Photonic de Broglie Waves

Joseph Jacobson,* Gunnar Björk, Isaac Chuang, and Yoshihisa Yamamoto†

ERATO Quantum Fluctuation Project, Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305

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In quantum optics an ensemble of photons is treated as a Bose condensate which has a de Broglie wavelength given by λ_0/N where λ_0 and N are the wavelength and average number of constituent photons, respectively. We describe an interferometer which is capable of measuring this de Broglie wavelength. We show specifically that a coherent state input has a de Broglie coherence length proportional to $1/\sqrt{N}$. Finally we show that the interferometer is Heisenberg limited in sensitivity ($\Delta\phi_{\min} = 1/N$) and thus forms a new class of interferometers operating at the fundamental quantum limit.

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The advent of quantum mechanics requires us to take an operational view of nature in which we describe properties of a system in terms of measurable observables. In this paper we show that the measured wavelength of an object is, fundamentally, dependent on the internal structure of the object being measured. In general, the wavelength is measured by splitting an incident object into a coherent superposition of states and interfering those states with each other as a function of phase delay. In order to treat the case of incident objects with internal structure we introduce the concept of "effective" beam splitters which imbue an effective internal structure such as binding between constituent parts of the object. In a simple case we show that such an effective internal structure may have dramatic effects, for instance, that incident states of definite energy do not necessarily have a well-defined de Broglie wavelength. We then extend this concept of effective beam splitters to construct an interferometer which is capable of measuring the de Broglie wavelength of an incident state of photons (light packet) as a whole.

Consider an interferometer for molecules as has been recently demonstrated [1]. The output of the interferometer oscillates as a function of the path length difference. The oscillation period is a measure of the molecule's de Broglie wavelength. In this paper we were motivated by considering what happens to the measured wavelength as a function of the binding energy of the molecule as follows: Assume an iodine molecule I_2 with mass $\approx 2m_I$ (ignoring the mass contribution of binding). The de Broglie wavelength of the molecule, which we expect to measure, is given by $h/2m_I v$ where v is the molecular velocity. However, in the limit that the binding energy between the two I atoms goes to zero the molecule's constituent atoms become disassociated and we effectively have two separate I atoms each with de Broglie wavelength $h/m_I v$ incident on the interferometer. In this case we expect the measured wavelength to indicate the de Broglie wavelength of the individual atoms. At what point does the molecular object make a transformation from a wavelength of $h/2m_I v$ to a wavelength of $h/m_I v$?

We may construct a simplified model of the molecular interferometer by considering a modified Mach-Zehnder interferometer with effective beam splitters [denoted by E in Fig. 1(inset)] which imbue an effective binding to the constituent particles of the incident state. The input state is written as $|\psi_0\rangle = |2,0\rangle$. The first label of the ket denotes the number of constituent particles incident on the horizontal port and the second label indicates the number of constituent particles incident on the vertical port. In our model the interaction Hamiltonian for the effective beam splitter is given by

$$\hat{H}_{\rm bs} = i\hbar\frac{\pi}{4}\chi[\hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{a}] + i\hbar\frac{\pi}{8}(1-\chi)[(\hat{a}^\dagger\hat{b})^2 - (\hat{b}^\dagger\hat{a})^2], \quad (1)$$

where $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$) are the lowering (raising) boson operators corresponding to the different paths of the interferometer. The unitary time propagation operator for the beam splitter is given by $\hat{U}_{\rm bs} = e^{-i\hat{H}_{\rm bs}t/\hbar}$. A 50/50 beam splitting is achieved for a propagation from $t = 0$ to $t = 1$.

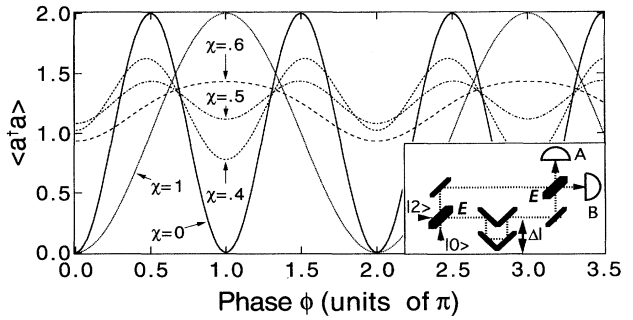

FIG. 1. (Inset) A modified Mach-Zehnder interferometer with effective beam splitters. (Plot) The expectation number $\langle\hat{a}^\dagger\hat{a}\rangle$ as a function of the phase ϕ for various values of the parameter χ. ($\chi = 1$, no binding) corresponds to a beam splitter which splits incident composite objects into constituent parts. ($\chi = 0$, tight binding) corresponds to a beam splitter which does not split incident composite objects into constituent parts. The incident state is $|2,0\rangle$.

