Effects of the Hyperfine Interaction on Orbital Electron Capture

L. M. Folan and V. I. Tsifrinovich
Physics Department, Polytechnic University, 6 Metrotech Center, Brooklyn, New York 11201
(Received 12 August 1994)

Electron capture from an unfilled atomic s orbital is investigated theoretically. Analyses are carried out for two simple situations, a mirror transition of a nucleus with a nucleon in an outer $s_{1/2}$ state and a Gamow-Teller transition decreasing the nuclear spin by one unit. It is shown that the hyperfine interaction between electron and nuclear spins has a great influence on the rate of electron capture at temperatures small compared to the hyperfine splitting. The possibility of inducing electron capture with resonant electromagnetic fields is discussed.

PACS numbers: 23.40.–s, 31.30.Gs

The techniques of atom [1] and ion [2,3] trapping and cooling provide wonderful opportunities to study systems that are well isolated from their surroundings. A cold trapped cloud provides a unique environment to investigate unusual aspects of orbital electron capture. In this Letter we discuss hyperfine effects on electron capture that may be observed in free atoms or ions at temperatures low compared to hyperfine splittings.

The orbital electron capture process, in which a nucleus absorbs a bound atomic electron and emits a neutrino, i.e.,

$$Z + e^- \rightarrow (Z - 1) + \nu_e , \tag{1}$$

is closely related to classic β decay [4–7]. It is well known that a classic β-decay rate does not depend on nuclear spin orientation. For polarized nuclei the probability of emission of a particle (electron or antineutrino) along the direction of nuclear polarization differs from the probability for emission in the opposite direction. However, the total rate of the β decay does not depend on nuclear spin orientation because of the isotropy of space. Our idea is that the situation is significantly different for orbital electron capture. We will show, for an electron-nucleus system coupled by the hyperfine interaction, that the total rate of decay depends strongly on the mutual orientation of the electron and nuclear spins, and hence on temperature.

Let us consider a free atom or ion with one electron in an outer s orbital. For our purposes, it is enough to use the simple, nonrelativistic theory of electron capture [5]. The rate of the process is given by the standard formula

$$W = (2\pi/\hbar)|H_{fi}|^2\rho_f , \tag{2}$$

where H_{fi} is the matrix element for the transition between the initial and final states, $\rho_f = (c\hbar)^{-3}E^2$ is the density of final states per unit energy range, and E is the neutrino energy (approximately the disintegration energy). Ignoring the hyperfine interaction, one can write the wave function of the initial state of the system as a product of the nuclear and electronic wave functions. In this case one obtains the well-known expression for the rate of the electron capture,

$$W = G(|\int 1|^2 + \lambda^2|\int \boldsymbol{\sigma}|^2) , \tag{3}$$

where $G = (2\pi/\hbar)\rho_f g^2 u^2(0)$, the weak interaction coupling constant $g \sim 10^{-62}$ J m^3, $u^2(0)$ is the electron density at the nucleus, $\int 1$ and $\int \boldsymbol{\sigma}$ are the matrix elements of the Fermi and Gamow-Teller interactions for the nuclear states, and the factor λ is close to 1. Taking into consideration the hyperfine interaction we have to deal with a superposition of electron and nuclear wave functions. We will consider here two simple cases to illustrate our idea.

(1) Mirror transition.—Consider a nucleus with one proton in the outer nuclear shell. In the electron capture process this proton transforms to a neutron without changing the spatial and spin components of its wave function (i.e., a "mirror transition"). For such a transition we can consider electron capture as approximately a two-particle nucleon-lepton process by neglecting the influence of other nucleons. The matrix element for such a process may be written in the form

$$H_{fi} = g\langle\psi_f|\tau_1\tau_2\{1 - \lambda\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\}\delta(\mathbf{r}_1 - \mathbf{r}_2)|\psi_i\rangle , \tag{4}$$

where τ_1 connects states of a proton and a neutron, τ_2 does the same for an electron and a neutrino, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli matrices for the nucleon and lepton spins, $\mathbf{r}_1$ and $\mathbf{r}_2$ are the nucleon and lepton coordinates, and ψ_i and ψ_f are the initial and final states of the nucleon-lepton system.

