Fast Hadronization of Supercooled Quark-Gluon Plasma

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A new rapid hadronization scenario is proposed based on the dynamical chiral model including quarks interacting with background meson fields. We estimate time scales and spatial characteristics of chiral-symmetry breaking instabilities in expanding, nonthermal quark-gluon plasma. The transition from the chirally symmetric to broken state proceeds through the formation of multiquark-antiquark clusters, surrounded by domains of the coherent chiral field.

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In the coming years serious experimental efforts are planned at the CERN Super Proton Synchrotron and Large Hadron Collider (LHC) and at the Brookhaven (BNL) Relativistic Heavy Ion Collider (RHIC) to produce quark-gluon plasma (QGP) in heavy ion reactions. The dynamics of the rehadronization of the expanding and cooling QGP was discussed recently [1,2] on the basis of the nucleation model. In the thermally overdamped limit the characteristic nucleation time was found to be $\approx$100 fm/c, which led to the conclusion about strong supercooling of plasma. Because of this large supercooling other faster processes must also play a role in the hadronization [3]. Nonthermal processes are possible if the hadronization happens simultaneously to the thermal freeze-out or later. In particular, it has been shown [3] that in this case the release of the latent heat does not necessarily lead to an overheated final hadronic phase, as it was thought earlier [4].

Recent lattice-QCD calculations [5] predict $T_c \approx 150-170$ MeV. Taking 20% supercooling, necessary to accelerate the nucleation, we get transition temperatures of 130 MeV. At such low temperatures parton collisions are already too rare to maintain thermodynamical equilibrium in the rapidly expanding plasma. Therefore, we expect freeze-out in the plasma phase, before hadronization. In this case the abundances and spectra of hadrons will reflect freeze-out conditions in the QGP.

This picture is in qualitative agreement with recent experimental data. Enhanced production of strange particles in central nucleus-nucleus collisions as compared to pp and pA reactions was observed in several experiments [6]. Heavy ion data show almost equal yields of $\overline{\Lambda}$ and $\overline{p}$ at midrapidity compared to their ratio of 0.2 in pp collisions [7]. Both these observations can hardly be explained in the equilibrium hadronic scenario. The data suggest also a high specific entropy and small chemical potential for strange particles [8]. This is also in agreement with the rapid hadronization of the QGP and no rescatterings of produced hadrons.

The dynamical chiral model.—Let us consider late stages of the QGP evolution when collisions between partons have already ceased and they interact only with the background fields. The evolution is governed by the interplay between the collective expansion and the intrinsic instabilities of the system. To describe such a system we use an effective field-theoretical model, the linear σ model, where quarks are moving in the background chiral field. Several works [9–12] have dealt recently with similar problems, however, without introducing quark degrees of freedom explicitly. The Lagrangian is

$$\mathcal{L} = \overline{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\vec{\pi})]q + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}) - U(\sigma,\vec{\pi}), \quad (1)$$

where $U(\sigma,\vec{\pi}) = \lambda^2/4(\sigma^2 + \vec{\pi}^2 - v^2)^2 - H\sigma$ is the so-called "Mexican hat" potential, q stands for the light (u and d) quark fields, while σ and $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ are the scalar and pseudoscalar pion fields which together form a four-component chiral field $(\sigma, \vec{\pi})$. Without the term $H\sigma$ this Lagrangian is invariant with respect to the $SU_L(2) \otimes SU_R(2)$ chiral transformations. The parameters in this Lagrangian are chosen in such a way that in normal vacuum chiral symmetry is spontaneously broken, and the expectation values of the meson fields are

$$\langle\sigma\rangle = f_\pi, \qquad \langle\vec{\pi}\rangle = 0, \quad (2)$$

where $f_\pi = 93$ MeV is the pion decay constant. To have the correct pion mass in vacuum, $m_\pi = 138$ MeV, one should take $v^2 = f_\pi^2 - m_\pi^2/\lambda^2$ and $H = f_\pi m_\pi^2$. The parameter λ^2 is related to the σ mass $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2$, which is usually chosen as $\approx$0.6 GeV (then $\lambda^2 \approx 20$). The remaining coupling constant g can be fixed by the requirement that the effective quark mass m in normal vacuum $m = g\langle\sigma\rangle = gf_\pi$ coincides with the constituent quark mass $m \approx m_N/3$, yielding $g \approx (m_N/3)/f_\pi \approx 3.3$.

