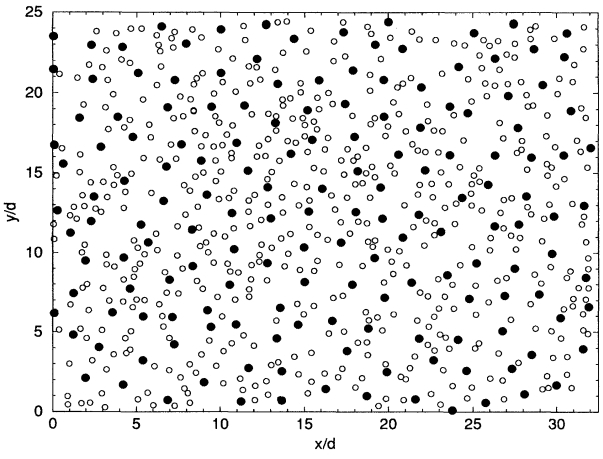

FIG. 1. Positions of empty columnar defects (open circles), and pins occupied by flux lines (filled circles), as obtained from the experiment ($f \approx 0.24$, $\lambda \approx 0.45a_0$).

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We consider the following model free energy for N flux lines, defined by their trajectories $\mathbf{r}_i(z)$ as they traverse the sample, with the magnetic field $\mathbf{B}$ aligned along the z axis (perpendicular to the Cu-O planes) [3],

$$F = \int_0^L dz \sum_{i=1}^{N} \left\{ \frac{\tilde{\epsilon}_1}{2} \left(\frac{d\mathbf{r}_i(z)}{dz} \right)^2 + \frac{1}{2} \sum_{j \neq i}^{N} V[r_{ij}(z)] + \sum_{k=1}^{N_D} V_D[\mathbf{r}_i(z) - \mathbf{R}_k] \right\}. \quad (1)$$

Here $r_{ij}(z) = |\mathbf{r}_i(z) - \mathbf{r}_j(z)|$, and $V(r) = 2\epsilon_0 K_0(r/\lambda)$ is the repulsive interaction potential between the lines; the modified Bessel function $K_0(x) \propto -\ln x$ for $x \to 0$, and $\propto x^{-1/2} e^{-x}$ for $x \to \infty$. Thus the (in-plane) London penetration depth λ defines the interaction range. The energy scale is set by $\epsilon_0 = (\phi_0/4\pi\lambda)^2$, and $\tilde{\epsilon}_1$ is the tilt modulus. Finally, the pinning energy is a sum of N_D z-independent potential wells V_D with average spacing d centered on the $\{\mathbf{R}_k\}$, whose diameters are typically $c_0 \approx 100$ Å, with a variation of $\delta c_k/c_0 \approx 15\%$, caused by the ion-beam dispersion. This induces some distribution P of the pinning energies U_k, which may be determined from the (interpolation) formula $U_k \approx (\epsilon_0/2) \ln[1 + (c_k/\sqrt{2}\,\xi)^2]$ (ξ is the coherence length) [3]. For example, for $\lambda \approx 4200$ Å one has $U_0 \approx 0.67\epsilon_0$, and $w = \sqrt{\langle \delta U_k^2 \rangle} \approx 0.1\epsilon_0$.

As is explained in detail in Ref. [3], the statistical mechanics of the model (1) can be formally mapped onto a two-dimensional zero-temperature quantum mechanical problem. In this boson analogy, the real temperature T assumes the role of $\hbar$ in the quantum problem, and the boson electric field and current density map on the superconducting current J and the true electric field $\mathcal{E}$, respectively (see Table I in Ref. [3]). Thus the roles of conductivity and resistivity become interchanged.