Now we assume, for simplicity, that the considered nucleon is in an $s_{1/2}$ state (e.g., ^{31}S decays to ^{31}P from an $s_{1/2}$ state and electron capture competes with e^+ emission [5]). In this case we can write the wave functions as products of spatial and spin parts

$$\psi_i \propto R_i(\mathbf{r}_1, \mathbf{r}_2)\chi_i \quad \text{and} \quad \psi_f \propto R_f(\mathbf{r}_1, \mathbf{r}_2)\chi_f . \tag{5}$$

Neglecting the variations in the lepton wave functions inside the nucleus we may put

$$R_i(\mathbf{r}_1, \mathbf{r}_2) \rightarrow R_i(\mathbf{r}_1)u(0) \quad \text{and} \quad R_f(\mathbf{r}_1, \mathbf{r}_2) \rightarrow R_f(\mathbf{r}_1) . \tag{6}$$

Taking into consideration that for mirror transitions $R_i(\mathbf{r}_1) = R_f(\mathbf{r}_1)$ we obtain

$$W = G|\langle\chi_f|(1 - \lambda\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)|\chi_i\rangle|^2 . \tag{7}$$

Let us calculate the value of W. Since the hyperfine interaction operator commutes with the operator of the

0031-9007/95/74(4)/499(3)$06.00

square of the total spin F^2, the spin function χ_i is an eigenfunction of F^2. We have four possible initial states,

$$\chi_i = 2^{-1/2}(\alpha_1\beta_2 - \beta_1\alpha_2) \quad \text{for } F = 0,$$
$$\chi_i = \alpha_1\alpha_2,\ \beta_1\beta_2,\ 2^{-1/2}(\alpha_1\beta_2 + \beta_1\alpha_2) \quad \text{for } F = 1, \tag{8}$$

where $\alpha = \binom{1}{0}$ and $\beta = \binom{0}{1}$. The matrix element in (7) connects only the same spin states of the nucleon-lepton system in accordance with the conservation of the total spin F and its projection F_z. Using the well-known relations for the Pauli matrices and putting $\epsilon = \lambda - 1$ yields

$$W = W_0 = G(4 + 3\epsilon)^2 \quad \text{for } F = 0,$$
$$W = W_1 = G\epsilon^2 \quad \text{for } F = 1. \tag{9}$$

Substituting $\epsilon \approx 0.26$ [8], we find that the rate of the electron capture from the state with $F = 0$ is approximately 340 times larger than the rate for the state with $F = 1$.

For arbitrary temperature T we have the general expression

$$W = \frac{a_0 W_0 \exp(-E_0/kT) + a_1 W_1 \exp(-E_1/kT)}{a_0 \exp(-E_0/kT) + a_1 \exp(-E_1/kT)}, \tag{10}$$

where E_0 and E_1 are the energies, a_0 and a_1 are the numbers of states; for total spin $F = 0$ and $F = 1$, respectively. In the limit of vanishing hyperfine interaction (i.e., $\Delta = |E_1 - E_0| \to 0$) and using $a_0 = 1$ and $a_1 = 3$ we get

$$W = G(4 + 6\epsilon + 3\epsilon^2) = G(1 + 3\lambda^2), \tag{11}$$

in agreement with the expression obtained using (3) with $|\int 1|^2 = 1$ and $|\int \boldsymbol{\sigma}|^2 = 3$ for the transition considered [5]. The same result ($W \approx 5.8G$) is, of course, obtained in the high temperature region, $kT \gg \Delta$, meaning the correlation between the orientations of the electron and nuclear spin is small in this limit.

