Symmetry Breaking Induced by Top Quark Loops

J. P. Fatelo, J.-M. Gérard, T. Hambye, and J. Weyers
Institut de Physique Théorique, Universite Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium
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It is argued that top quark loops trigger symmetry breaking in the standard electroweak model. The Higgs boson is then expected to be lighter than 400 GeV. Further speculations on this dynamical mechanism even suggest a Higgs boson observable at the CERN e^+e^- collider LEP 200.

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Recent experimental evidence [1] for a top quark with a mass around 174 GeV implies that the strongest force in the electroweak sector of the standard model is due to the Yukawa coupling of the Higgs scalar to the top quark. Indeed, with $m_t = gv/\sqrt{2}$ and the vacuum expectation value $v \simeq 247$ GeV, the coupling constant g is close to 1.

In this Letter we argue that symmetry breaking (SB) of the gauge group $SU(2)_L \times U(1)$ is a dynamical effect driven by this Yukawa force. More precisely, in the framework of an effective electroweak theory, our suggestion is that SB does not have to be put in by hand at tree level but is induced by top quark loops. One immediate consequence is a range of allowed values for the Higgs boson mass $30 \lesssim m_H \lesssim 400$ GeV depending on the values of the physical cutoff Λ. Furthermore, if we try to implement the idea that top one-loop effects exhaust the physics of SB by making the effective scalar interactions as small as possible, we find $\Lambda \approx 1$ TeV and $m_H \approx 80$ GeV.

In a completely different context, namely that of a fundamental renormalizable electroweak theory, a similar SB mechanism can apply and appears to be an attractive alternative to the usual Higgs mechanism. Here again, since top quark loops trigger SB it is tempting to neglect scalar loops or, more technically, to assume that λ_R, the renormalized quartic scalar coupling, is quite small if not zero. Should one take $\lambda_R = 0$ at the scale v one would find the same rather small value for the Higgs boson mass, namely around 80 GeV.

I. Effective electroweak theory.—Let us first consider the Lagrangian

$$\mathcal{L} = \overline{\Psi} i \partial \Psi + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\mu^2}{2}\phi^2 + \frac{g}{\sqrt{2}}\overline{\Psi}\Psi\phi, \quad (1)$$

where Ψ is the top quark field and ϕ the neutral component of the standard Higgs doublet.

Equation (1) is nothing but the Lagrangian of a massless fermion interacting with a massive scalar ($\mu^2 > 0$). It is the relevant part of the standard model Lagrangian for symmetry breaking, as we shall see. We explicitly assume no quartic scalar self-interactions. This is perfectly acceptable in the context of an effective theory, i.e., a theory cutoff at some physical scale Λ.

The one-loop quantum corrections to the tree-level potential $V^{(0)}(\phi) = \frac{1}{2}\mu^2\phi^2$ are obtained from the infinite series of Feynman diagrams given in Fig. 1. The resulting effective potential reads [2,3]

$$\begin{aligned} V^{(1)} &= \frac{\mu^2\phi^2}{2} - \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} q^2\,dq^2 \ln\left(1 + \frac{g^2\phi^2}{2q^2}\right) \\ &= \frac{\mu^2\phi^2}{2} - \frac{N_c}{16\pi^2}\left[\Lambda^4 \ln\left(1 + \frac{g^2\phi^2}{2\Lambda^2}\right) + \frac{g^2\phi^2\Lambda^2}{2} - \frac{g^4\phi^4}{4}\ln\left(1 + \frac{2\Lambda^2}{g^2\phi^2}\right)\right], \end{aligned} \quad (2)$$

with N_c the number of colors.

From Eq. (2) it is clear that the classical minimum $\langle\phi\rangle = 0$ of the tree-level potential can be turned into a maximum by the one-loop corrections. A new minimum then appears at $\langle\phi\rangle = v \neq 0$, while the potential of Eq. (2) remains bounded from below. Indeed, with $m_t = gv/\sqrt{2}$, the extremum condition on $V^{(1)}$ admits the unique nontrivial solution

$$\frac{m_t^2}{\Lambda^2}\ln\left(1 + \frac{\Lambda^2}{m_t^2}\right) = 1 - \frac{8\pi^2\mu^2}{N_c g^2 \Lambda^2}, \quad (3)$$

if $0 < \mu^2 < N_c g^2 \Lambda^2/8\pi^2$. In words, for quantum SB to occur, the scalar mass term at tree level *must* be genuine. The change of sign of the second derivative of the potential at the origin is entirely due to the one-loop corrections.