We are interested in the low-temperature properties of flux lines pinned to columnar defects, with filling fraction $f = N/N_D = (d/a_0)^2 < 1$, in the Bose glass phase, where all the vortices are bound to the pinning centers. For T less than a characteristic temperature $T_1 \approx 0.9T_c$ for $Bi_2Sr_2CaCu_2O_8$ [3], one arrives at the classical limit of the corresponding boson problem ($\hbar \to 0$), and as the vortices are now well separated, the Bose statistics become irrelevant. Furthermore, in this limit thermal wandering is suppressed, and the flux lines will be essentially straight; hence the tilt energy in Eq. (1) can be neglected. The ground state is then determined by minimizing a z-independent two-dimensional effective Hamiltonian

$$H = \frac{1}{2} \sum_{i \neq j}^{N_D} n_i n_j V(r_{ij}) + \sum_{i=1}^{N_D} n_i t_i, \quad (2)$$

where $i, j = 1, \ldots, N_D$ denote the defect sites, randomly distributed on the x-y plane. $n_i = 0, 1$ is the corresponding site occupation number, and the t_i are random-site energies (originating in the varying pin diameters), whose distribution may be chosen to be centered at $\bar{t} = 0$, with width w [for simplicity, we assume a flat distribution $P(t_i) = 1/2w$ for $|t_i| \leq w$, and $P(t_i) = 0$ otherwise].

The Hamiltonian (2) has the form studied in the context of localized charge carriers in doped semiconductors (Coulomb glass) [4,7,8]. It is equivalent to a two-dimensional random-site, random-field Ising antiferromagnet with long-range exchange interactions, and has eluded successful analytical approaches going beyond simplifying mean-field considerations [4], and phenomenological scaling arguments [9]. Therefore one has to resort to numerical studies using suitable Monte Carlo algorithms, as described in Refs. [4,7].

A fraction f of a given distribution of N_D defect sites is occupied, and the corresponding site energies $\epsilon_i = \partial H/\partial n_i = \sum_{j \neq i}^{N_D} n_j V(r_{ij}) + t_i$ are calculated. The initial configuration is relaxed by moving "particles" to empty places, until precisely the N lowest energy levels are filled ($\epsilon^1_{\max} = \max_{n_i=1} \epsilon_i < \epsilon^0_{\min} = \min_{n_i=0} \epsilon_i$). The ensuing state is then probed against all possible single-particle hops from filled to empty sites; i.e., if any associated energy change $\Delta_{i \to j} = \epsilon_j - \epsilon_i - V(r_{ij})$ is negative, the move from site i to j is performed, and thus the total energy decreased. Afterwards the entire process may have to be iterated. Finally, the chemical potential is determined (approximately) by $\mu = (\epsilon^0_{\min} + \epsilon^1_{\max})/2$ [10].

By repeating this procedure for different initial configurations, one may then obtain the distribution of pinning energies $g(\epsilon)$ from the site energy statistics, normalized according to $\int g(\epsilon)\, d\epsilon = 1/d^2$. [In the corresponding Coulomb gap problem in semiconductors, the function $g(\epsilon)$ would be the electronic density of states [4].] Previous investigations have shown that minimizing the total energy with respect to only single-particle hops already yields at least qualitatively reliable estimates for the energy level distribution [4,7,8]. Details of our investigations for a variety of filling fractions f and values of λ/d will be reported elsewhere [11]. In this Letter, we shall instead use these techniques to calculate the distribution of pinning energies *directly* from experimental data.

The positions of columnar defects and flux lines in a $Bi_2Sr_2CaCu_2O_8$ (BSCCO) crystal were determined simultaneously by a combined chemical etching/magnetic decoration approach (for details, see Ref. [6]). A digitized scanning electron microscopy image of the positions of $N = 162$ flux lines and $N_D = 686$ columnar defects (hence $f \approx 0.24$) for a BSCCO sample irradiated with corresponding matching field $B_\phi = 118$ G ($d \approx 4400$ Å) and decoration field $B = 27$ G ($a_0 \approx 9400$ Å) is shown in Fig. 1. The critical and irreversibility temperature under these conditions are $T_c = 87$ K and $T_{\rm irr} = 81$ K, respectively. Assuming the flux line distribution is frozen in at $T_{\rm irr}$, we estimate that the effective London penetration depth is $\lambda(T_{\rm irr}) \approx 4200$ Å $\approx 0.45a_0$ [12], and $\lambda/d \approx 0.96$.

