Self-Induced Transparency in Bragg Reflectors: Gap Solitons near Absorption Resonances

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We show that pulse transmission through near-resonant media embedded within periodic dielectric structures can produce self-induced transparency (SIT) in the band gap of such structures. This SIT constitutes a principally new type of gap soliton.

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Self-induced transparency (SIT), namely, solitary propagation of electromagnetic (EM) pulse in near-resonant media, irrespective of the carrier-frequency detuning from resonance, is one of the most striking and important effects of nonlinear optics [1,2]. It reflects the essence of driven two-level atom dynamics, which is described in the soliton frame by a pendulum equation for the pulse area θ (the sine-Gordon equation). If the pulse duration is much shorter than the transition (spontaneous-decay) lifetime (T_1) and dephasing time (T_2), and θ is a multiple of 2π, then pulse-area conservation gives rise to SIT, corresponding to reemission of the absorbed radiation in phase with the driving field.

One of the standard tacit requirements for SIT is uniformity of the medium. Indeed, one would expect that partial reflection of the field in a nonuniform, e.g., layered, medium should destroy SIT, because the pulse area is then split between the forward and backward (reflected) waves and is no longer conserved for each wave. This expectation seems to be supported by treatments of a single thin resonant film [3] or a periodic array of such films [4], which yield bistable (two-valued) transmission of the incident pulse, because of the coupling of the forward and backward waves. Should we then anticipate severely hampered transmission through a medium whose resonance lies in a reflective spectral domain (photonic band gap) of a periodically layered structure (a Bragg reflector)? In this Letter we show that, contrary to such expectations, it is possible for the pulse to overcome the band-gap reflection and produce SIT in a near-resonant medium embedded in a Bragg reflector.

The predicted SIT propagation is a principally new type of gap soliton, which does not obey any of the familiar soliton equations, such as the nonlinear Schrödinger equation (NLSE) or the sine-Gordon equation. Its spatiotemporal form, intensity dependence, and transmission mechanism are shown here to be quite unique. Nevertheless, it shares several common features with the extensively studied gap solitons [5–8] or with ultrashort pulses [9] in Kerr-nonlinear Bragg reflectors.

We consider the propagation of an EM pulse through a medium consisting of two-level systems (TLS) embedded within a one-dimensional periodic dielectric structure, e.g., a multilayered dielectric mirror. Our starting point is the Maxwell equation for the field E

$$c^2\frac{\partial^2 E}{\partial z^2}-\frac{\partial^2 E}{\partial t^2}=\frac{\partial^2 P_{\rm tot}}{\partial t^2} \quad (1)$$

driven by the current of the total polarization $P_{\rm tot}=P_{\rm lin}+P_{\rm nl}$. The linear part of the polarization $P_{\rm lin}=\chi_{\rm lin}E$ is characterized by a linear refractive index $n_{\rm lin}=\epsilon_{\rm lin}^{1/2}=(1+4\pi\chi_{\rm lin})^{1/2}$ with fundamental period d, $\epsilon_{\rm lin}=\epsilon_0+\sum_{m=1}^{\infty}\Delta\epsilon_m\cos(2mkz)$, where $k=\pi/d$ and $\Delta\epsilon_m$ is the variation of the mth harmonic of the dielectric index. The nonlinear polarization $P_{\rm nl}$ is the near-resonant response of the TLS.

We may, analogous to the theory of distributed feedback lasers [10–12], decompose the total field and nonlinear polarization into forward (F) and back-reflected (B) components

$$E(z,t)=[\mathcal{E}_F(z,t)e^{ikz}+\mathcal{E}_B(z,t)e^{-ikz}]e^{-i\omega_{gc}t}+\text{c.c.}, \quad (2a)$$

$$P_{\rm nl}(z,t)=[\mathcal{P}_F(z,t)e^{ikz}+\mathcal{P}_B(z,t)e^{-ikz}]e^{-i\omega_{gc}t}+\text{c.c.}, \quad (2b)$$

assuming that the carrier frequency ω lies near the center of the lowest fundamental band gap $\omega_{gc}=kc/n_0$, where $n_0=\epsilon_0^{1/2}$. The small detuning from the gap center $|\omega_{gc}-\omega|\ll\omega$ will be considered as phase modulation of the complex amplitude $\mathcal{E}_{F(B)}$. Under the weak-reflection assumption ($|\Delta\epsilon_m|\ll\epsilon_0$) we may drop spatially fast-varying components (varying on the scale of a wavelength) of E and $P_{\rm nl}$, and, consistently, $m>1$ terms of $\epsilon_{\rm lin}$. We then obtain the following coupled-mode equations for the Rabi frequencies corresponding to the slow-varying field amplitudes, $\Omega_{F(B)}=\mu\mathcal{E}_{F(B)}/\hbar$, where μ is the dipole moment of the TLS transition [12]:

$$\left(\pm\frac{c}{n_0}\frac{\partial}{\partial z}+\frac{\partial}{\partial t}\right)\Omega_{F(B)}=ic\kappa/n_0\Omega_{B(F)}+\tau_c^{-2}\mathcal{P}_{F(B)}. \quad (3)$$