In addition to the normal vacuum state (2) the Lagrangian (1) has one more stationary point on the top of the potential, U. In the chiral limit $m_\pi \to 0$ it corresponds to

$$\langle\sigma\rangle = 0, \qquad \langle\vec{\pi}\rangle = 0. \quad (3)$$

This state becomes a true ground state of the matter at high density and/or temperature signaling the restoration of chiral symmetry. The difference between the energy densities of the symmetric and broken vacuum states, i.e., the bag constant, is $B \approx (\lambda^2/4)f_\pi^4 \sim m_\pi^4$ in this model. We assume that the system is initially in the chirally symmetric phase.

We perform calculations within the mean-field approximation, ignoring all loop contributions. Therefore, we consider σ and $\vec{\pi}$ as classical fields obeying the equations of motion:

$$\partial_\mu \partial^\mu \sigma(x) + \lambda^2[\sigma^2(x) + \vec{\pi}^2(x) - v^2]\sigma(x) - H = -g\rho_S(x),$$
$$\partial_\mu \partial^\mu \vec{\pi}(x) + \lambda^2[\sigma^2(x) + \vec{\pi}^2(x) - v^2]\vec{\pi}(x) = -g\rho_P(x). \tag{4}$$

Here $\rho_S = \langle \bar{q} q \rangle$ and $\rho_P = i\langle \bar{q}\gamma_5 \vec{\tau} q \rangle$ are scalar and pseudoscalar quark densities, which should be determined self-consistently from the motion of q and $\bar{q}$ in background meson fields.

Relativistic kinetic equation.—Using the Wigner function formalism one can derive the relativistic transport equation for the σ-model Lagrangian (1). Analogous calculations have been performed for the Walecka model [13] and for the Nambu–Jona-Lasinio model [14]. Disregarding spin polarization effects one can represent the Wigner matrix for quarks in the form

$$W(x,p) = f(x,p)[\sigma(x) - i\gamma_5 \vec{\tau}\vec{\pi}(x) + p_\mu \gamma^\mu]. \tag{5}$$

In the quasiclassical approximation the scalar part of the Wigner function $f(x,p)$ obeys the relativistic Vlasov equation

$$\left[p^\mu \partial_\mu + \tfrac{1}{2}[\partial^\mu m^2(x)]\frac{\partial}{\partial p^\mu}\right] f(x,p) = 0, \tag{6}$$

where the quark (antiquark) effective mass $m(x)$ is obtained self-consistently,

$$m^2(x) = g^2[\sigma^2(x) + \vec{\pi}^2(x)]. \tag{7}$$

This expression for m can be justified also by chiral-symmetry arguments. The vanishing collision term on the right-hand side of the transport equation (6) reflects the fact that we are describing the evolution of the system after freeze-out.

It is well known (see, for instance, Ref. [13]) that in Eq. (6) the 4-vectors x and p should be treated as independent variables. Nevertheless, only those solutions are physically meaningful which are finally projected on the mass shell, $p^\mu p_\mu = m^2(x)$.

According to Eq. (5), the scalar and pseudoscalar densities can be represented as

$$\rho_S(x) = a(x)\sigma(x), \qquad \rho_P(x) = a(x)\vec{\pi}(x), \tag{8}$$

where $a(x)$ is expressed in terms of the momentum distribution function $f(x,p)$,

$$a(x) = \nu_q \int \frac{d^4p}{(2\pi\hbar)^3} 2\delta(p^\mu p_\mu - m^2(x)) f(x,p)$$
$$\to \frac{\nu_q}{(2\pi\hbar)^3} \int \frac{d^3p}{E(x,\mathbf{p})}[n_q(x,\mathbf{p}) + n_{\bar{q}}(x,\mathbf{p})]. \tag{9}$$

Here ν_q is the degeneracy factor of quarks $E(x,\mathbf{p}) = \sqrt{m^2(x) + \mathbf{p}^2}$, $n_q(x,\mathbf{p})$ and $n_{\bar{q}}(x,\mathbf{p})$ are the occupation numbers of valence quarks and antiquarks.