In Fig. 2 the structure factor $S(q)$ for the flux lines is shown, as obtained from $S(\mathbf{q}) = \frac{1}{N} \sum_{i,j}^{N} e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$ by

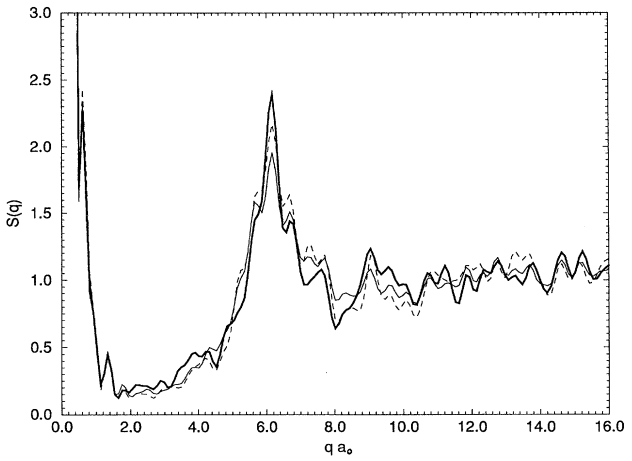

FIG. 2. Vortex structure function $S(q)$ as obtained from experiment (thick line), and from simulations with $w = 0$ (dashed), and $w = 0.1\epsilon_0$ (thin line), averaged over 100 different assignments of random-site energies.

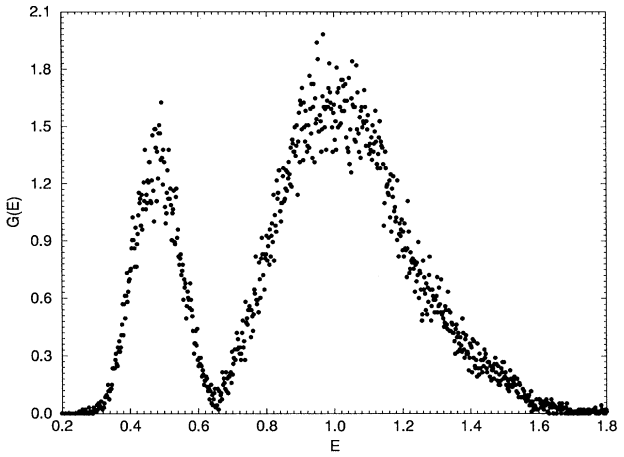

FIG. 3. Normalized distribution of pinning energies $G(E) = 2\epsilon_0 d^2 g(\epsilon)$ as a function of the site energies $E = \epsilon/2\epsilon_0$, averaged over 100 runs ($\lambda/d = 0.96$, $w = 0.1\epsilon_0$). The mean chemical potential is $\mu/2\epsilon_0 = 0.65$.

averaging over directions in Fourier space (thick line). Clearly the vortex distribution is highly correlated, with $S(q)$ displaying a peak at $q_0 a_0 \approx 2\pi$. We have used the experimental defect and flux line positions as an initial state for the Monte Carlo routine. In order to minimize boundary effects, we have kept the configuration fixed in a frame extending 10% ($\approx 3\lambda$) inwards from each of the rectangular boundaries. This leaves 464 sites and 106 displaceable particles for the simulation. For vanishing site randomness ($w = 0$), about 20% of the flux lines are moved in the course of the energy minimization process. Using the more realistic value $w = 0.1\epsilon_0$, we find typically 40% changes with respect to the original experimental distribution. But in both cases the highly correlated character of the flux line distribution is preserved (Fig. 2). Substantially stronger disorder would destroy the peak in $S(q)$. Similarly, for considerably lower values of λ ($\lambda/d \leq 0.2$), the spatial correlations also disappear, because the site randomness would then dominate over the interactions. The correlations we do find strongly support the assumption that the effective London penetration depth is $\lambda(T_{\rm irr}) > \lambda_0$ [12]. Although there is no immediate translation from $S(q)$ into the distribution of pinning energies, the presence of spatial correlations suggests the possibility of correlations in $g(\epsilon)$.

