Quantum Decay of One-Dimensional Supercurrent: Role of Electromagnetic Field

Ji-Min Duan
Department of Physics, University of California at San Diego,
9500 Gilman Drive, La Jolla, California 92093
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The quantum decay of a one-dimensional small supercurrent is a charge non-neutral process which involves an electromagnetic field. I take proper account of magnetic and electric screening, respectively, and find that the electromagnetic field contributes an exponentially small *multipling* factor $e^{-A/\alpha} \sim e^{-100}$ to the quantum decay rate with α the fine structure constant and A a numerical constant of order unity. This means that a circulating supercurrent lives forever. Possible relevance with several recent experiments is discussed.

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One of the most fundamental questions about superconductivity is on the stability of circulating supercurrent. Shortly after Kamerlingh Onnes discovered superconductivity, it was found that once the circulating supercurrents were set up they were observed to flow without measurable decrease for a year [1]. Since a supercurrent is due to the gradient of the phase of the superconducting order parameter, and the phase can only change an integer times 2π when it goes once around a singly connected loop, a metastable supercurrent corresponds to the phenomenon of fluxoid quantization associated with an integer winding number [2]. When a supercurrent decays, it can only decay by a discrete amount due to the changing of the winding number by an integer, meanwhile overcoming a topological barrier. For a thin superconducting wire the topological barrier is smaller than that for a thick wire because of reduced coherence volume. In the 1960s Little [3] realized that in order for a supercurrent to decay part of the wire of a size of the coherence length must become normal due to thermal fluctuation. This was followed by a beautiful theory of Langer, Ambegaokar, McCumber, and Halperin (LAMH) [4] on the thermal decay of a supercurrent. The theoretical prediction agrees with experiments within about a factor of 2 in the exponent of thermal activation [5]. Typically, for a wire with a width of several thousand angstroms, the thermally broadened resistive transition region is of the order of 10^{-3} K.

However, the following question remains. Namely, for temperatures sufficiently low that the thermal decay rate of supercurrent is negligibly small, what is the supercurrent stability against quantum mechanical fluctuations? This question is of current relevance since there exists several more or less conflicting experimental reports [6–8] of the resistive transitions of superconducting wires as thin as several hundred angstroms. For the earlier experiment by Giordano, a crossover was observed from the LAMH region to a previously unknown region which was suggested [6] as due to a new macroscopic quantum tunneling phenomenon in the spirit of Caldeira and Leggett [9]. This has stimulated theoretical investigations on quantum phase slips in various dimensions [10–14]. For a superconducting wire the quantum dynamics of the order parameter in one-dimensional space plus one-dimensional imaginary time (1 + 1)D [11] is similar to Maki's soliton-antisoliton model [15] for charge-density waves (however, see Ref. [14]). The electromagnetic (EM) field accompanying the quantum phase slippage outside the core spreads into three-dimensional space and should be treated in (3 + 1)D, similar to Zhang's superconducting cosmic string model [16], in which electric and magnetic fields were treated equally. In a realistic superconducting wire, the roles of magnetic and electric fields are rather different: the London penetration depth λ_L for current screening can be one or more orders of magnitude larger than the Debye shielding length λ_D for charge screening. In this paper I study the influence of the EM field on the quantum decay rate of a one-dimensional supercurrent, with a quite surprising result: For a small supercurrent density J (reasonably smaller than the critical current density J_c of the wire), the EM field contributes to the quantum decay rate a minus exponent which is of the order of the inverse fine structure constant. This effect seriously reduces the possibility of experimental observation of quantum decay of 1D supercurrent. I will give an intuitive explanation of this result and discuss the relevance or irrelevance with experiments [6–8]. This work was briefly reported earlier [17].