The situation changes drastically at low temperatures, $\Delta \gg kT$. In this region the capture rate depends critically on the sign of the hyperfine constant A. If $A > 0$ (i.e., $F = 0$ is the ground state of the system), then the rate of capture is $W \approx 23G$, approximately 4 times the value at high T. If $A < 0$ then $W \approx 0.068G$, i.e., capture of the unpaired electron practically stops.

(2) Gamow-Teller transition.—Now let us consider electron capture with a decrease in the nuclear spin by one unit. In this case, in contrast to the previous one, the transition is caused by the Gamow-Teller interaction only. We will show that the influence of the hyperfine interaction on the electron capture rate may be caused by angular momentum conservation alone. Suppose we have a nuclear spin I ($I \geq 1$) in the initial state and spin $I - 1$ in the final state [e.g., decay of ^{7}Be ($I = \frac{3}{2}$) to the excited state ^{7}Li* ($I = \frac{1}{2}$) or ^{119}Sb ($I = \frac{5}{2}$) to the excited state ^{119}Sn* ($I = \frac{3}{2}$) [9,10]].

In the initial state of the system we have a nucleus and an electron, and in the final state there are the transformed nucleus and a neutrino. The total angular momentum F of the initial state may take two values, $F = I + \frac{1}{2}$ and $F = I - \frac{1}{2}$. For the final state we have $F = I - \frac{1}{2}$ or $F = I - \frac{3}{2}$ (only one value, $F = \frac{1}{2}$, if $I = 1$). It is clear that electron capture from the state with $F = I + \frac{1}{2}$ is forbidden by angular momentum conservation. So only the transition from the state with $F = I - \frac{1}{2}$ is possible. Let the rate of this $\Delta F = 0$ transition be W_0. Then for arbitrary temperature T the capture rate is again described by (10) with $a_0 = 2I$, $a_1 = 2(I + 1)$, and $W_1 = 0$. For high temperatures, $kT \gg \Delta$, W is given by

$$W = W_0 I/(2I + 1). \tag{12}$$

At low temperatures, $\Delta \gg kT$, the rate W again depends on the sign of the hyperfine interaction constant A. If $A > 0$ then $W = W_0$, i.e., the rate of the process is increased $2 + 1/I$ times compared to (12). In the opposite case, the capture of the unpaired electron is again practically halted.

In summary, we have shown that the rate of electron capture from an unfilled s orbital of a free atom or ion may change drastically when the temperature is decreased so that kT is small in comparison to the hyperfine splitting Δ. We have considered two simple situations, a mirror transition for a nucleus in the $s_{1/2}$ state and a Gamow-Teller transition with a decrease in nuclear spin of 1. For both cases, with $\Delta \gg kT$, the rate of the capture W depends on the sign of the hyperfine constant A. For positive A the value of W is increased somewhat in comparison to the high temperature region. For negative A, electron capture practically ceases. In the latter situation it is possible to induce electron capture by applying an electromagnetic field at the resonant frequency $\Delta/\hbar$. The most dramatic effects are expected for hydrogenlike ions, because of the absence of competing decays from other bound electrons.

The authors believe that cold trapped ions can be used to investigate the effects described above. Trapping techniques have developed to the extent that it is now possible to trap all ions of the hydrogen isoelectronic sequence to U^{91+} [11]. Experiments at a storage ring have recently demonstrated the existence of bound state β decay, using completely stripped ions [12]. Charged particles can now be confined for long periods of time and a number of fundamental physical quantities have been measured, with great precision, using ion traps [13,14].

The experimental challenge in observing large hyperfine effects on electron capture is in the preparation of a sample of hydrogenlike ions whose temperature is considerably less than the hyperfine splitting of the ions. High Z hydrogenlike ions trapped in an electron beam trap [3,11] or storage ring [12] are very promising systems, because of their large hyperfine splittings and rapid radiative relaxation of the hyperfine levels. As an example, consider the decay of ^{119}Sb to the excited state ^{119}Sn*, with a decrease in nuclear spin of one unit. This decay is followed by

emission of a single, characteristic γ ray. The magnetic moment of ^{119}Sb is $+3.45\mu_N$ [9,10] and the lower hyperfine level of the ground state in $^{119}Sb^{50+}$ will be $F = 2$. For this case, $\Delta \gg kT$ at room temperature and the population in the upper hyperfine level is negligible. Application of cw resonant electromagnetic fields ($\omega = \Delta/\hbar$) would allow the populations in the hyperfine states to be equalized, with a 2.4 times reduction in the nuclear decay rate. Changes in the decay rate could be monitored using the characteristic γ ray.

