Nonlinear Theory and Simulations of Stimulated Brillouin Backscatter in Multispecies Plasmas

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(Received 16 December 1994)

We present the first detailed particle-in-cell simulation studies of stimulated Brillouin scattering (SBS) in an underdense plasma composed of multi-ion species. In particular, collisionless nonlinear saturation mechanisms are well modeled, and are found to depend critically on the plasma composition. The effect of a self-consistently evolved non-Maxwellian ion distribution can be observed in detail. He, H, C_5H_{12}, C_5D_{12}, CO_2, and various mixtures of these gases are studied in these simulations. The reflectivities due to the SBS instability show many of the trends seen in experiments with these gases.

PACS numbers: 52.65.Rr, 52.35.Nx, 52.40.Nk, 52.60.+h

The effects of adding a light ion species to a plasma composed primarily of a heavier, more abundant, ion species was first investigated experimentally by Alexeff, Jones, and Montgomery (AJM) [1]. In particular, they showed that the damping on the ion wave in the heavier ion plasma increased as trace amounts of a lighter species were introduced into the plasma. This result was shown to be in agreement with a linear ion Landau damping model given by Fried and Gould [2], which was later improved upon by Fried, White, and Samec [3]. Experiments by Clayton *et al.* [4] using a CO_2 laser confirmed many of these predictions in the linear regime. There are now several works that have carried this research further [5,6]. However, questions concerning nonlinear aspects of the addition of a light species to a heavier ion plasma have been neglected. Since recent experiments [7–9] with intense laser light and relatively high plasma density ($\sim\frac{1}{10}n_{\rm cr}$, where $n_{\rm cr}$ is the critical density) are thought to operate in a nonlinearly saturated state, some insight into this state is important.

We have used a particle ion, fluid electron simulation code (electron-fluid particle ion code or EPIC) to study various kinetic nonlinearities for saturation of the ion wave associated with stimulated Brillioun scattering (SBS). An interesting correlation to linear theory is observed; however, there are significant differences. We find that particle trapping can substantially lower the ion wave amplitude, when a light ion species is present in a predominantly heavy ion plasma, in agreement with simple nonlinear theories. It is found that even in the nonlinear state trace amounts of light ions can substantially reduce the amount of reflectivity due to SBS in a heavy ion plasma.

A brief explanation of the simulation code will illustrate the physics accessible in this study. EPIC resolves the electromagnetic (EM) wave associated with the laser light, both spatially and temporally. The algorithm used for this is known as the Langdon-Dawson advection scheme [10]. In 1D, the light can be broken up into right-going (E_+) and left-going (E_-) waves that advect exactly at the speed of light, c,

$$\left(\frac{\partial}{\partial t} \pm c\,\frac{\partial}{\partial x}\right)E_\pm = -\frac{1}{2}J_y = \frac{1}{2}n_e e v_y\,, \tag{1}$$

where J_y is the transverse current and n_e is the electron plasma density. In the presence of a plasma, the transverse velocity (v_y) of the electrons serves as the source for the EM waves. The ponderomotive potential $U_p = \frac{1}{2}mv_y^2$, where m is the electron mass, is saved every time step. This is essential component of the feedback loop in the SBS instability, as it couples the reflected light wave to the ion density fluctuations (at wave number $2k_0$ and frequency $\omega_{ia} = 2k_0C_s$, where k_0 is the wave number of the incident laser, $C_s^2 = ZT_e/M_i$ is the square of the ion sound speed, T_e is the electron temperature, Z the charge state of the ion, and M_i is the ion mass) which then grow exponentially, according to linear theory. Since the ions move slowly on the time scale of the laser light, they are moved every 200th time step or so by an amount determined by a time average of the ponderomotive potential over this time interval. EPIC handles the ions as particles, and the force on these ions is the electrostatic field obtained from the electron momentum equation. The electrons are taken to be a massless fluid, since they respond to bulk ion motion essentially instantaneously on the ion time scale. Hence,

$$E_x = -\frac{1}{e}\left(\frac{\partial\langle U_p\rangle}{\partial x} - \frac{1}{n_e}\frac{\partial(n_e)T_e}{\partial x}\right), \tag{2}$$

where $\langle U_p\rangle$ is the time averaged ponderomotive potential. Once this electric field is known, the ions are then moved on the grid according to

$$\frac{dv_i}{dt_i} = \frac{Ze}{M_i}E_x\,, \quad \frac{dx_i}{dt_i} = v_i\,. \tag{3}$$