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χ is a parameter on [0, 1] which changes the beam splitter continuously from one which imbues an effective binding (i.e., does not split composite objects into constituent parts; $\chi = 0$) to one which does not imbue an effective binding (i.e., normal beam splitter which does split composite objects into constituent parts; $\chi = 1$).

The phase shift operator is written simply as the free space propagator for one arm of the interferometer and is given by

$$\hat{U}_\phi = e^{-i\omega_0 \hat{a}^\dagger \hat{a} t} = e^{-i\phi \hat{a}^\dagger \hat{a}}, \tag{2}$$

where ω_0 is the frequency of the constituent particles ($= 2\pi c/\lambda_0$) and Δl is the path length difference. $\phi = \omega_0 \Delta l/c$.

Figure 1 shows the output (expectation value of the particle number $\langle \hat{a}^\dagger \hat{a} \rangle$) as a function of the phase ϕ for several different values of the functional splitting parameter χ. It may be discerned from the figure that as the beam splitter is transformed from a normal beam splitter which splits incident composite objects into constituent parts ($\chi = 1$, no binding) to an effective beam splitter which does not split incident composite objects into constituent parts ($\chi = 0$, tight binding) the measured wavelength is halved (e.g., the measured frequency is doubled). We note that at intermediate "binding energies," despite the fact that the incident object has a well-defined energy, the wavelength is nonetheless ill defined.

Recently, Davidovich *et al.* [2] described an ingenious quantum switch for creating a superposition between an arbitrary incident state of light (e.g., coherent state light packet) and the vacuum by means of a microwave cavity whose resonance is dependent upon the state of a single atom in the cavity. In this paper we use two such switches to construct an interferometer for states as a whole.

Consider the schematic setup of Fig. 2. Here an atom incident on two cavities controls the optical properties of these cavities depending on whether or not the atom is in the ground or upper state due to a state dependent index of refraction. In our setup we consider a situation where an atom in the upper state causes the incident photon field to be reflected, whereas an atom in the ground state allows the incident field to be transmitted. Consider an upper-state atom incident on the first cavity. The atom is subjected to a $\pi/2$ pulse which transforms an excited state atomic wave function according to $|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and a ground state atomic wave function according to

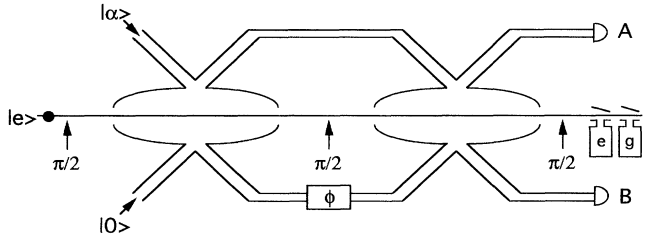

FIG. 2. Schematic of a Mach-Zehnder interferometer for measuring the de Broglie wavelength of an optical pulse.

$|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$. Simultaneously, a coherent state $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n (\alpha^n/n!^{1/2})|n\rangle$ and the vacuum state $|0\rangle$ are incident on the two optical input ports of the cavity.

We may write our incident wave function as $|\psi_0\rangle = |\alpha, 0, e\rangle$ in the basis $|a, b, x\rangle$, where a and b are the two photon modes and x denotes the state of the atom (e excited, g ground). Therefore after both the initial $\pi/2$ pulse and the first cavity we have the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0, \alpha, e\rangle + |\alpha, 0, g\rangle)$, where the labels in the first ket on the right side have been exchanged due to the action of the excited state atom in the cavity in which the presence of an excited state atom in the cavity causes the incident photon state to be exchanged from mode a to mode b and vice versa (i.e., reflected), whereas the presence of a ground state atom causes no mode exchange (i.e., transmitted).