Here and hereafter the upper (lower) sign corresponds to the first (second) subscript. The first term on the right hand side of (3) describes the forward-to-backward wave coupling via Bragg reflection with characteristic reflection (attenuation) length $1/\kappa$, where $\kappa=k\Delta\epsilon_1/4\epsilon_0$.

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The second term is proportional to the TLS polarization current, and $\tau_c^{-2} = 2\pi\omega_{12}\mu^2\sigma/n_0^2\hbar$ is the inverse square of the cooperative resonant absorption time τ_c, σ being the density of the TLS. Note that κ is positive or negative for TLS embedded in regions with the higher or lower refractive index ($\Delta\epsilon_1 > 0$ or $\Delta\epsilon_1 < 0$), respectively.

In treatments of bidirectional field propagation in media with arbitrary spatial distribution of near-resonant atoms [13,14], the Bloch equations for the population inversion $w(z,t)$ and polarization $P_{\rm nl}$ are entangled in a fashion which leads to an infinite hierarchy of equations for successive spatial harmonics. The truncation of this hierarchy can only be justified by phenomenological arguments. Here we avoid this complication by restricting the near-resonant species distribution to thin layers (much thinner than the resonant wavelength) with the same periodicity d as the dielectric structure. Then the nonlinear polarization envelopes may be decomposed as [4]

$$\mathcal{P}_{F(B)} = \sum_j P_{\rm nl}(z_j,t)\delta(z - z_j)e^{\mp ikz}. \tag{4}$$

Here $P_{\rm nl}(z_j,t)$ is the *nonlinear* polarization of the TLS in the jth layer ($z_j = jd$). The Bloch equations then assume the form

$$\partial_t P_{\rm nl}(z_j,t) = w(z_j,t)(\Omega_F e^{ikz_j} + \Omega_B e^{-ikz_j}) - i(\omega_{gc} - \omega_{12})P_{\rm nl}(z_j,t), \tag{5a}$$

$$\partial_t w(z_j,t) = -\frac{1}{2}P^*_{\rm nl}(z_j,t)(\Omega_F e^{ikz_j} + \Omega_B e^{-ikz_j}) + \text{c.c.}, \tag{5b}$$

where w is the population inversion and $\omega_{gc} - \omega_{12}$ is the detuning of the TLS resonance frequency ω_{12} from the gap center. The periodicity of the TLS positions which satisfy the Bragg condition, i.e., $\exp(\pm 2ikz_j) = 1$, yields $\mathcal{P}_F = \mathcal{P}_B = \mathcal{P}$ in (4). Upon summing over all layers j, the Bloch equations (5) can be written under the Bragg condition in the following closed form, without resorting to harmonic expansion:

$$\partial_t\mathcal{P}(z,t) = w(z,t)(\Omega_F + \Omega_B) - i(\omega_{gc} - \omega_{12})\mathcal{P}(z,t), \tag{6a}$$

$$\partial_t w(z,t) = -\frac{1}{2}[\mathcal{P}^*(z,t)(\Omega_F + \Omega_B) + \text{c.c.}]. \tag{6b}$$

In an attempt to further simplify the Maxwell-Bloch equations, we first convert the variables to the dimensionless form $\tau = t/\tau_c$, $\zeta = n_0 z/c\tau_c$, $\eta = \kappa\tau_c$, and $\delta = (\omega_{gc} - \omega_{12})\tau_c$. We now rewrite Eqs. (3) and (6) for the sum and difference of the forward and backward field envelopes $\Sigma_+ = \tau_c(\Omega_F + \Omega_B)$ and $\Sigma_- = \tau_c(\Omega_F - \Omega_B)$, and obtain by simple manipulations

$$\left[\partial_\tau^2 - \partial_\zeta^2\right]\Sigma_+ = 2\partial_\tau\mathcal{P} + i2\eta\mathcal{P} - \eta^2\Sigma_+, \tag{7a}$$

$$\left[\partial_\tau^2 - \partial_\zeta^2\right]\Sigma_- = -2\partial_\zeta\mathcal{P} - \eta^2\Sigma_-. \tag{7b}$$