Let us consider a homogeneous superconducting wire with cross-sectional area σ. Assume the width $\sqrt{\sigma}$ of the wire is smaller than or comparable to the temperature-dependent coherence length ξ so that it is quasi-one-dimensional. $\sqrt{\sigma}$ is also smaller than or comparable to λ_L such that a supercurrent is homogeneously distributed across the wire. Experimentally [6,7] $\sqrt{\sigma}$ ranges from a few hundred to a couple of thousand angstroms ($\sqrt{\sigma} \approx \xi, \lambda_L$). Neglecting effects due to weak links and grain boundaries and choosing the wire along the z axis, the Euclidean action S_E for the dynamics of the system is described by a phenomenological time-dependent Ginzburg-

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Landau theory [10–18]:

$$S_E = \int dx\,dy\,dz\,d\tau \Big\{ \sigma \delta^2(x,y) \Big[\nu \Big(\frac{\partial}{\partial \tau} + \frac{e^*}{\hbar} \varphi \Big) \Psi^* \Big(\frac{\partial}{\partial \tau} - \frac{e^*}{\hbar} \varphi \Big) \Psi + \gamma \Big(\vec{\nabla} + \frac{ie^*}{\hbar c} \mathbf{A} \Big) \Psi^* \Big(\vec{\nabla} - \frac{ie^*}{\hbar c} \mathbf{A} \Big) \Psi - a \Psi^* \Psi + \frac{b}{2} (\Psi^* \Psi)^2 \Big] + \frac{\mathbf{E}^2 + \mathbf{H}^2}{2} \Big\}. \quad (1)$$

Here τ is imaginary time and $\delta^2(x,y)$ is a two-dimensional delta function. $(\varphi, \mathbf{A})$ are potentials of EM fields $(\mathbf{E}, \mathbf{H})$. Ψ is the order parameter functional which includes both amplitude and phase. $e^* = 2e$ is the Cooper pair charge, a, b are usual Ginzburg-Landau coefficients, and $a/b = \Delta_0^2$ with Δ_0 the equilibrium value of the energy gap. $\xi = \sqrt{\gamma/a}$ and $\sqrt{\gamma/\nu} = v_F/\sqrt{3}$ with v_F the Fermi velocity. The intrinsic resistance of the superconducting wire [4] is proportional to the rate of quantum phase slippage:

$$\Gamma = F e^{-\Delta S_E/\hbar}, \quad (2)$$

where the fluctuation prefactor F is less important and will not be discussed here. ΔS_E is the difference between a topologically nontrivial saddle point action and that of the metastable current carrying state. To get a better physics picture I choose a dimensionless frame $(\vec{\rho}, \rho_4) = (\rho_1, \rho_2, \rho_3, \rho_4)$ in which space $\vec{\rho}$ is measured in units of ξ and imaginary time ρ_4 in units of $\sqrt{3}\,\xi/v_F$. Dimensionless potentials $\mathbf{A}' = \mathbf{A}\sqrt{\gamma/a}/e^*$, $A_4' = \varphi\sqrt{\nu/a}/e^*$, and the dimensionless order parameter is measured in units of Δ_0. To see the basic topology let us first consider a neutral superfluid (setting $e^* = 0$) where the EM field is not involved. The saddle point picture is a vortex-antivortex pair [11,16] in dimensionless (1 + 1)D, sitting in the background of a dimensionless current J/J_c (see Fig. 1). The separation between the pair is 2ρ and $\rho \approx J_c/J$. Within the vortex core (of radius unity) the amplitude of the order parameter drops to zero, contributing to the action an amount

$$S_{\rm core} \cong 4\pi\sigma\sqrt{\nu\gamma}\, a/b\,. \quad (3)$$

Outside the core region the amplitude of the order parameter is basically constant while the phase Θ of the order parameter varies. The time derivative and space gradient of Θ correspond to changes of the superfluid density and supercurrent density from the metastable state, respectively. Now consider charge superconductors. One can write

$$\Delta S_E = S_{\rm core} + S_{\rm EM}\,, \quad (4)$$

where $S_{\rm core}$ in Eq. (3) from amplitude variations of the order parameter remains unchanged [18] and $S_{\rm EM}$ is the EM field contribution from phase vortices outside the core region. Using the Lorentz gauge and choosing $A_1 = A_2 = 0$ from the symmetry of the system [16], the extreme condition $\delta S_E/\delta(\mathbf{A}, \varphi) = 0$ leads to Maxwell equations $(i = 3, 4)$