Boost-invariant expansion.—First we study the boost-invariant expansion of the plasma in homogeneous background fields. Let us define the proper time and space-time rapidity coordinates as $\tau = \sqrt{t^2 - z^2}$, and $\eta = \frac{1}{2}\ln[(t+z)/(t-z)]$, where the z axis is chosen along the beam direction. Besides, we introduce the transverse momentum $p_\perp$ and the rapidity of quarks $y = \frac{1}{2}\ln[(E+p_z)/(E-p_z)]$. We assume that the meson fields and the quark densities depend on the proper time τ only. We also assume that the flow rapidity of the matter is equal to the local rapidity coordinate, consequently the local four-velocity of the flow is $u^\mu = (t,0,0,z)/\tau$ [15].

Under these assumptions one can prove that the Vlasov equation (6) is satisfied by an function $f(s,p_\perp)$, which besides $p_\perp$ depends on only one scaling variable, $s = \pm(\tau/\tau_0)\sqrt{(p^\mu u_\mu)^2 - m^2(\tau) - p_\perp^2}$. On the mass shell $s = (\tau/\tau_0)p_\parallel$, where $p_\parallel = \sqrt{m^2(\tau) + p_\perp^2}\sinh(y - \eta)$ is the longitudinal momentum in the local rest frame.

At freeze-out $\tau = \tau_0$ the quark and antiquark occupation numbers can be approximated by the Fermi-Dirac distribution,

$$n_q(x,\mathbf{p}) = \left[\exp\left(\frac{\sqrt{m^2(\tau_0) + p_\perp^2 + p_\parallel^2} - \mu_0}{T_0}\right) + 1\right]^{-1} \tag{10}$$

and $n_{\bar{q}}(x,\mathbf{p}) = n_q(x,\mathbf{p}; \mu_0 \to -\mu_0)$. Here $\mu_0 = \mu(\tau_0)$ and $T_0 = T(\tau_0)$ are the chemical potential and temperature at the time of freeze-out. The scaling solution of the Vlasov equation at $\tau > \tau_0$ can be obtained now from Eq. (10) by simply changing $p_\parallel$ to $p_\parallel \tau/\tau_0$.

The post-freeze-out evolution of the momentum distribution of quarks and antiquarks is characterized by two features: (i) it becomes narrower due to the growing effective mass $m(\tau)$ with increasing τ and (ii) an anisotropy is developed in the momentum space, because the $p_\parallel$ distribution additionally shrinks as τ_0/τ. The scaling solution for the case $m = 0$ was found and discussed in Ref. [16].

Now one can calculate $a(x)$, Eq. (9), as a function of τ and m in a straightforward way. After substituting ρ_S and ρ_P into the equations of motion (4) for meson fields one can solve them for a specified initial condition at $\tau = \tau_0$.

Some features of the dynamics are similar to those found in Ref. [12], where pure mesodynamics was considered without quarks. At some state of the evolution the $q/\bar{q}$ density drops so much that the point $\sigma \approx 0$, $\vec{\pi} \approx 0$ becomes a local maximum of the effective potential. Then the fields start rolling down along the potential well towards their new equilibrium values given by Eq. (2). Because of the large mass the scalar field relaxes rapidly to its new equilibrium value f_π, but the pion field experiences long oscillations around zero. Below we discuss the characteristic time scales for this transition.

Unstable mode analysis.—Let us study the onset of instability associated with the chiral-symmetry breaking transition. We apply here a linear response method which was used earlier for analyzing spinodal instability in nuclear matter [17]. We consider small perturbations around the homogeneous solution $\sigma = 0$, $\vec{\pi} = 0$. Initially all directions in the isospin space are nearly equivalent and one can, for instance, introduce a perturbation $\delta\sigma$ along the σ axis, $\sigma = \langle\sigma\rangle + \delta\sigma$, with $\langle\sigma\rangle = 0$. The linearized equation of motion for $\delta\sigma$ is

$$[\partial_\mu \partial^\mu - \lambda^2 v^2 + g^2 a(\tau)]\delta\sigma = 0. \qquad (11)$$

In the lowest order it is sufficient to calculate $a(\tau)$ at $m = 0$. For the scaling solution $f(s, p_\perp)$ with $m = 0$ one can perform the momentum integration in Eq. (9) analytically yielding

$$a(\tau)|_{m=0} = \nu_q \left(\frac{T_0^2}{12} + \frac{\mu_0^2}{2\pi^2} \right) \frac{\tau_0}{\tau} h_1\left(\frac{\tau_0}{\tau}\right), \qquad (12)$$

where $h_1(x) = \arcsin(\sqrt{1-x^2})/\sqrt{1-x^2}$. In two limiting cases one has $h_1(1) = 1$ and $h_1(0) = \pi/2$. Below we consider the baryon-free plasma where the quark chemical potential vanishes, $\mu_0 = 0$.