Although Σ_+ and Σ_- now obey separate equations, they are still coupled via $\mathcal{P}$, which satisfies the Bloch equations

$$\partial_\tau\mathcal{P} = w\Sigma_+ - i\delta\mathcal{P}, \tag{8a}$$

$$\partial_\tau w = -\tfrac{1}{2}(\mathcal{P}^*\Sigma_+ + \mathcal{P}\Sigma_+^*). \tag{8b}$$

We emphasize again the crucial role of the assumption that the TLS layers are much thinner than a wavelength and satisfy the Bragg condition. Without this assumption we could not have obtained (7) and (8), which are closed in $\mathcal{P}$ and w (in contrast to Refs. [13,14]).

Equations (7) and (8) cannot be reduced to any familiar soliton equation. Our main idea is to try for the above equations a *phase-modulated* 2π-soliton SIT solution

$$\Sigma_+ = A_0\frac{\exp[i(\alpha\zeta - \Delta\tau)]}{\cosh[\beta(\zeta/u - \tau)]}, \tag{9}$$

where $\Delta = (\omega - \omega_{gc})\tau_c$, A_0 is the amplitude of the solitary pulse, β its width, and u its group velocity (normalized to c).

Substituting $\partial_\tau\mathcal{P}$ from Eq. (8a) into Eq. (7a), we may express $\mathcal{P}$ in terms of Σ_+ and the population inversion w. Then, upon eliminating $\mathcal{P}$ and using Eq. (9), we can integrate Eq. (8b) for the population inversion w, obtaining

$$w = -1 - \frac{A_0^2(\Delta - \alpha/u)}{2(\delta - \kappa)}\frac{1}{\cosh^2[\beta(\zeta/u - \tau)]}. \tag{10}$$

Using these explicit expressions for $\mathcal{P}$ and w in (7a) and (8a), we reduce our system to a set of algebraic equations for the coefficients α, Δ that determine the spatial and temporal phase modulation, and the pulse width β as functions of the velocity u. The soliton amplitude is then found to satisfy $|A_0| = 2\beta$, exactly as in the case of the usual SIT [2]. This implies, by means of Eq. (9), that the area under the Σ_+ envelope is 2π.

Let us consider the most illustrative case, when the atomic resonance is exactly at the center of the optical gap, $\delta = 0$. Then the solutions for the above parameters are [15]

$$\alpha = -\frac{\eta}{2u}\frac{1 - 3u^2}{1 - u^2}, \quad \Delta = \frac{\eta}{2}\frac{1 + u^2}{1 - u^2}, \tag{11a}$$

$$\beta^2 = |A_0|^2/4 = \frac{8u^2(1 - u^2) - \eta^2(1 + u^2)^2}{4(1 - u^2)^2}. \tag{11b}$$

In the frame moving with the group velocity of the pulse, $\zeta' = \zeta - u\tau$, the temporal phase modulation will be $(\alpha u - \Delta)\tau$, which is found from Eq. (11) to be equal to $-\eta\tau$. Since $\eta = \kappa\tau_c$ is the (dimensionless) gap width, this means that the frequency is detuned in the moving frame exactly to the band-gap edge. The band-gap edge corresponds (by definition) to a standing wave, whence this result demonstrates that such a pulse is indeed a soliton, which does not disperse in its group-velocity frame.

The allowed range of the solitary group velocities may be determined from Eq. (11b) through the condition $\beta^2 > 0$ for a given η, as illustrated in Fig. 1. The same condition implies $|\eta| < \eta_{\max}$, where

$$\eta_{\max}^2 = 8u^2(1 - u^2)/(1 + u^2)^2. \quad (12)$$

It follows from (12) that the condition for SIT is $|\eta| < 1$, $\eta_{\max} = 1$ corresponding to $u = 1/\sqrt{3}$. This condition means that the cooperative *absorption length* $c\tau_c/n_0$ should be *shorter than the reflection* (attenuation) *length* in the gap $1/\kappa$, i.e., that the incident light should be absorbed by the TLS before it is reflected by the Bragg structure. In addition, both these lengths should be much longer than the light wavelength for the weak-reflection and slow-varying approximation to be valid.