$$\eta_1^2 \frac{\partial^2}{\partial \rho_4^2} A_i' + \vec{\nabla}'^2 A_i' = -2\eta_2 \delta^2(\rho_1, \rho_2)\, \alpha_i J_i(\rho_3, \rho_4)\,, \quad (5)$$

and

$$J_i = \partial\Theta/\partial\rho_i - \alpha_i A_i'(\rho_1 = \rho_2 = 0, \rho_3, \rho_4) \quad (6)$$

are the dimensionless current $(i = 3)$ and charge $(i = 4)$ densities. Here $\eta_1 = v_F/\sqrt{3}\,c$, $\eta_2 = \sigma\gamma a/be^{*2}$, $\alpha_3 = e^{*2}/\hbar c = 4\alpha$, and $\alpha_4 = \sqrt{3}\,e^{*2}/\hbar v_F$. These coefficients are related by $\alpha_3/\alpha_4 = \eta_1$, $2\eta_2\alpha_3^2 = \sigma/\lambda_L^2$, $2\eta_2\alpha_4^2 = \sigma/\lambda_D^2$, and $\lambda_D/\lambda_L = v_F/\sqrt{3}\,c$. Inserting Eq. (5) back into the Euclidean action, one has

$$S_{\rm EM} = \frac{\sigma a\sqrt{\nu\gamma}}{b} \int d\rho_3\, d\rho_4 \sum_{i=3}^{4} \frac{\partial\Theta}{\partial\rho_i} \times J_i(\rho_1 = \rho_2 = 0, \rho_3, \rho_4)\,. \quad (7)$$

First let us look at the structure of a single phase vortex in (1 + 1)D due to EM coupling. For a vortex or antivortex centered at the origin, $\Theta(\rho_3, \rho_4)$ is determined by $(\partial^2/\partial\rho_3^2 + \partial^2/\partial\rho_4^2)\Theta = \pm 2\pi\delta^2(\rho_3, \rho_4)$. Using this relation and making a Fourier transformation $(\rho_1, \rho_2, \rho_3, \rho_4) \leftrightarrow (q_1, q_2, q_3, q_4)$, one can solve Eqs. (5) and (6) and find the action for a single vortex:

$$\frac{S_{\rm EM}^{\rm single}}{\hbar} = \frac{\sigma a\sqrt{\nu\gamma}}{b\hbar} \int dq_3\, dq_4 \frac{1}{q_3^2[1 + (\sigma/\lambda_D^2) M(q_3, q_4)] + q_4^2[1 + (\sigma/\lambda_L^2) M(q_3, q_4)]} \cong \frac{\pi}{8\alpha} \ln\ln\frac{L}{\xi}\,, \quad (8)$$

where $M(q_3, q_4) = \ln[(q_c^2 + q_3^2 + v_F^2 q_4^2/3c^2)/(q_3^2 + v_F^2 q_4^2/3c^2)]$ and $q_c \cong 1$ is the ultraviolet cutoff due to the vortex core. L is the length of the wire and ξ/L serves as the infrared cutoff. When deriving the last equation in (8), I have made a transformation $q_4' = (\lambda_D/\lambda_L)\, q_4 = (v_F/\sqrt{3}\,c)\, q_4$, which leads to a "squeezed" phase vortex in dimensionless (1 + 1)D due to anisotropic electric and magnetic screenings (see Fig. 2). The anisotropy of this squeezed vortex is $v_F/\sqrt{3}\,c \sim O(1/500)$. It is easy to understand this anisotropy since we know that, in a superconductor, current around a vortex spreads over a region of size λ_L and charge should spread over a region of size λ_D.

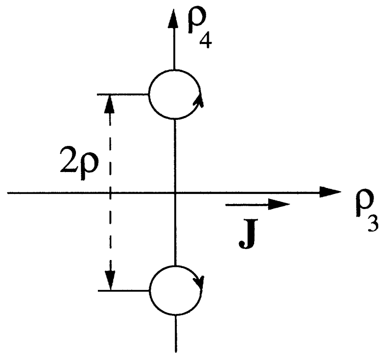

FIG. 1. A vortex-antivortex pair in (1 + 1)D as the saddle point picture for an uncharged superfluid.