When the ions are at the new positions, the ion density n_i is computed. This is done for all species of ions, usually two or three species with different masses $M_j = A_jM_p$ and charge states Z_j. Here M_p is the mass of the proton, and A_j (Z_j) is the atomic number (charge state) of the jth ion. Finally, a new electron density is found by

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setting $n_e = \sum(n_i)_j$ on each grid. The EM waves are then advected for another 200 time steps, and the process is repeated.

The approximation for the electron density amounts to neglecting the $k\lambda_{\mathrm{De}}$ correction in the electric field, a good approximation when $k\lambda_{\mathrm{De}}$ is much less than 1. For the parameters of the simulations discussed here, when $n_{\mathrm{cr}} = 0.25$, $k\lambda_{\mathrm{De}} \sim 0.25$, and when $n_{\mathrm{cr}} = 0.1$, $k\lambda_{\mathrm{De}} \sim 0.45$. Neglect of the ion wave dispersion could lead to larger levels of higher harmonics in strongly driven simulations. However, it has been shown experimentally [11] that harmonic generation does not play a key role in nonlinear saturation of ion waves. In addition, strong ion wave damping disrupts the harmonic generation.

One motivation for these simulations is a set of recent experimental results obtained by MacGowan *et al.* [7] on the Nova laser using gasbags, as well as by Fernandez *et al.* [8] and Turner *et al.* [9] using gas-filled hohlraums. Briefly described, these experiments measured the reflectivity due to SBS from a number of large scale length ($\sim$1 mm), low density ($\sim\frac{1}{10}n_{\mathrm{cr}}$) plasmas composed of a variety of gases, such as neopentane (C_5H_{12}), deuterated neopentane (C_5D_{12}), and carbon dioxide (CO_2) with T_e measured to be about 3 keV and T_e/T_i estimated from theory to be 5. The simulations presented here attempt to describe some of the trends seen in the experimental data. The electron temperature for all simulations is $T_e = 3$ keV, the density either $\frac{1}{10}n_{\mathrm{cr}}$ or $\frac{1}{4}n_{\mathrm{cr}}$, and the ratio of the electron temperature to the initial ion temperature is always in the range $3 \leq T_e/T_i \leq 10$. The laser wavelength is $\lambda_0 = \frac{1}{3}$ μm, and the intensity is $I = 2.7 \times 10^{15}$ W/cm^2, as in the experiment. Roughly 1×10^6 particles per ion species are used for the longer system sizes. The reflected wave grows from a noise level (I_{noise}), set by scattering of the incident wave from ion density fluctuations. In these simulations, I_{noise}/I_0 is of the order 1%, but can be as low as 0.01%.

It is important to point out some limitations of the simulations. These are one-dimensional simulations with initially uniform plasma and significant noise levels. The plasma is relatively small in extent (typically 1–2 speckle lengths for an $f/4$ lens) and the simulations last for about 10–20 ps, which is sufficient to reach a quasi-steady-state. However, one might use insight from the particle-in-cell (PIC) simulations to limit the maximum level of ion density fluctuations in the reduced descriptions of the interaction, which can also include realistic 3D models of the laser beam. The PIC simulations can also help us understand important trends determined by nonlinear effects.

We now consider the recent experiments with mixed species. In order to compare to relative trends observed in experiments, we present three different mixed species runs, CO_2, C_5H_{12}, and C_5D_{12}. The first set of simulations had parameters identical to those given above, except that $n/n_{\mathrm{cr}} = 1/4$, and $L_x = 159\lambda_0$. The reflected light leaves the plasma in short "bursts" which, when averaged over times like 300 fs, usually reached some steady-state value after an initial transient period. CO_2 gave a reflectivity of 60%, while C_5H_{12} gave 6% and C_5D_{12} gave $10\% < r < 30\%$. The second set of simulations had parameters identical to those given above, except that $n/n_{\mathrm{cr}} = 1/10$, and $L_x = 330\lambda_0$. This time CO_2 gave a reflectivity of 40%, while C_5H_{12} gave 1% and C_5D_{12} gave $4\% < r < 6\%$. It is clear from the distribution function of the H in the C_5H_{12} case that although a quasi-steady-state is achieved, there is a severe modification to the distribution near the phase velocity of the ion wave associated with SBS.