The distribution of pinning energies for the 464 "inner" sites, as obtained from averaging over 100 runs with different assignments of random-site energies, drawn from the same distribution $P(t_i)$ with width $w = 0.1\epsilon_0$, and using $\lambda(T_{\rm irr})$ for the interaction range, is shown in Fig. 3. (Results for the distribution of pinning energies are quite insensitive to the precise value of w.) The Coulomb gap in $g(\epsilon)$ is remarkably wide, even though $\lambda \approx 0.45 a_0$ is relatively small. Its width amounts to $\approx 0.5\epsilon_0$ at half maximum [$2\epsilon_0 d^2 g(\epsilon) \approx 0.8$], i.e., almost a *third* of the total width of $g(\epsilon)$. Near its minimum, this soft gap may be described approximately by the formula

$$g(\epsilon) \propto |\epsilon - \mu|^{s_{\rm eff}}, \tag{3}$$

with an effective gap exponent $s_{\rm eff} \approx 1.2$. In the immediate vicinity of the chemical potential, this power law is smeared out, and actually $g(\mu) > 0$ (but very small) due to the finite range of the interactions. In the limit $\lambda \to \infty$ and for small filling, $s_{\rm eff} \approx 3$ may be reached [11].

From this distribution of pinning energies, we may now infer the transport properties in the variable-range hopping regime. Since we used $\lambda(T_{\rm irr}) \approx 4200$ Å in determining $g(\epsilon)$, our results apply to transport slightly below $T_{\rm irr} = 81$ K, where the vortices first localize in a Bose glass with very slow dynamics. Note that for magnetic fields $B < \phi_0/\lambda^2$, i.e., $\lambda < a_0$, $\tilde{\epsilon}_1 \approx \epsilon_0$ and thermal renormalizations of pinning energies become relevant only for $T_1 \approx 0.9 T_c$ (≈ 78 K here) [3]. Thus, all temperatures $T < T_1 \approx T_{\rm irr}$ may be considered as "low."

For low currents, the most important excitation is a fluxon sending out a pair of superkinks of separation Z to another columnar defect at distance R, such that the tunneling probability between the pins is optimized [3]. Thus, we must minimize the free energy of two superkinks of size R and separation Z, $\delta F_{\rm SK} = 2E_K R/d + Z\Delta(R) - f_L RZ$ [see Eq. (4.13) of Ref. [3]]. Here, the first term consists of the energy of the double kink, with $E_K = \sqrt{\tilde{\epsilon}_1 U_0}\, d$, and the third one derives from the Lorentz force $f_L = \phi_0 J/c$ induced by the current J. The second contribution stems from the fact that for a hop of distance R, the available energy states lie in the interval $[\mu, \mu + \Delta(R)]$, where $\Delta(R)$ is determined by the equation $\int_\mu^{\mu+\Delta} g(\epsilon) d\epsilon = R^{-2}$. Minimizing first for $J = 0$ gives the longitudinal extent of the kink to be $Z^* = -2E_K/d(\partial\Delta/\partial R)_{R^*}$. To first order in J, one subsequently arrives at $J\phi_0/c = \Delta(R^*)/R^*$, and thus

$\delta F^* = 2E_K R^*(J)/d$ is the result for the optimized free energy. The latter finally enters the resistivity ρ as an energy barrier for thermal activation, $\mathcal{E} = \rho_0 J \exp(-\delta F^*/k_B T)$.

In the regime where Eq. (3) holds, the final result from these considerations for the highly nonlinear current-voltage characteristics may be cast in the form

$$\mathcal{E} \approx \rho_0 J \exp[-(2E_K/k_B T)(J_0/J)^p], \quad (4)$$

where p is an exponent generalizing Mott's law ($p_0 = 1/3$), which is valid in the case of vanishing interactions. For long-range interactions producing a Coulomb gap of the form (3), one finds $p = (s + 1)/(s + 3)$. Figure 4 shows a log-log plot of $R^*(J)/d$ vs $j = J\phi_0 d/2\epsilon_0 c$, derived from the distribution of pinning energies in Fig. 3, as compared to a similarly calculated curve with the vortex repulsion being switched off. While in the latter case the result is indeed a straight line with slope $-1/3$, interactions *considerably* enhance the pinning by raising the effective Mott exponent to $p_{\text{eff}} \approx 0.5$. For $J \to 0$, however, the cutoff of the interaction at λ reduces p_{eff} somewhat.