Let us look at the action due to a vortex-antivortex pair (see Fig. 1). Using linear superposition of the phase, it is straightforward to write

$$\frac{S_{\rm EM}^{\rm pair}}{\hbar} = \frac{\sigma a\sqrt{\nu\gamma}}{b\hbar}\int dq_3\, dq_4 \times \frac{4\sin^2(\rho q_4)}{q_3^2[1 + (\sigma/\lambda_D^2)M(q_3,q_4)] + q_4^2[1 + (\sigma/\lambda_L^2)M(q_3,q_4)]}. \tag{9}$$

Now the important length scale for the problem is the distance between the pair, and ξ/L is no longer the relevant infrared cutoff. For current J satisfying the condition

$$\frac{2\sigma}{\lambda_L^2}\ln\frac{J_c}{J} \geq 1, \tag{10}$$

I get

$$\frac{S_{\rm EM}^{\rm pair}}{\hbar} \approx \frac{1}{\alpha}\frac{\pi}{4}\ln\ln\frac{J_c}{J} = \frac{A}{\alpha}, \tag{11}$$

where $A = (\pi/4)\ln\ln(J_c/J)$ for any reasonable current J. This apparent universal result is independent of Ginzburg-Landau coefficients and the details of the wire (e.g., σ). Equation (10) is easily satisfied since $\sqrt{\sigma}$ is comparable to λ_L in the experiments (cf., below). The fine structure constant $\alpha = 1/137$. For $J_c/J = 10^1-10^4$, $S_{\rm EM}^{\rm pair}/\hbar$ ranges from 90 to 239. So the EM field contributes an exponentially small *multiplying* factor,

$$e^{-S_{\rm EM}^{\rm pair}/\hbar} \sim e^{-\hbar c/e^2} \sim e^{-100}, \tag{12}$$

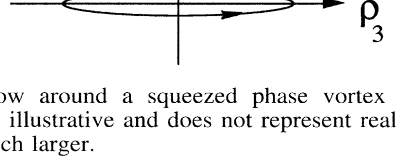

FIG. 2. Flow around a squeezed phase vortex centered at origin. It is illustrative and does not represent real anisotropy, which is much larger.

to the quantum decay rate of supercurrent. This effect essentially eliminates the possibility of observing quantum decay of 1D supercurrent experimentally, let alone core contributions [Eq. (3)]. So I conclude that *the lifetime of a small supercurrent in a superconducting ring at very low temperature is longer than can be measured.*

Now I give some qualitative and physical explanations. First let us look at the condition of small current. Consider a unit length of the superconducting wire with n_s and v_s the density and velocity of the superconducting electrons, respectively. The supercurrent density $J = n_s e v_s$ and the total current is $I = \sigma n_s e v_s$. The kinetic energy of the condensate is $\sigma n_s m v_s^2/2 = mI^2/2n_s e^2\sigma$ with m the electron mass. The magnetic field around the wire is $H \sim I/r$, where r is the distance from the center of the wire. The inductive energy (magnetic field energy) is the integral of H^2 and is roughly $I^2\ln(r_l/r_s)$, where r_l and r_s are the long distance and short distance cutoffs, respectively. Remembering that λ_L^2 is $\sim m/n_s e^2$, so the condition that inductive energy is larger than the kinetic energy gives $(2\sigma/\lambda_L^2)\ln(r_l/r_s) > 1$, which is basically our small current condition [Eq. (10)] since there is an inverse relation between distance and current. So small current means that the inductive energy of the wire is larger than the kinetic energy of the superfluid, or equivalently one can say that *the inertia of the EM field is larger than the inertia of the condensate.* Dressed with the inertia of the EM field, it is difficult for the phase field to tunnel through the topological barrier. This seems to be some kind of Coulomb blockade effect. Another intuitive explanation is that, during the tunneling process, a large amount of the EM field energy is dissipated, and this overdamping tends to prevent the phase "particle" from rolling down the "washboard" potential whose slope is determined by the metastable current.