We assume that the overall expansion is slow and consider plane wave solutions with a wave vector $\mathbf{k}$ along the beam axis: $\delta\sigma(t,z) = \delta\sigma_{\omega,\mathbf{k}} e^{-i\omega t + ikz}$. In an expanding system all wavelengths will be uniformly stretched in accordance with the growing linear scale: $k \to k\tau_0/\tau$. Then from Eq. (11) one obtains the dispersion relation between the frequency and the wave number of the fluctuation:

$$\omega^2 = k^2(\tau_0/\tau)^2 - k_{\max}^2, \qquad (13)$$

where

$$k_{\max} = \sqrt{\lambda^2 v^2 - g^2 \nu_q \left(\frac{T_0^2}{12} + \frac{\mu_0^2}{2\pi^2} \right) \frac{\tau_0}{\tau} h_1\left(\frac{\tau_0}{\tau}\right)}.$$

Unstable modes correspond to the solutions with $\mathrm{Im}\,\omega > 0$. They appear at $k\tau_0/\tau < k_{\max}$. The value of $k_{\max}$ depends on the freeze-out parameters and on the time of the subsequent post-freeze-out expansion.

In numerical estimates we use the standard parameters $g = 3.3$, $\nu_q = 12$, and $\lambda^2 = 20$. In a thermally equilibrated system, the model predicts the instability of the chiral-symmetric state (3) at $T = m_\sigma/g\sqrt{2} \approx 130$ MeV. According to Ref. [3] the rapid freeze-out may start at $\tau_0 = 6-10$ fm/c and $T_0 = 140-100$ MeV, respectively. Therefore, the onset of instability is close to the thermal freeze-out. Below we take $T_0 = 130$ MeV and $\tau_{\rm th} \approx \tau_0 = 7$ fm/c. The increments of unstable modes $\mathrm{Im}\,\omega$ calculated for these parameters are shown in Fig. 1. The characteristic growth time is obviously equal to $1/\mathrm{Im}\,\omega$.

One can see that the long wavelength fluctuations ($k \to 0$) grow most rapidly, while shorter ones grow at a smaller rate, going to zero at $k \to k_{\max}\tau/\tau_0$. Therefore, new shorter and shorter wavelength modes become unstable with increasing τ.

The possibility for instability opens at some threshold time $\tau_{\rm th}$ determined from the transcendental equation (13) at $\omega = k = 0$. At $\tau = \tau_{\rm th}$ the instability just becomes possible for $k = 0$, and the corresponding growth time is infinitely long. When the system expands further $\mathrm{Im}\,\omega$ grows rapidly. Quarks and antiquarks stabilize the system and make the transition less sharp. Nevertheless, as Fig. 1 shows, the fluctuations of 1 fm size have a characteristic growth time of the order of a few fm/c soon after the freeze-out. At later stages of the expansion the characteristic growth time of the intrinsic instability, $1/\mathrm{Im}\,\omega \approx \sqrt{2}/m_\sigma = 0.47$ fm/c, is much shorter than the typical time scale of the expansion, ~ 10 fm/c. Therefore, the process under consideration can be indeed responsible for the rapid hadronization of QGP in heavy ion collisions.

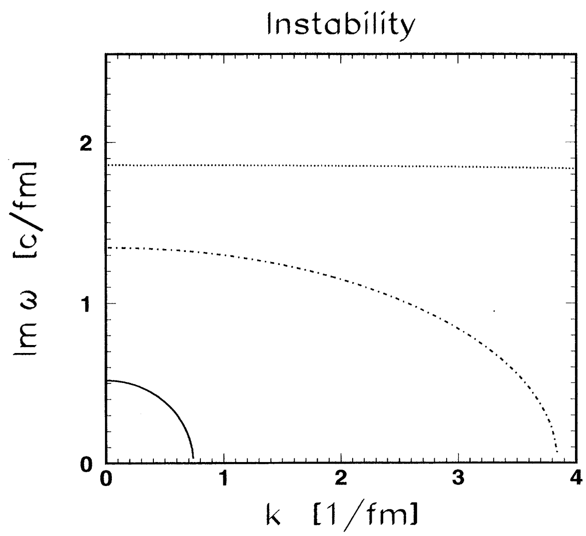

FIG. 1. The increment of instability $\mathrm{Im}\,\omega$ as a function of the wave number k for different times after the freeze-out: $\tau = 10$ fm/c (full line), $\tau = 20$ fm/c (dash-dotted line), and $\tau = 100$ fm/c ($\approx\infty$, dotted line).