Linear theory predicts that trace amounts of light ions substantially increase the ion Landau damping of the ion wave. There are several reasons for this effect, the first of which is obvious. If we assume that the ion temperatures are the same, the larger thermal velocity for the light ion species leads to more ions in resonance with the wave. The second effect is that the phase velocity is a function of the plasma composition. To illustrate this, consider a simple model (valid for $k^2\lambda_{\mathrm{De}}^2 \ll 1$) which gives a physical picture of the interaction. (A more rigorous kinetic treatment can be found in Williams *et al.* and Vu *et al.* in Ref. [6].) We start with the linearized ion continuity and momentum equations for the two fluids,

$$\frac{\partial n_{i1}}{\partial t} + n_{i10}\frac{\partial u_{i1}}{\partial x} = 0, \quad \frac{\partial n_{i2}}{\partial t} + n_{i20}\frac{\partial u_{i2}}{\partial x} = 0, \tag{4}$$

$$\frac{\partial u_{i1}}{\partial t} = -\frac{Z_1}{M_1}kT_e\frac{1}{n_{e0}}\frac{\partial n_e}{\partial x}, \quad \frac{\partial u_{i2}}{\partial t} = -\frac{Z_2}{M_2}kT_e\frac{1}{n_{e0}}\frac{\partial n_e}{\partial x}. \tag{5}$$

By combining these equations into one equation for the ion wave, an effective phase velocity for the ion wave can be found to be

$$v_p^2 = \left[\frac{n_{i10}Z_1^2kT_e/M_1 + n_{i20}Z_2^2kT_e/M_2}{Z_1n_{i10} + Z_2n_{i20}}\right]. \tag{6}$$

This phase velocity is plotted in Fig. 1, for the specific case of carbon and hydrogen at $T_e = 3$ keV, as a function of the percentage of hydrogen in the gas mixture. The limit in which only carbon ions are present smoothly connects to the case for only hydrogen present. Note that the phase velocity falls rapidly from the pure hydrogen case, essentially because the phase velocity is determined by the heavy species when the fraction of hydrogen is roughly 50%.

The physics can be clarified by considering the velocity distribution functions $f(v)$ of a gas composed of He and H. Consider first pure He, where only a small number of particles at the phase velocity of the ion wave $C_{s\mathrm{He}}$ ($= v_p$) are available to damp the SBS generated ion wave. Now consider pure H, at the same temperature. The slope at this point is somewhat larger than that for the case of pure

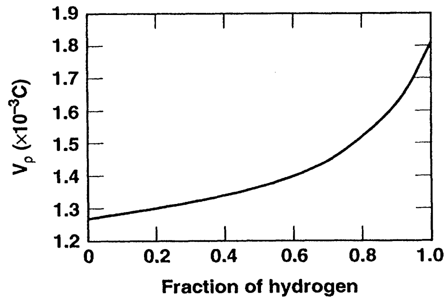

FIG. 1. A plot of the acoustic wave phase velocity as a function of percentage of hydrogen present in a CH plasma.

He, and there are more particles at the phase velocity $C_{s\mathrm{H}}$ (which is greater than $C_{s\mathrm{He}}$). Finally, consider a small amount of H present in a mostly He plasma (20%-80% mixture, say.) Now, v_p is now given in accordance with Eq. (6), and is approximately equal to $C_{s\mathrm{He}}$. In this case, a number of H ions can now damp even more efficiently the ion wave than in the pure H case, as the phase velocity has moved further into the bulk of the distribution function. Thus, linear theory predicts that the damping for the mixture will actually be more than for H alone (lowest reflectivity), and that pure helium will have the smallest damping (largest reflectivity). In fact, simulations bear this out; pure He gives a reflectivity $r \sim 30\%$, pure H a reflectivity $r \sim 10\%$, and the 80-20 mixture gives $r \sim 5\%$ for parameters given above.