From Eq. (7b) we find

$$\Sigma_- = \frac{1}{u}\Sigma_+, \quad |\Omega_{F(B)}| = \frac{1}{2\tau_c}\left|\left(1 \pm \frac{1}{u}\right)\Sigma_+\right| \quad (13)$$

and the equation for population inversion, obtained from Eq. (10)

$$w = -1 + \left(\frac{1}{u^2} - 1\right)\frac{\beta^2}{\cosh^2[\beta(\zeta/u - \tau)]}. \quad (14)$$

The envelopes of both waves (forward and backward) propagate in the same direction; therefore the group velocity of the backward wave is in the direction opposite to its phase velocity. This is analogous to climbing a descending escalator.

Analogous to Kerr-nonlinear gap solitons [5,6], the real part of the nonlinear polarization $\mathrm{Re}\, P_{\mathrm{nl}}$ creates a traveling "defect" in the periodic Bragg reflector structure which allows the propagation at band-gap frequencies. The real part of the nonlinear polarization is governed by the frequency detuning from the TLS resonance. Exactly on resonance (which we here take to coincide with the gap center) $\Delta = \delta = 0$, $\mathrm{Re}\, P_{\mathrm{nl}} = 0$, and our solutions (11) yield imaginary values of the velocity u and modulation coefficient α. The forward field envelope then decays with the same κ exponent as in the absence of TLS in the structure. Because of this mechanism, SIT exists only on one side of the band-gap center, depending on the sign of κ in Eqs. (3), i.e., on whether the TLS are in the region of the higher or the lower linear refractive index. This result may be understood as the addition of a near-resonant nonlinear "refractive index" to the modulated index of refraction of the gap structure. When this addition compensates the linear modulation, then soliton propagation is possible (Fig. 2). On the "wrong" side of the band-gap center, *soliton propagation is forbidden even in the allowed zone,* because the nonlinear polarization then cannot compensate even for a very weak loss of the forward field due to reflection.

The soliton amplitude and velocity dependence on frequency detuning from the gap center (which coincides with atomic resonance) are illustrated in Fig. 1. They demonstrate that forward soliton propagation is allowed well within the gap, for Δ satisfying $(1 - \sqrt{1 - \eta^2})/\eta < \Delta < (1 + \sqrt{1 - \eta^2})/\eta$. In addition to frequency detuning from resonance, the near-resonant gap soliton possesses another unique feature: spatial self-phase modulation $\alpha\zeta$ of both the forward and backward field components.

To check our analytical solutions (9)–(13) we have compared them with numerical simulations of Eq. (3) using the numerical method developed in [4]. As the launching condition, we take the incident wave in the form $\mathcal{E}_F = A\exp[i(\omega - \omega_{gc})(t - t_0)]/\cosh[\beta(t - t_0)/\tau_c]$ without a backward wave ($\mathcal{E}_B = 0$) at $z = 0$. By varying the de-

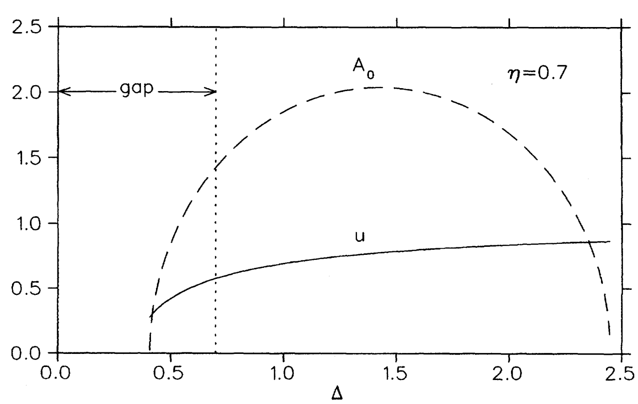

FIG. 1. Dependence of the solitary pulse velocity (solid line) and amplitude (dashed line) on frequency detuning from the gap center for $\eta = 0.7$. At the gap edge (dotted line) $u = 1/\sqrt{3}$ and $|E_F|/|E_B| = (\sqrt{3} + 1)/(\sqrt{3} - 1)$.