As before the phase shift operator is given by Eq. (2). Thus after the initial $\pi/2$ pulse, the first cavity and a phase shift ϕ along mode a we have that the state if given by $|\psi\rangle = \frac{1}{\sqrt{2}}(|0, \alpha, e\rangle + |\alpha e^{-i\phi}, 0, g\rangle)$, where we have used $e^{-i\phi \hat{a}^\dagger \hat{a}}|\alpha\rangle = |\alpha e^{-i\phi}\rangle$.

Using the rules above we have that after traversing the rest of the apparatus (Fig. 2), consisting of another $\pi/2$ pulse followed by the second cavity and finally followed by a finishing $\pi/2$ pulse, the final state is given by

$$\begin{aligned} |\psi_f\rangle = \frac{1}{2\sqrt{2}} (&|\alpha, 0, e\rangle + |\alpha, 0, g\rangle + |0, \alpha, g\rangle - |0, \alpha, e\rangle \\ &+ |\alpha e^{-i\phi}, 0, g\rangle - |\alpha e^{-i\phi}, 0, e\rangle \\ &- |0, \alpha e^{-i\phi}, e\rangle - |0, \alpha e^{-i\phi}, g\rangle). \end{aligned} \tag{3}$$

Consider the case where we measure the final state of the atom to be in the upper state $|e\rangle$. We have the projected photon field to be

$$|\psi_f\rangle_e = \frac{1}{2}(|\alpha, 0\rangle - |0, \alpha\rangle - |\alpha e^{-i\phi}, 0\rangle - |0, \alpha e^{-i\phi}\rangle), \tag{4}$$

and the expected output at port A for our interferometer is

$${}_e\langle\psi_f|\hat{a}^\dagger \hat{a}|\psi_f\rangle_e = \frac{N}{2}[1 - e^{N(\cos\phi - 1)} \cos(\phi + N \sin\phi)], \tag{5}$$

where here the average photon number $N = |\alpha|^2$. As an aside we note that if we were simply to trace over the final atomic state, instead of conditioning the photon output at port A on a specific atom state as we have done, the output of the interferometer would be ${}_e\langle\psi_f|\hat{a}^\dagger \hat{a}|\psi_f\rangle_e + {}_g\langle\psi_f|\hat{a}^\dagger \hat{a}|\psi_f\rangle_g = N$ yielding no interference.

The upper-state conditioned interferometer output of Eq. (5) is plotted as a function of ϕ in Figs. 3(a, solid line) and 3(b). In these plots the average photon number is $N = 100$. From Fig. 3(a), solid line, we see that the output has an oscillation period of $2\pi/N$ indicating a measurement of the de Broglie wavelength (λ_0/N) of the light packet as a whole. We may understand the decay of the interference for long phase delays [large ϕ—see Fig. 3(a), solid line] by recalling that the de Broglie wavelength is given by the mean number of photons, N, and therefore the "coherence