At the one-loop level, the Higgs boson mass can be identified as

$$\left.\frac{\partial^2 V^{(1)}}{\partial\phi^2}\right|_{\phi=v} = m_H^2 = \frac{N_c g^2}{4\pi^2}\left\{\ln\left(1 + \frac{\Lambda^2}{m_t^2}\right) - \frac{\Lambda^2}{\Lambda^2 + m_t^2}\right\} m_t^2, \quad (4)$$

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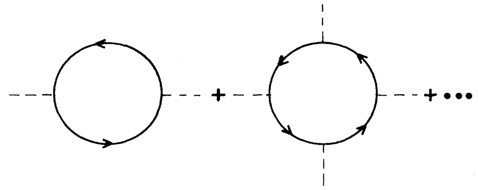

FIG. 1. Massless top quark contributions to the one-loop effective potential. Directed lines represent the top, dashed lines the scalar.

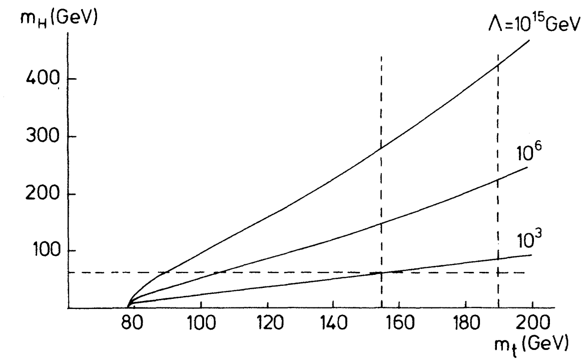

FIG. 2. Higgs boson mass as a function of top quark mass for different values of the cutoff Λ. The horizontal line corresponds to the present experimental lower bound $m_H \gtrsim 60$ GeV and the vertical lines to a reasonable range of values for m_t.

while the fourth derivative of $V^{(1)}$ at the vacuum expectation value reads

$$\left.\frac{\partial^4 V^{(1)}}{\partial \phi^4}\right|_{\phi=v} = \frac{3N_c g^4}{8\pi^2}\left\{\ln\left(1+\frac{\Lambda^2}{m_t^2}\right)+\frac{9m_t^2}{\Lambda^2+m_t^2} - \frac{8m_t^4}{(\Lambda^2+m_t^2)^2}+\frac{8}{3}\frac{m_t^6}{(\Lambda^2+m_t^2)^3}-\frac{11}{3}\right\}. \tag{5}$$

It is not difficult to include one-loop gauge boson contributions to Eq. (2). One then obtains

$$V^{(1)} = \frac{1}{2}\mu^2\phi^2 - \frac{1}{32\pi^2}\int_0^{\Lambda^2} dq^2 \times q^2\left\{4N_c \ln\left[1+\frac{g^2\phi^2}{2q^2}\right] - 6\ln\left[1+\frac{g_2^2\phi^2}{4q^2}\right] - 3\ln\left[1+\frac{(g_1^2+g_2^2)\phi^2}{4q^2}\right]\right\}, \tag{6}$$

with g_1 and g_2 the U(1) and $SU(2)_L$ gauge couplings, respectively. In our normalization the W and Z masses are given by $M_W^2 = g_2^2 v^2/4$ and $M_Z^2 = (g_1^2 + g_2^2)v^2/4$. It is then straightforward to compute the corresponding modifications to Eqs. (3) and (4).

Notice that the gauge boson contributions alone would lead to $\partial^2 V^{(1)}/\partial\phi^2|_{\phi=v} < 0$. A real Higgs boson mass requires (see Fig. 2)

$$m_t \gtrsim \left(\frac{6M_W^4 + 3M_Z^4}{4N_c}\right)^{1/4} \approx 78\ \text{GeV}. \tag{7}$$

A heavy top quark is therefore an *essential* ingredient for the quantum SB mechanism advocated in this Letter. Furthermore, with m_t around 174 GeV, the contributions of the gauge bosons to the effective potential are numerically quite small, at most of the order of a few percent. They are included in Fig. 2 where we plot m_H as a function of m_t for different values of the cutoff Λ. If we take Λ as small as the vacuum expectation value v, we find the lower limit $m_H \cong 30$ GeV. However, the experimental bound [4] for the Higgs boson mass $m_H \gtrsim 60$ GeV together with the range allowed data from the CERN e^+e^- collider LEP and from the Collider Detector at Fermilab [5] for the top mass require Λ to be greater than about 500 GeV. On the other hand, taking Λ smaller than the grand-unified-theory (GUT) scale implies the upper bound

$$m_H \lesssim 400\ \text{GeV}. \tag{8}$$

In fact for values of Λ of the order of or larger than the GUT scale, the one-loop approximation breaks down since one expects the ratio of the two-loop to the one-loop top quark contributions to be of the order of $(N_c g^2/16\pi^2)\ln(\Lambda^2/m_t^2)$.