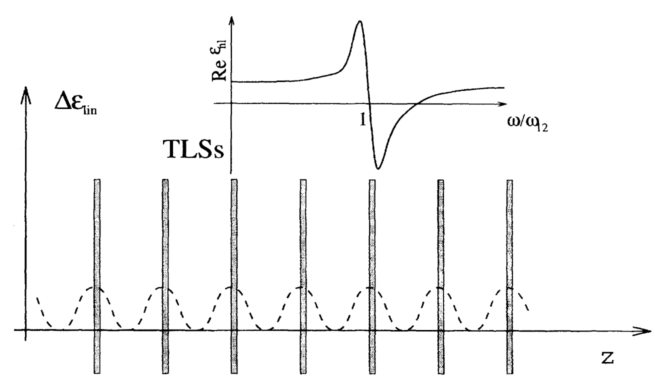

FIG. 2. The first-harmonic modulation $\Delta\epsilon_l \cos 2kz$ of the linear refractive index (dashed curve) in a structure of periodically alternating layers. This modulation can be canceled by the near-resonant nonlinear response $\mathrm{Re}\epsilon_{\mathrm{nl}}$ (inset), if it has the opposite sign to $\Delta\epsilon_l$ at the TLS positions.

tuning $\omega - \omega_{gc}$ and amplitude A we investigate the field evolution inside the structure. When these parameters are close to those allowed by Eqs. (11) and (12), we observe the formation and lossless propagation of both forward and backward solitonlike pulses with amplitude ratios predicted by our solutions [Fig. 3(a)]. In contrast, exponential decay of the forward pulse in the gap is numerically obtained in the absence of TLS [Fig. 3(b)].

The observation of the predicted SIT at band-gap frequencies requires high values of the electric dipole moment μ and high density of the TLS, in order to achieve short τ_c along with large T_2 and T_1 times (to avoid dephasing and energy losses by incoherent processes). The most adequate system for experimental observation of this effect appears to be excitons in semiconductors. Let us consider a periodic array of 12-nm-thick GaAs quantum wells ($\lambda = 806$ nm) separated by $\lambda/2$ nonresonant AlGaAs layers [16]. In this system $\mu \sim 10^{-18}$ cm electrons and area density concentration $\sigma \sim 10^8$ cm^{-2} are achievable (corresponding to an average bulk density of $\sim 10^{13}$ cm^{-3}), which yields $\tau_c \simeq 10^{-13}$ s. The relaxation time (at 2 K) is then $T_2 \sim 10$ ps [16]. A solitary pulse duration of the order of $\lesssim 1$ ps, i.e., much shorter than the dephasing time T_2, corresponds to a pulse width $\beta \gtrsim 0.1$. From Eq. (11) we find that SIT in this structure requires that $\eta \lesssim 0.99$, corresponding to a band-gap reflection length $1/\kappa \gtrsim 100\lambda$, i.e., $\Delta\epsilon_1/\epsilon_0 \lesssim 0.01$. An alternative choice may be bound I_2 excitons in CdS, which yield similar τ_c and T_2 at lower exciton densities (controlled by the donor impurity concentration) and are therefore more appropriate for the TLS description.

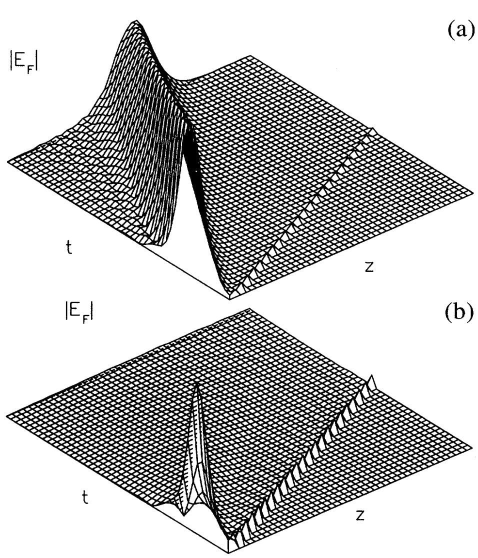

FIG. 3. Numerical simulations of the intensities of "forward" waves in the gap (a) when Eqs. (11) and (12) are obeyed ($\eta = 0.7$, group velocity $u \sim 0.3$), (b) without TLS [same η and incident pulse as in (a)].

To conclude, we have demonstrated the possibility of solitary pulse transmission through a one-dimensional band-gap region, by means of near-resonant polarization effects. In comparison with Kerr-nonlinear gap solitons, near-resonant gap solitons have several unique features: temporal and spatial phase modulations, the detuning conditions, and the resulting velocity and amplitude thresholds. Their salient advantage is stability with respect to absorption. In contrast, strong absorption is a severe problem associated with a large nonlinear Kerr coefficient. Regarding applications, the sensitivity to launching conditions on the carrier frequency and pulse shape can make the predicted gap SIT an effective filter for signal transmission.

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