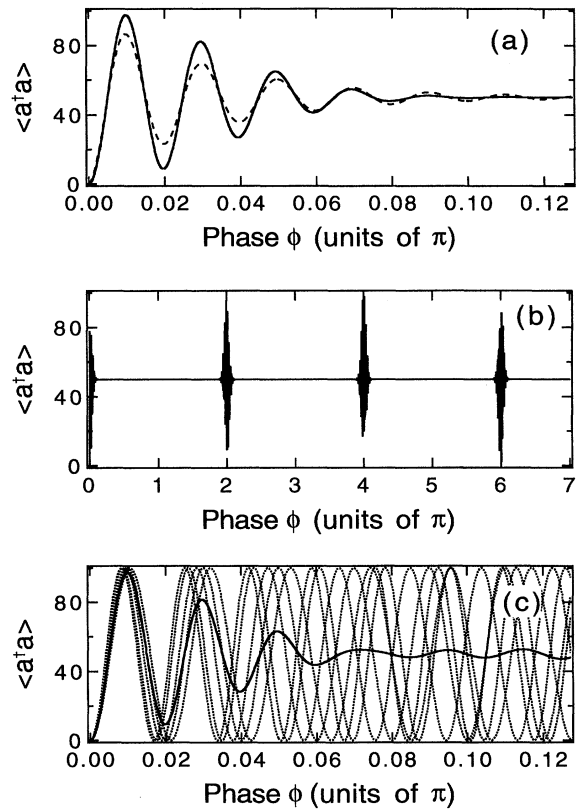

FIG. 3. (a, solid line), (b) The expectation number $\langle \hat{a}^\dagger \hat{a} \rangle$ output of the de Broglie wavelength Mach-Zehnder interferometer for an incident coherent state with average photon number $N = 100$. (a, dashed line) An approximate model of the output indicating the coherence length of the coherent state; see text. (c, dashed) The expectation number output of the de Broglie wavelength Mach-Zehnder interferometer for several different incident Fock states. (c, solid) The ensemble average output for 500 incident Fock states with photon numbers distributed according to a Poisson distribution centered at $N = 100$.

length" of the light packet, Δl_c, is a function of the fluctuations in photon number, ΔN, which for a coherent state is given by $\sqrt{N}$. The dashed line in Fig. 3(a) is a plot of the function $\frac{N}{2}(1 - e^{-\sqrt{N}\phi}\cos N\phi)$ and indicates that the coherence length of the coherent state light packet is given approximately by $\Delta l_c \approx c/\omega_0\sqrt{N}$.

Figure 3(b) shows that all of the wavelength components of a coherent state light packet are rephased at periods of 2π. This is to be contrasted with the revivals in the Jaynes-Cummings system, where the revival time there is dependent on the square root of the photon number, $\sqrt{N}$ [3]. We further note that, whereas there are also oscillations as a function of time delay in the Jaynes-Cummings experiment, that experiment does not represent a measure of the de Broglie wavelength, since the oscillation period is proportional to the square root of the photon number $\sqrt{N}$ whereas in our case it is proportional to the photon number N directly. We do note that for less than or equal to two photons three-level systems, such as the two photon micromaser [4], may be treated as effective two-level systems with a Rabi frequency proportional to N [5]. The effective Hamiltonian which governs these systems is similar to the second part of our Eq. (1).

It is also interesting to consider the (normal beam splitter) Mach-Zehnder interferometer with Fock states ($|n, n\rangle$) incident on the two input ports [6,7]. An exact solution of that interferometer generalized for arbitrary n [8] finds that the probability that the difference in photocount between the two output ports is zero (i.e., the perfect coincidence rate) is given by $P_n^0(\phi) = [L_n(\cos\phi)]^2$, where L_n are the Legendre polynomials defined by $L_n(x) = (1/2^n n!)(d^n/dx^n)(x^2 - 1)^n$. While the Legendre functions contain many frequency components and thus cannot be considered a pure measure of the de Broglie wavelength of the incident Fock states (recall that the Fock state has only a single de Broglie frequency component at $n\omega_0$), such dual Fock input interferometers do feature components of their coincidence rate at the de Broglie frequency. For the special case of single photon inputs $n = 1$ the Legendre function becomes a pure cosine at the de Broglie frequency.

Referring back to Fig. 3(a) it is interesting to note that the decayed interference at large ϕ may be revived from data which have already been recorded. This may be accomplished by correlating the list of measured values of photon number at exit port A with the total photon number measured after the interferometer as the sum of counts at A and B. We recall that the Fock state has an infinitely long de Broglie coherence length, and thus if we condition the recorded interferometer data on the total photon count we may again recover interference even though the conglomerate set of data shows a decay of the interference. Figure 3(c), dashed lines, shows the output of the interferometer for several incident Fock states each of different photon number. Figure 3(c), solid line, is the average interferometer output for a statistical mixture of incident Fock states whose numbers are distributed according to a Poisson distribution of average photon number $N = 100$. We note the similarity with Fig. 3(a).