We have discussed the effect of plasma composition on linear damping of the ion acoustic wave. However, the simulations show that nonlinear effects are important when the reflectivity is significant. We will now examine the simplest nonlinear effect, which is ion trapping. In a one species plasma, when the velocity of the fastest ions reaches the phase velocity of the ion wave, these ions become trapped and the amplitude of the density perturbation associated with the ion wave will saturate. For one species, this limiting amplitude (neglecting temperature effects) is the well known result of $\delta n/n = 1/2$ [12]. For two species, assuming cold ions ($T_i \sim 0$), the limiting amplitude of the density perturbation is lower by a factor of 2, when the light species is hydrogen. This factor of 2 arises from the fact that the hydrogen ions are the particles being trapped (with $Z/A = 1$) in a wave with a phase velocity determined predominantly by the heavy fluid (typically with a $Z/A = 1/2$). The assumption here is that only a small fraction of gas mixture is composed of light ions.

Next in order of complexity are ion thermal effects. If one considers that the trapping velocity is determined by the trapping of the light ion species, and the phase velocity of the wave is given by Eq. (6), then the $\delta n/n$ can be considerably reduced. In fact, a simple estimate that neglects the thermal velocity of the heavier ion species gives

$$\left(\frac{\delta n}{n}\right)_{\mathrm{tr}} \approx \frac{1}{2T_e}\left(\frac{M_\mathrm{H}}{Z_\mathrm{H}}\right)\left(v_p - \sqrt{\frac{3T_i}{M_\mathrm{H}}}\right)^2, \tag{7}$$

where the phase velocity is found from the equation

$$v_p^4 + v_p^2\left(\omega_{p\mathrm{H}}^2 \lambda_{\mathrm{De}}^2 + \omega_{p\mathrm{C}}^2 \lambda_{\mathrm{De}}^2 + 3v_\mathrm{H}^2\right) + 3v_{p\mathrm{H}}^2 \omega_{p\mathrm{C}}^2 \lambda_{\mathrm{De}}^2 = 0, \tag{8}$$

where $\omega_{p\mathrm{H}}^2 = 4\pi n_{i\mathrm{H}} e^2/M_\mathrm{H}$, $\omega_{p\mathrm{C}}^2 = 4\pi n_{i\mathrm{C}} Z_\mathrm{C}^2 e^2/M_\mathrm{C}$, and v_H is the thermal velocity of the hydrogen (light species). Of course, a kinetic description is required for more quantitative results for v_p. Here we have chosen the light ion species to be hydrogen and the heavy ions to be carbon. For a temperature ratio of $T_i/T_e = 1/10$, Eq. (7) predicts a $\delta n/n \sim 5\%$. Thus, when the phase velocity becomes smaller, ions are trapped at a lower $\delta n/n$. Simulations confirm this reduction. For all cases, a mixture of carbon and hydrogen gave a lower value for the nonlinear saturated value of the $\delta n/n$.

The nonlinear state, as seen in EPIC simulations, is very rich. For example, for the case of C_5H_{12}, the hydrogen ion distribution function near v_p initially flattens due to the ion trapping discussed above. However, a number of subsequent effects follow. First, the distribution function, inside the trapping width, oscillates at the bounce frequency ω_B as predicted by O'Neil [13] (see Fig. 2). As the distribution function evolves in time, there is a nonlinear frequency shift [14] due to modifications that the trapped ions make on the ion wave that reduces v_p. This allows the instability to access a "fresh" slope on the distribution function. An additional effect that works to widen the plateau region is the transport of the faster, heated ions. This allows for a redevelopment of the slope, which allows for additional damping. One concern might be the fact that if collisions are taken into account, they could cause the distribution to stay Maxwellian. However, collisional damping, though enhanced in a multispecies plasma [15], is not important in these plasmas with ion temperatures of order 1 keV. We estimate that