Similar to the gap index s, from which it is derived, p should not be understood as a universal number, but rather as some effective exponent p_{eff} conveniently describing the I-V characteristics. Its value in general depends on both the filling f and the interaction range λ/d; its maximum value $p_{\text{eff}} \approx 0.68$ is reached for $\lambda \to \infty$ and small f [11]. These results clearly rule out the mean-field estimate, which would yield $s = 2/\sigma - 1$ and $p = 1/(1 + \sigma)$ for a potential $V(r) \propto r^{-\sigma}$ ($\sigma < 2$; a logarithmic interaction is recovered for $\sigma \to 0$). They seem to be more consistent with the scaling analysis by Fisher, Tokuyasu, and Young for the supposedly equivalent gauge glass model, which yields $2/3 \leq p \leq 4/5$ [9]. Yet, even the large-system simulations by Möbius and Richter for the $1/r$ potential have apparently not reached the fully asymptotic regime [8], if it exists at all. Also, our derivation of $R^*(J)$ from $g(\epsilon)$ constitutes an approximation which neglects subtle correlations, e.g., the spatial clustering of those sites which are energetically close to μ [7,11], and may be subject to corrections in the limit $J \to 0$.

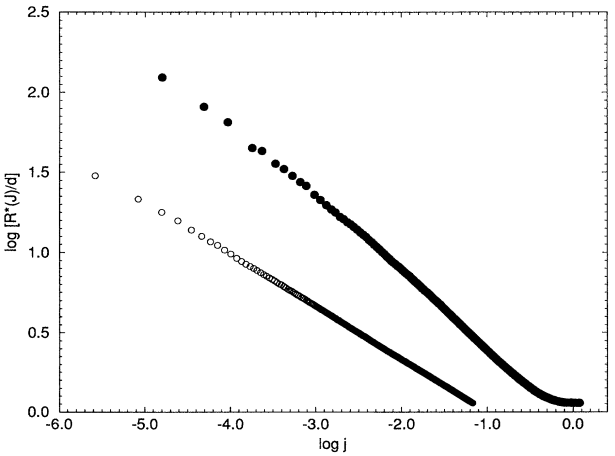

FIG. 4. Log-log plot of $R^*(J)/d$ vs $j = J\phi_0 d/2\epsilon_0 c$, as obtained from $g(\epsilon)$ in Fig. 3 (filled circles), compared to the non-interacting case (open circles).

Pinning to point defects is subject to much stronger thermal renormalization than pinning to correlated disorder. For very low currents, additional point defects may, in fact, trap the spreading of the double-superkink configuration considered above; however, this becomes effective only on unphysically large length scales [3], and thus should not alter our results for realistic samples.

In summary, we have demonstrated that the vortex-vortex repulsion can lead to remarkable correlations both in real space and in the distribution of pinning energies, even for $\lambda \approx a_0$. An important consequence of these correlation effects is the drastic enhancement of flux line pinning to columnar defects in the Bose glass phase, whenever $\lambda \geq a_0$.

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[10] Formally, a term $-\mu \sum_i n_i$ is to be added to Eq. (2); the chemical potential is related to the external magnetic field H by $\mu = \epsilon_0 \ln(\lambda/\xi) - H\phi_0/4\pi - \langle U_k \rangle$, where $\langle U_k \rangle$ may include a small thermal renormalization.

[11] U. C. Täuber and D. R. Nelson (unpublished).

[12] Assuming $\lambda_0 \approx 2100$ Å and the two-fluid formula, $\lambda(T) = \lambda_0/[1 - (T/T_c)^4]^{1/2}$, we find that $\lambda(T_{\text{irr}}) \approx 2\lambda_0 \approx 4200$ Å.