Below the GUT scale and for quite an extended range of the cutoff the one-loop approximation should give a reasonable description of the physics involved, provided higher order scalar contributions remain small. The one-loop scalar contribution vanishes by assumption since $V^{(0)}$ does not contain a self-interacting ϕ^4 term or, equivalently, since $\partial^4 V^{(0)}/\partial\phi^4|_{\phi=0} = 0$. Should we impose that $\partial^4 V^{(1)}/\partial^4\phi|_{\phi=v}$ remains zero—scalar self-interactions vanish at the true vacuum—the cutoff would turn out to be around 1 TeV [see Eq. (5)] and the one-loop approximation looks trustworthy. Combining Eq. (4) with Eq. (5), this further constraint on $V^{(1)}$ implies then

$$m_H = \frac{2}{\pi}\frac{m_t^2}{v} + O\left(\frac{m_t^2}{\Lambda^2}\right), \tag{9}$$

i.e., a Higgs boson mass of about 80 GeV.

To put it differently our proposal for SB in the standard model is that all the physics of the phenomenon are well described by single top quark loops. Pushing the idea to the limit, namely *making sure that all other contributions remain negligible,* leads to a rather low cutoff Λ ($\approx$ 1 TeV) and a light elementary Higgs scalar.

At this point it is enlightening to compare our approach with a top condensation model [6]. Clearly the form of our classical Lagrangian (1) implies that Eq. (3) is equivalent to the gap equation. Since the tree-level kinetic term for the scalars generates the W and Z boson masses, top condensation is not necessary in our model and the usual fine-tuning problem can be avoided with a small enough cutoff Λ.

On the other hand, it is also straightforward to check that with Λ around the TeV scale, a sizable vacuum expectation value $\langle t\bar{t}\rangle$ cannot be generated by the Yukawa interaction. Of course if $\langle t\bar{t}\rangle$ were large, which occurs only for high values of Λ, we should anyway go beyond the one-loop approximation and take into account the mixing between the fundamental and composite scalar modes [7].

II. Renormalizable electroweak theory.—It is worthwhile to repeat the calculations of the previous section in the framework of a renormalizable theory. The effective potential at the one-loop level reads [2,3]

$$V_R = \frac{1}{2}\mu_R^2\phi^2 + \frac{1}{4!}\lambda_R\phi^4 + \frac{1}{(4\pi)^2}\phi^4\left(\frac{\lambda_R^2}{12} - \frac{N_c}{4}g_R^4\right) \times \left(\ln\frac{\phi^2}{M^2} - \frac{25}{6}\right). \quad (10)$$

In Eq. (10) μ_R^2 and λ_R are finite renormalized parameters defined by $\mu_R^2 = \partial^2 V_R/\partial\phi^2|_{\phi=0}$ and $\lambda_R = \partial^4 V_R/\partial\phi^4|_{\phi=M}$ with M an arbitrary scale.

We have included the would-be Goldstone boson contributions in Eq. (10); without them the term $\lambda_R^2/12$ would have been $\lambda_R^2/16$. Gauge boson contributions have been neglected because, once again, their effect is at most of a few percent.

To simplify the discussion let us choose the renormalization scale $M = v$. In that case a nontrivial minimum of V_R can occur only if $\mu_R^2 < 0$. Defining as usual $m_H^2 = \partial^2 V_R/\partial\phi^2|_{\phi=v}$, we obtain

$$m_H^2 = \left(\frac{N_c g_R^4}{3\pi^2} + \frac{\lambda_R}{3} - \frac{\lambda_R^2}{9\pi^2}\right)v^2. \quad (11)$$

For small λ_R (at the scale v), the Higgs boson mass is dominated by the contribution of the Yukawa coupling and, again, one finds

$$m_H = \frac{2}{\pi}\frac{m_t^2}{v} + O(\lambda_R) \approx 80\text{ GeV}. \quad (12)$$

In view of the present uncertainties on m_t, it is not worthwhile to include QCD corrections. Also we are well aware of triviality and stability issues [8] concerning the effective potential when $\lambda_R \cong 0$. We simply recall that they are again related to the (non)appearance of new physics at some scale Λ, but we will not discuss these points here. Our main point, much like in the previous section, is that top quark loop effects can be responsible for a significant fraction if not all of the Higgs boson mass.

To summarize, our suggestion in this Letter is that the large Yukawa coupling responsible for a heavy top quark plays a most important dynamical role in the standard model: We have shown that single top quark loops alone are enough to trigger symmetry breaking at the quantum level. In the effective Lagrangian approach of Section I, this scenario $m_H \lesssim 400$ GeV. Pushing our suggestion to the extreme, namely assuming that all the relevant physics are given by the top one-loop effective potential, seems to us an attractive and economical alternative to the usual Higgs mechanism. It then requires scalar self-interactions to be as small as possible. In the effective as well as in the renormalizable case, this assumption leads to $m_H = (2/\pi)m_t^2/v \approx 80$ GeV, which is in a range accessible [9] at LEP 200.

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