Multiquark-antiquark clusters.—In the next order approximation one should take into account the finite quark effective mass in regions with nonzero meson fields. From Eq. (9) one can see that the quantity $a(x)$, determining the quark densities through Eq. (8), is a decreasing function of m. In other words, the propagation of quarks and antiquarks into the regions with large meson fields is suppressed by the large effective mass. Therefore, the nonlinear effects amplify the inhomogeneity leading to the spatial decomposition of the system into the domains with reduced and enhanced $q/\bar{q}$ density. This will result in the formation of multiquark-antiquark clusters or droplets surrounded by the regions of excited vacuum. Inside the clusters the chiral symmetry is preserved and quarks or antiquarks have small, current masses (large q-$\bar{q}$ bags).

At some stage the clusters will decouple from the overall expansion. We expect that this happens when the energy density inside the clusters drops below that in ordinary hadrons. Such clusters are very unstable and later on decay into usual hadrons. Each cluster should produce a bump in the rapidity distribution of emitted hadrons, their typical transverse momenta are of the order of 300 MeV$/c$. Since hadron formation takes place already in the dilute system, this should not lead to the hadron rescatterings and reheating of the hadronic phase.

Coherent pion field.—The evolution of vacuum to the normal, spontaneously broken ground state goes through the generation of the time-dependent chiral field. One can expect that after an incoherent excitation of many modes of instability, later only few, most unstable, long wavelength modes will dominate. Because of the large amplitude and the coherence they may be treated as classical field configurations (chiral condensates) [18–20]. Numerical simulations [9,11,12] show that the pion field oscillations persist as long as $\tau = 20-40$ fm$/c$.

A striking feature of the coherent pion field is the isospin alignment. Therefore, its decay products will have a large isospin imbalance between charged and neutral pions [18–20]. In different domains the isospin orientation could be different leading to the disoriented chiral condensates [20]. Events of this kind, Centauros (with $\pi^{\pm}$ excess) and anti-Centauros (with π^0 excess), have been observed in cosmic ray experiments [21]. The observation of Centauro-like events in relativistic heavy ion collisions would be a clear signature of the chiral phase transition.

Because of the collective expansion the decomposition of matter goes faster in the longitudinal direction compared to the transverse one. Therefore, initially, domains may look like "pancakes" perpendicular to the beam axis. At a later stage the instability will also develop in the transverse direction. The characteristic size of domains is at present a matter of debate. The growth dynamics and the size of domains of the coherent chiral condensate were studied in Refs. [9–11,22] in the quench approximation and in Ref. [12] within a more realistic model incorporating expansion. A small domain size will reduce to a large extent the signal of the coherent pion field. If the domains are large, of 3–7 fm size as predicted in Ref. [12], then in addition to the isospin alignment, the emitted pions should have very low relative momenta, say $p_t < 50$ MeV$/c$. We would like to emphasize, however, that this does not affect significantly our hadronization scenario.

In conclusion, we proposed a new scenario for rapid hadronization of the QGP at RHIC and LHC energies. The hadronization is associated with the transition from the initial, chirally symmetric state to the spontaneously broken final state. This process leads to the formation of q-$\overline{q}$ clusters surrounded by domains of excited vacuum with oscillating scalar and pion fields. The characteristic decomposition time is about a few fm$/c$, i.e., much shorter than the time scale of homogeneous nucleation. The fast hadronization process should manifest itself by short emission times in Hanbury-Brown–Twiss measurements. The specific signals of the chiral transition, i.e., large fluctuations in isospin and rapidity of produced pions, can be observed only on an event-by-event basis. Other signals, such as an excess of low p_t pions, enhanced strangeness production, and vanishing in-medium effects should be seen even in event-averaged data.

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