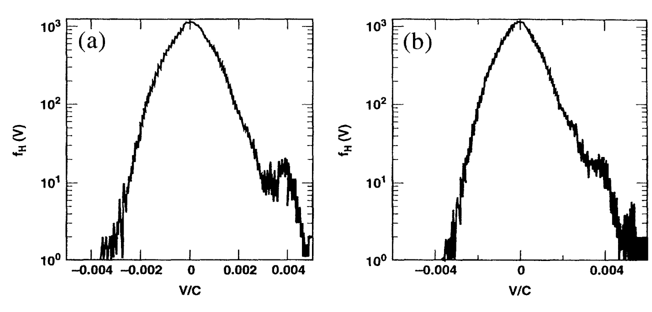

FIG. 2. Ion distribution for hydrogen component in a C_5H_{12} plasma showing extreme nonlinear modification and development of slope near the phase velocity of the ion wave associated with SBS. The parameters are as follows: $n/n_{\mathrm{cr}} = 0.25$, $T_e = 3$ keV, $T_e/T_i = 5$, $L_x = 159\lambda_0$, $I = 2.5 \times 10^{16}$ W/cm^2, and $\lambda_0 = \frac{1}{3}$ μm.

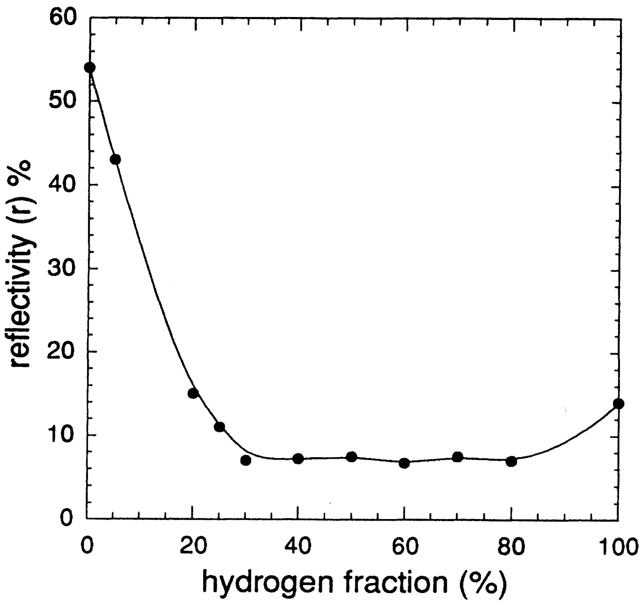

FIG. 3. Simulation results of reflectivity as a function of percentage of hydrogen present in the plasma. The parameters are as follows: $n/n_{cr} = 0.25$, $T_e = 3$ keV, $T_e/T_i = 5$, $L_x = 159\lambda_0$, $I = 2.7 \times 10^{15}$ W/cm^2, and $\lambda_0 = \frac{1}{3}$ μm.

$\omega\tau_i \gg 1$ and $k\lambda_i \gg 1$, where τ_i is the ion collision time and λ_i the corresponding mean free path.

Finally, we present EPIC simulations with parameters as given above. Here, amounts of light ion (hydrogen) were added to a pure C plasma, and the reflectivity was measured. Experimentally, it was found [7–9] that by increasing the amount of hydrogen in various gas mixtures, the reflectivity as a function of hydrogen present was found to decrease dramatically. This phenomenon was seen in previous experiments using a CO_2 laser at UCLA by Clayton *et al.* [4]. Plotted in Fig. 3 are results of EPIC simulations, which show a similar trend. Keep in mind that these simulations are in a large gain limit, where nonlinear effects are expected to limit the reflectivity. Finally, Fig. 3 shows clearly that the reflectivity goes up slightly for the pure hydrogen case, but not as high as for pure carbon. This can be understood by looking at Fig. 1. For the pure hydrogen case, we see that the ion wave phase velocity is deeper into the distribution; thus, the damping is greater, and the trapping amplitude lower, than for the pure carbon case.

To conclude, we have presented a detailed study of important nonlinear processes associated with SBS in a multi-ion species plasma. The trends seen in simulation are consistent with those observed in experiment. Additionally, we presented the modifications to ion trapping, and found that the addition of a light species could significantly decrease the amplitude of the ion wave at which trapping will occur. Finally, these kinetic simulations show that distortion in the ion distribution functions plays an important role in determining the amount of reflectivity due to SBS.

We are happy to acknowledge useful conversations with M. Rosen, J. Lindl, E. M. Campbell, H. Rose, B. Cohen, and R. Berger. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48.

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