Therefore, for a small current the inductive energy dominates and the wire can be regarded as an inductor with inductance L_0. On the other hand, the wire has a capacitance C_0 and one effectively has a L_0C_0 circuit. Generally speaking, the probability of quantum tunneling is roughly [9] $P \sim \exp(-V/\hbar\omega_0)$, where V is the effective barrier height and ω_0 is the intrinsic oscillation frequency of the circuit, $\omega_0 \sim (L_0C_0)^{-1/2}$. Phase slip is equivalent to a change of one flux quantum $\phi_0 = hc/2e$ at some length scale (presumed of order ξ). So the barrier height V is approximately $\phi_0^2/2L_0$. The relative barrier height $V/\hbar\omega_0 \sim (hc/2e)^2(L_0C_0)^{1/2}/2\hbar L_0 \sim (C_0/L_0)^{1/2}\hbar c^2/e^2 \sim \hbar c/e^2$. This is because for a line conductor $(C_0/L_0)^{1/2} \sim 1/c$. Hence the quantum tunneling rate is $P \sim \exp(-\hbar c/e^2)$, in agreement with Eq. (12).

In the light of the theory presented above, let us discuss experimental results. In a careful experiment by Sharifi, Herzog, and Dynes [7] for homogeneous *in situ* grown Pb wires (free of grains), no macroscopic quantum tunneling region at small current was observed for wires with a width as thin as 220–550 Å. The only resistive transition observed is the LAMH region. This result actually agrees with my conclusion of unobservability of the quantum decay of 1D supercurrent.

Next let us turn to the experiment by Giordano [6] who claimed to have observed a macroscopic quantum tunneling phenomenon. There are three major disagreements between this current theory and the experiment which are detailed below. (i) For superconducting In $\lambda_L(0)$ is about 300 Å. The experimental quantum decay region extends down to 0.6 K below T_c ($T_c \sim 3$ K). The cross section σ of the wire is about (500 Å)^2. Since $\lambda_L^2(T)/\lambda_L^2(0) \sim n_s(0)/n_s(T) \sim 1/(1 - T/T_c)$, we have $\sigma/\lambda_L^2(T) \sim 0.5$. Equation (10) is easily satisfied for his small measuring current, and so we believe the quantum decay rate should be too small to be detected. (ii) If one ignores the effect from EM field and only compares the core contribution [Eq. (3)], the absolute value of the theoretical exponent is about 50 times larger than that of the experiment although the temperature dependence $(1 - T/T_c)$ agrees with each other. That simply means that the experimentally observed decay rate is still far too large. (iii) Even if one ignores the above two points, there is yet another qualitative disagreement between theory and the experiment. For any fluctuation theories [Ref. [4] and Eq. (3) here], the exponent of the decay rate due to the vortex core contribution is proportional to the cross section σ. So the ratio of the two exponents for wires of $\sigma_1 = (410$ Å$)^2$ and $\sigma_2 = (505$ Å$)^2$ should be proportional to $\sigma_1/\sigma_2 \sim 0.66$. But experimentally (Fig. 1 in Ref. [6]) the two lines in the quantum decay region have the same slope even to the naked eye (hence the ratio of the two exponents is 1). Taken together, these disagreements suggest that the effect observed by Giordano is probably not the macroscopic quantum decay phenomenon he claimed. Point (iii) subtly suggests that some intrinsic length scale, shorter than the width of the wires, has caused the observed effect. Indeed there are grains of size 100–200 Å in the wires, as reported in Ref. [6]. This indicates a possible crossover from the LAMH region to one dominated by Josephson weak links between the grains.

This possibility is corroborated by a recent experiment [8] which observed similar effects as in Ref. [6] for thin type-II superconducting wires inside magnetic fields. The Josephson type barrier height was observed and the results were interpreted as quantum creep of preexisting vortex lines [19] between grain boundaries. Experimental observations of quantum tunneling of vortices between pinning sites were also reported for thin superconducting films [20,21].

In summary, I have shown that the EM field contributes an exponentially small multiplying factor of the order of $\sim e^{-1/\alpha}$ to the quantum decay rate of 1D supercurrent. In case weak link effects and preexisting vortices are absent, a small 1D supercurrent is unlikely to decay at very low temperatures, and there is such a thing as a one-dimensional superconductor.

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