Low Z, hydrogenlike ions have small hyperfine splittings and low radiative relaxation rates and techniques are needed to deplete the upper hyperfine level in an experiment. This probably can be achieved using, for example, exchange collision techniques [15]. A rather interesting system for this type of experiment is $^7Be^{3+}$. The nuclear magnetic moment of ^{7}Be has not been measured, but it is expected to be about $-1.3\mu_N$ [16]. Approximately 10% of ^{7}Be nuclei decay to ^{7}Li*, with a decrease in spin of one unit, followed by emission of a single, experimentally detectable, γ ray. If $F = 2$ is, in fact, the lower hyperfine level of the ground state, then with $\Delta \gg kT$ the decay to ^{7}Li* would be forbidden and an ion cloud would emit no γ rays. This provides the exciting possibility of controlling and switching a nuclear decay using hyperfine transitions. Driving the population into the $F = 1$ state with a resonant electromagnetic π pulse will induce decay to ^{7}Li* followed by emission of γ rays.

Smaller effects are expected in nonhydrogenlike ions with one electron in an outer s orbital, due to competition from inner electrons. Nevertheless, the effects should be detectable, particularly in $^7Be^+$. In this case, optical pumping of the $2s$-$3p$ transition or microwave pumping of the $2s$-$2p$ transition may be used to deplete the upper hyperfine level of the ground state [17]. Subsequent periodic sequences of resonant π pulses could be used to modulate the hyperfine populations. This would cause a modulation of the electron capture rate and consequently the γ-ray emission rate. Small amplitude modulations of a γ-ray emission could be detected with great sensitivity using techniques such as phase sensitive detection. Experimental and further theoretical work on these phenomena is currently underway at the Polytechnic.

The authors gratefully acknowledge very useful discussions with colleagues, especially with V. A. Sheverev.

[1] S. Chu and C. Wieman, J. Opt. Soc. Am. B **6**, 2020 (1989).
[2] R. Blatt, P. Gill, and R. C. Thompson, J. Mod. Opt. **39**, 193 (1992).
[3] R. E. Marrs, P. Beiersdorfer, and D. Schneider, Phys. Today **47**, No. 10, 27 (1994).
[4] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, MA, 1962).
[5] C. S. Wu and S. A. Moszkowski, *Beta Decay* (John Wiley & Sons, New York, 1966).
[6] W. Bambynek *et al.*, Rev. Mod. Phys. **49**, 77, 961 (1977).
[7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Springer-Verlag, New York, 1979).
[8] P. Bopp *et al.*, Phys. Rev. Lett. **56**, 919 (1986).
[9] *Table of Isotopes*, edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).
[10] N. E. Holden, in *CRC Handbook of Chemistry and Physics*, edited by D. R. Lide (CRC Press, Boca Raton, 1994–1995), 75th ed.
[11] R. E. Marrs, S. R. Elliott, and D. A. Knapp, Phys. Rev. Lett. **72**, 4082 (1994).
[12] M. Jung *et al.*, Phys. Rev. Lett. **69**, 2164 (1992).
[13] L. S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986).
[14] D. A. Church, Phys. Rep. **228**, 253 (1993).
[15] E. N. Fortson, F. G. Major, and H. G. Dehmelt, Phys. Rev. Lett. **16**, 221 (1966).
[16] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1937).
[17] D. J. Wineland, W. T. Itano, and R. S. Van Dyck, in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Bederson (Academic Press, New York, 1983), Vol. 19, p. 135.