In order to calculate sensitivity we must calculate fluctuations in the output signal. We have that the expectation value for the number operator squared is

$$_e\langle\psi_f|\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}|\psi_f\rangle_e = \frac{N}{2}\{(N+1) + e^{-2N\sin^2(\phi/2)} \times [N\sin\phi\sin(\phi + N\sin\phi) - (1 + N\cos\phi)\cos(\phi + N\sin\phi)]\}. \quad (6)$$

Defining the interferometer signal as $\langle \hat{a}^\dagger \hat{a} \rangle_{\phi+\delta\phi} - \langle \hat{a}^\dagger \hat{a} \rangle_\phi$ and the interferometer noise as $\sqrt{\langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle_\phi - \langle \hat{a}^\dagger \hat{a} \rangle_\phi^2}$ we may solve for the minimum detectable phase shift $\Delta\phi_{\min}$ by requiring that signal/noise ≥ 1.

Figure 4(a) plots $\Delta\phi_{\min}$ as a function of the phase ϕ for an incident coherent state with average photon number $N = 100$. For a phase setting of ϕ close to zero the interferometer is Heisenberg limited in sensitivity, such that $\Delta\phi_{\min} = 1/N$ for $N > 1$ and $\Delta\phi_{\min} = 1/\sqrt{N}$ for $N < 1$. Figure 4(b) shows that the ideal performance is maintained from the region of much less than one average photon to much more than one average photon. This establishes that the device forms a new class of interferometers operating at the fundamental quantum limit of sensitivity [9].

The above result may be understood when recalling that a normal interferometer when fed a single photon in one input port and the vacuum in the other port is both Heisenberg limited and standard quantum limited as $1/N = 1/\sqrt{N}$ for $N = 1$. Therefore the incident coherent state photon packet is in essence a single de Broglie particle when incident on the special beam splitters of the de Broglie wave interferometer. It is interesting to note that in the case of the dual Fock state interferometer [7], as we have mentioned, the coincidence rate there is given by the Legendre polynomials. While, as noted, these functions contain many frequency components, if one chooses the operating point near $\phi = 0$ the dominant component becomes the de Broglie frequency and thus the Heisenberg limited operation of that interferometer can also be understood in terms of the de Broglie wave interferometer.

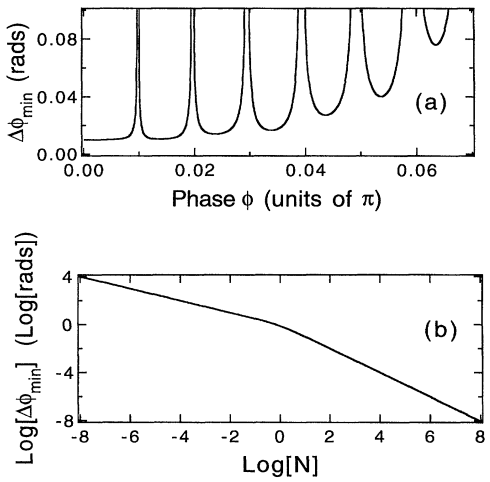

FIG. 4. The minimum phase $\Delta\phi_{\min}$ measurable by the de Broglie wavelength Mach-Zehnder interferometer for incident coherent state pulses: (a) As a function of the phase ϕ for an average photon number of $N = 100$. (b) For the optimal phase setting as a function of average photon number N.

In conclusion, we have shown that wavelength is an internal structure dependent quantity. Specifically, the outcome of a wavelength measurement is dependent on the interaction which takes place at the beam splitters present inside the wavelength measurement apparatus. The interaction at the beam splitters may be a property of the interaction between constituent particles in an incident composite object as in an interferometer for molecules or in other bound boson or bound photon systems such as in optical solitons or diphoton systems [10] or the interaction may be a function of the beam splitters themselves as in the case of the effective beam splitters introduced in this paper.

We show that such effective beam splitters may be used to construct a de Broglie wave interferometer for optical states as a whole. In the specific case of an coherent state optical pulse we show that there is a finite coherence length proportional to $1/\sqrt{N}$ due to the uncertainty in photon number. Furthermore, there is a revival of the interference at phase periods of 2π due to the discrete nature of the constituent number states. A calculation of noise properties establishes that the interferometer is Heisenberg limited in sensitivity and thus forms a new class of interferometers operating at the fundamental quantum limit.

*Electronic address: jacobson@sierra.stanford.edu
†Also affiliated with NTT Basic Research Laboratories, Atsugishi, Kanagawa, Japan.

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